

Chapter
2

DIFFERENTIATION

EXERCISE 2.1

Q.1 Find by definition, the derivatives w.r.t 'x' of the following functions defined as:

- (i) $2x^2 + 1$ (ii) $2 - \sqrt{x}$ (iii) $\frac{1}{\sqrt{x}}$ (iv) $\frac{1}{x^3}$
(v) $\frac{1}{x-a}$ (vi) $x(x-3)$ (vii) $\frac{2}{x^4}$ (viii) $x^2 + \frac{1}{x^2}$
(ix) $(x+4)^3$ (x) $x^{\frac{3}{2}}$ (xi) $x^{\frac{5}{2}}$ (xii) x^m
(xiii) $\frac{1}{x^m}$, $m \in \mathbb{N}$ (xiv) x^{40} (xv) x^{-100}

Solution:

(i) $2x^2 + 1$ (Lahore Board 2011)

Let $y = 2x^2 + 1$

$$y + \delta y = 2(x + \delta x)^2 + 1$$

$$\delta y = 2(x + \delta x)^2 + 1 - y$$

$$\delta y = 2(x^2 + \delta x^2 + 2x \delta x) + 1 - (2x^2 + 1) \quad \therefore y = 2x^2 + 1$$

$$\delta y = 2x^2 + 2\delta x^2 + 4x\delta x + 1 - 2x^2 - 1$$

$$\delta y = 2\delta x^2 + 4x\delta x$$

$$\delta y = 2\delta x(\delta x + 2x)$$

Dividing both sides by δx .

$$\frac{\delta y}{\delta x} = \frac{2\delta x(\delta x + 2x)}{\delta x}$$

$$\frac{\delta y}{\delta x} = 2(\delta x + 2x)$$

Taking limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} [2(\delta x + 2x)]$$

$$\frac{dy}{dx} = 2(0 + 2x)$$

$$\frac{dy}{dx} = 4x$$

$\frac{d}{dx} (2x^2 + 1) = 4x$

Ans.

(ii) $2 - \sqrt{x}$

Let

$$\begin{aligned}
 y &= 2 - \sqrt{x} \\
 y + \delta y &= 2 - \sqrt{x + \delta x} \\
 \delta y &= 2 - \sqrt{x + \delta x} - y \\
 \delta y &= 2 - \sqrt{x + \delta x} - (2 - \sqrt{x}) \quad \because y = 2 - \sqrt{x} \\
 \delta y &= 2 - \sqrt{x + \delta x} - 2 + \sqrt{x} \\
 \delta y &= \sqrt{x} - \sqrt{x + \delta x} \\
 \delta y &= x^{\frac{1}{2}} - \left[x \left(1 + \frac{\delta x}{x} \right) \right]^{\frac{1}{2}} \\
 \delta y &= x^{\frac{1}{2}} - x^{\frac{1}{2}} \left(1 + \frac{\delta x}{x} \right)^{\frac{1}{2}} \\
 \delta y &= x^{\frac{1}{2}} \left[1 - \left(1 + \frac{\delta x}{x} \right)^{\frac{1}{2}} \right] \\
 \delta y &= x^{\frac{1}{2}} \left[1 - \left\{ 1 + \frac{1}{2} \left(\frac{\delta x}{x} \right) + \frac{\frac{1}{2} (\frac{1}{2} - 1)}{2!} \left(\frac{\delta x}{x} \right)^2 + \dots \right\} \right] \\
 \delta y &= x^{\frac{1}{2}} \left[1 - 1 - \frac{1}{2} \left(\frac{\delta x}{x} \right) - \frac{\frac{1}{2} (\frac{1}{2} - 1)}{2!} \cdot \left(\frac{\delta x}{x} \right)^2 - \dots \right] \\
 \delta y &= x^{\frac{1}{2}} \left[-\frac{1}{2} \left(\frac{\delta x}{x} \right) - \frac{\frac{1}{2} (\frac{1}{2} - 1)}{2!} \cdot \left(\frac{\delta x}{x} \right)^2 - \dots \right]
 \end{aligned}$$

$$\delta y = x^{\frac{1}{2}} \frac{\delta x}{x} \left[\frac{-1}{2} - \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} \cdot \frac{\delta x}{x} - \dots \right]$$

Dividing both sides by δx .

$$\frac{\delta y}{\delta x} = \frac{\delta x}{x^{1-\frac{1}{2}}} \left[\frac{-1}{2} - \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} \cdot \frac{\delta x}{x} - \dots \right]$$

$$\frac{\delta y}{\delta x} = \frac{1}{x^{\frac{1}{2}}} \left[\frac{-1}{2} - \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} \cdot \frac{\delta x}{x} - \dots \right]$$

Taking limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} \frac{1}{x^{1/2}} \left[\frac{-1}{2} - \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} \cdot \frac{\delta x}{x} - \dots \right]$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{x}}$$

$$\boxed{\frac{dy}{dx} (2 - \sqrt{x}) = \frac{-1}{2\sqrt{x}}}$$

Ans.

(iii) $\frac{1}{\sqrt{x}}$ (L.B 2007)

Let

$$y = \frac{1}{\sqrt{x}}$$

$$y = x^{-1/2}$$

$$y + \delta y = (x + \delta x)^{-1/2}$$

$$\delta y = (x + \delta x)^{-1/2} - y$$

$$\delta y = (x + \delta x)^{-1/2} - x^{-1/2} \quad \because y = x^{-1/2}$$

$$\delta y = \left[x \left(1 + \frac{\delta x}{x} \right) \right]^{-1/2} - x^{-1/2}$$

$$\delta y = x^{-1/2} \left(1 + \frac{\delta x}{x} \right)^{-1/2} - x^{-1/2}$$

$$\delta y = x^{-1/2} \left[\left(1 + \frac{\delta x}{x} \right)^{-1/2} - 1 \right]$$

$$\delta y = x^{-1/2} \left[1 + \left(\frac{-1}{2} \right) \left(\frac{\delta x}{x} \right) + \frac{-1/2 (-1/2 - 1)}{2!} \left(\frac{\delta x}{x} \right)^2 + \dots - 1 \right]$$

$$\delta y = x^{-1/2} \frac{\delta x}{x} \left[\frac{-1}{2} + \frac{-1/2 (-1/2 - 1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

Dividing both sides by δx .

$$\frac{\delta y}{\delta x} = \frac{\delta x}{x^{1/2}} \left[\frac{-1}{2} + \frac{-1/2 (-1/2 - 1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

$$\frac{\delta y}{\delta x} = \frac{1}{x^{3/2}} \left[\frac{-1}{2} + \frac{-1/2 (-1/2 - 1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

Taking limit $\delta x \rightarrow 0$.

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} \frac{1}{x^{3/2}} \left[\frac{-1}{2} + \frac{-1/2 (-1/2 - 1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

$$\frac{dy}{dx} = \frac{-1}{2 x^{3/2}}$$

$$\boxed{\frac{d}{dx} \left(\frac{1}{\sqrt{x}} \right) = \frac{-1}{2 x^{3/2}}}$$

Ans.

(iv) $\frac{1}{x^3}$

Let

$$y = \frac{1}{x^3} \text{TALEEMCITY.COM}$$

$$y = x^{-3}$$

$$y + \delta y = (x + \delta x)^{-3}$$

$$\delta y = (x + \delta x)^{-3} - y$$

$$\delta y = (x + \delta x)^{-3} - x^{-3} \quad \because y = x^{-3}$$

$$\delta y = \left[x \left(1 + \frac{\delta x}{x} \right) \right]^{-3} - x^{-3}$$

$$\delta y = x^{-3} \left(1 + \frac{\delta x}{x} \right)^{-3} - x^{-3}$$

$$\delta y = x^{-3} \left[\left(1 + \frac{\delta x}{x} \right)^{-3} - 1 \right]$$

$$\delta y = x^{-3} \left[1 + (-3) \left(\frac{\delta x}{x} \right) + \frac{(-3)(-3-1)}{2!} \cdot \left(\frac{\delta x}{x} \right)^2 + \dots - 1 \right]$$



$$\delta y = x^{-3} \cdot \frac{\delta x}{x} \left[-3 + \frac{(-3)(-3-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

Dividing both sides by δx .

$$\frac{\delta y}{\delta x} = \frac{\delta x}{x^{1+3}} \left[-3 + \frac{(-3)(-3-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

$$\frac{\delta y}{\delta x} = \frac{1}{x^4} \left[-3 + \frac{(-3)(-3-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

Taking limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} \frac{1}{x^4} \left[-3 + \frac{(-3)(-3-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

$$\frac{dy}{dx} = \frac{-3}{x^4}$$

$$\boxed{\frac{d}{dx} \left(\frac{1}{x^3} \right) = \frac{-3}{x^4}}$$

Ans.

(v) $\frac{1}{x-a}$

Let



$$\begin{aligned}
 y &= \frac{1}{x-a} \\
 y &= (x-a)^{-1} \\
 y + \delta y &= (x+\delta x-a)^{-1} \\
 \delta y &= (x-a+\delta x)^{-1} - y \\
 \delta y &= \left[(x-a) \left(1 + \frac{\delta x}{x-a} \right) \right]^{-1} - (x-a)^{-1} \quad \because y = (x-a)^{-1} \\
 \delta y &= (x-a)^{-1} \left(1 + \frac{\delta x}{x-a} \right)^{-1} - (x-a)^{-1} \\
 \delta y &= (x-a)^{-1} \left[\left(1 + \frac{\delta x}{x-a} \right)^{-1} - 1 \right] \\
 \delta y &= (x-a)^{-1} \left[1 + (-1) \left(\frac{\delta x}{x-a} \right) + \frac{(-1)(-1-1)}{2!} \cdot \left(\frac{\delta x}{x-a} \right)^2 + \dots - 1 \right] \\
 \delta y &= (x-a)^{-1} \cdot \frac{\delta x}{x-a} \left[-1 + \frac{(-1)(-1-1)}{2!} \cdot \frac{\delta x}{x-a} + \dots \right]
 \end{aligned}$$

Dividing both sides by δx

$$\frac{\delta y}{\delta x} = \frac{\delta x}{\delta x (x-a)^{1+1}} \left[-1 + \frac{(-1)(-1-1)}{2!} \cdot \frac{\delta x}{x-a} + \dots \right]$$

$$\frac{\delta y}{\delta x} = \frac{1}{(x-a)^2} \left[-1 + \frac{(-1)(-1-1)}{2!} \cdot \frac{\delta x}{x-a} + \dots \right]$$

Taking limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} \frac{1}{(x-a)^2} \left[-1 + \frac{(-1)(-1-1)}{2!} \cdot \frac{\delta x}{x-a} + \dots \right]$$

$$\frac{dy}{dx} = \frac{-1}{(x-a)^2}$$

$\frac{d}{dx} \left(\frac{1}{x-a} \right) = \frac{-1}{(x-a)^2}$

Ans.

(vi) $x(x-3)$

Let

$$y = x(x-3)$$

$$y = x^2 - 3x$$

$$y + \delta y = (x + \delta x)^2 - 3(x + \delta x)$$

$$\delta y = x^2 + \delta x^2 + 2x\delta x - 3x - 3\delta x - y$$

$$\delta y = x^2 + \delta x^2 + 2x\delta x - 3x - 3\delta x - x^2 + 3x$$

$$\delta y = \delta x(\delta x + 2x - 3)$$

Dividing both sides by δx .

$$\frac{\delta y}{\delta x} = \frac{\delta x(\delta x + 2x - 3)}{\delta x} = \delta x + 2x - 3$$

Taking limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} (\delta x + 2x - 3)$$

$$\frac{dy}{dx} = 2x - 3$$

$\frac{dy}{dx} [x(x-3)] = 2x - 3$

Ans.

(vii) $\frac{2}{x^4}$

Let $y = \frac{2}{x^4}$

$$y = 2x^{-4}$$

$$\begin{aligned}
 y + \delta y &= 2(x + \delta x)^{-4} \\
 \delta y &= 2(x + \delta x)^{-4} - y \\
 \delta y &= 2 \left[x \left(1 + \frac{\delta x}{x} \right) \right]^{-4} - 2x^{-4} \quad \therefore y = 2x^{-4} \\
 \delta y &= 2x^{-4} \left(1 + \frac{\delta x}{x} \right)^{-4} - 2x^{-4} \\
 \delta y &= 2x^{-4} \left[\left(1 + \frac{\delta x}{x} \right)^{-4} - 1 \right] \\
 \delta y &= 2x^{-4} \left[1 + (-4) \left(\frac{\delta x}{x} \right) + \frac{(-4)(-4-1)}{2!} \left(\frac{\delta x}{x} \right)^2 + \dots - 1 \right] \\
 \delta y &= 2x^{-4} \frac{\delta x}{x} \left[-4 + \frac{(-4)(-4-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]
 \end{aligned}$$

Dividing both sides by δx .

$$\begin{aligned}
 \frac{\delta y}{\delta x} &= \frac{2\delta x}{\delta x \cdot x^{1+4}} \left[-4 + \frac{(-4)(-4-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right] \\
 \frac{\delta y}{\delta x} &= \frac{2}{x^5} \left[-4 + \frac{(-4)(-4-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]
 \end{aligned}$$

Taking limit $\delta x \rightarrow 0$

$$\begin{aligned}
 \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} &= \lim_{\delta x \rightarrow 0} \frac{2}{x^5} \left[-4 + \frac{(-4)(-4-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right] \\
 \frac{dy}{dx} &= \frac{2}{x^5} (-4) = \frac{-8}{x^5}
 \end{aligned}$$

$$\boxed{\frac{d}{dx} \left(\frac{2}{x^4} \right) = \frac{-8}{x^5}} \quad \text{Ans.}$$

(viii) $x^2 + \frac{1}{x^2}$

Let $y = x^2 + \frac{1}{x^2}$

$$y = x^2 + x^{-2}$$

$$y + \delta y = (x + \delta x)^2 + (x + \delta x)^{-2}$$

$$\delta y = x^2 + \delta x^2 + 2x\delta x + \left[x \left(1 + \frac{\delta x}{x} \right) - y \right]^{-2} - y$$

$$\delta y = x^2 + \delta x^2 + 2x\delta x + x^{-2} \left(1 + \frac{\delta x}{x} \right)^{-2} - (x^2 + x^{-2}) \quad \therefore y = x^2 + x^{-2}$$

$$\begin{aligned}
 \delta y &= x^2 + \delta x^2 + 2x\delta x + x^{-2} \left(1 + \frac{\delta x}{x}\right)^{-2} - x^2 - x^{-2} \\
 \delta y &= \delta x^2 + 2x\delta x + x^{-2} \left[\left(1 + \frac{\delta x}{x}\right)^{-2} - 1 \right] \\
 \delta y &= \delta x(\delta x + 2x) + x^{-2} \left[1 + (-2)\left(\frac{\delta x}{x}\right) + \frac{(-2)(-2-1)}{2!} \cdot \left(\frac{\delta x}{x}\right)^2 + \dots - 1 \right] \\
 \delta y &= \delta x(\delta x + 2x) + x^{-2} \frac{\delta x}{x} \left[-2 + \frac{(-2)(-2-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right] \\
 \delta y &= \delta x \left[\delta x + 2x + \frac{1}{x^{1+2}} \left\{ -2 + \frac{(-2)(-2-1)}{2!} \frac{\delta x}{x} + \dots \right\} \right]
 \end{aligned}$$

Dividing both sides by δx .

$$\begin{aligned}
 \frac{\delta y}{\delta x} &= \frac{\delta x}{\delta x} \left[\delta x + 2x + \frac{1}{x^3} \left\{ -2 + \frac{(-2)(-2-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right\} \right] \\
 \frac{\delta y}{\delta x} &= \delta x + 2x + \frac{1}{x^3} \left[-2 + \frac{(-2)(-2-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]
 \end{aligned}$$

Taking limit $\delta x \rightarrow 0$

$$\begin{aligned}
 \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} &= \lim_{\delta x \rightarrow 0} \left[\delta x + 2x + \frac{1}{x^3} \left\{ -2 + \frac{(-2)(-2-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right\} \right] \\
 \frac{dy}{dx} &= 2x - \frac{2}{x^3}
 \end{aligned}$$

$$\boxed{\frac{d}{dx} \left(x^2 + \frac{1}{x^2} \right) = 2x - \frac{2}{x^3}}$$

Ans.

(ix) $(x + 4)^{1/3}$ (Guj. Board 2003)

Let $y = (x + 4)^{1/3}$

$$\begin{aligned}
 y + \delta y &= (x + \delta x + 4)^{1/3} \\
 \delta y &= (x + 4 + \delta x)^{1/3} - y \\
 \delta y &= \left[(x + 4) \left(1 + \frac{\delta x}{x + 4} \right) \right]^{1/3} - (x + 4)^{1/3} \quad \because y = (x + 4)^{1/3} \\
 \delta y &= (x + 4)^{1/3} \left(1 + \frac{\delta x}{x + 4} \right)^{1/3} - (x + 4)^{1/3} \\
 \delta y &= (x + 4)^{1/3} \left[\left(1 + \frac{\delta x}{x + 4} \right)^{1/3} - 1 \right] \\
 \delta y &= (x + 4)^{1/3} \left[1 + \frac{1}{3} \left(\frac{\delta x}{x + 4} \right) + \frac{1/3(1/3-1)}{2!} \left(\frac{\delta x}{x + 4} \right)^2 + \dots - 1 \right]
 \end{aligned}$$

$$\delta y = (x+4)^{1/3} \cdot \frac{\delta x}{x+4} \left[\frac{1}{3} + \frac{1/3(1/3-1)}{2!} \cdot \frac{\delta x}{x+4} + \dots \right]$$

Dividing both sides by δx .

$$\frac{\delta y}{\delta x} = \frac{\delta x}{\delta x(x+4)x^{1-\frac{1}{3}}} \left[\frac{1}{3} + \frac{1/3(1/3-1)}{2!} \cdot \frac{\delta x}{x+4} + \dots \right]$$

$$\frac{\delta y}{\delta x} = \frac{1}{(x+4)^{2/3}} \left[\frac{1}{3} + \frac{1/3(1/3-1)}{2!} \cdot \frac{\delta x}{x+4} + \dots \right]$$

Taking limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} \frac{1}{(x+4)^{2/3}} \left[\frac{1}{3} + \frac{1/3(1/3-1)}{2!} \cdot \frac{\delta x}{x+4} + \dots \right]$$

$$\frac{dy}{dx} = \frac{1}{3(x+4)^{2/3}}$$

$$\frac{d}{dx} [(x+4)^{1/3}] = \frac{1}{3(x+4)^{2/3}}$$

Ans.

(x) $x^{3/2}$

Let $y = x^{3/2}$

$$y + \delta y = (x + \delta x)^{3/2}$$

$$\delta y = (x + \delta x)^{3/2} - y$$

$$\delta y = \left[x \left(1 + \frac{\delta x}{x} \right)^{3/2} - x^{3/2} \right] \because y = x^{3/2}$$

$$\delta y = x^{3/2} \left(1 + \frac{\delta x}{x} \right)^{3/2} - x^{3/2}$$

$$\delta y = x^{3/2} \left[\left(1 + \frac{\delta x}{x} \right)^{3/2} - 1 \right]$$

$$\delta y = x^{3/2} \left[1 + \frac{3}{2} \left(\frac{\delta x}{x} \right) + \frac{3/2(3/2-1)}{2!} \cdot \left(\frac{\delta x}{x} \right)^2 + \dots - 1 \right]$$

$$\delta y = x^{3/2} \cdot \frac{\delta x}{x} \left[\frac{3}{2} + \frac{3/2(3/2-1)}{2!} \cdot \frac{\delta x}{x} \right]$$

Dividing both sides by δx .

$$\frac{\delta y}{\delta x} = \frac{x^{\frac{3}{2}-1} \cdot \delta x}{\delta x} \left[\frac{3}{2} + \frac{3/2(3/2-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

$$\frac{\delta y}{\delta x} = x^{1/2} \left[\frac{3}{2} + \frac{3/2(3/2-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

Taking limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} x^{1/2} \left[\frac{3}{2} + \frac{3/2(3/2-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

$$\frac{dy}{dx} = x^{1/2} \cdot \frac{3}{2}$$

$\frac{d}{dx}(x^{1/3}) = \frac{3}{2}\sqrt{x}$

Ans.

(xi) $x^{5/2}$

Let $y = x^{5/2}$

$$y + \delta y = (x + \delta x)^{5/2}$$

$$\delta y = (x + \delta x)^{5/2} - y$$

$$\delta y = \left[x \left(1 + \frac{\delta x}{x} \right) \right]^{5/2} - x^{5/2} \quad \because y = x^{5/2}$$

$$\delta y = x^{5/2} \left(1 + \frac{\delta x}{x} \right)^{5/2} - x^{5/2}$$

$$\delta y = x^{5/2} \left[\left(1 + \frac{\delta x}{x} \right)^{5/2} - 1 \right]$$

$$\delta y = \left[1 + \frac{5}{2} \left(\frac{\delta x}{x} \right) + \frac{5/2(5/2-1)}{2!} \cdot \left(\frac{\delta x}{x} \right)^2 + \dots - 1 \right]$$

$$\delta y = x^{5/2} \cdot \frac{\delta x}{x} \left[\frac{5}{2} + \frac{5/2(5/2-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

Dividing both sides by δx .

$$\frac{\delta y}{\delta x} = \frac{x^{\frac{5}{2}-1} \cdot \delta x}{\delta x} \left[\frac{5}{2} + \frac{5/2(5/2-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

$$\frac{\delta y}{\delta x} = x^{3/2} \left[\frac{5}{2} + \frac{5/2(5/2-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

Taking limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} x^{3/2} \left[\frac{5}{2} + \frac{5/2(5/2-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

$$\frac{dy}{dx} = \frac{5}{2} x^{3/2}$$

$\frac{d}{dx}(x^{5/2}) = \frac{5}{2} x^{3/2}$

Ans.

(xii) x^m

Let $y = x^m$

$$y + \delta y = (x + \delta x)^m$$

$$\delta y = \left[x \left(1 + \frac{\delta x}{x} \right) \right]^m - y$$

$$\delta y = x^m \left(1 + \frac{\delta x}{x} \right)^m - x^m \quad \because y = x^m$$

$$\delta y = x^m \left[\left(1 + \frac{\delta x}{x} \right)^m - 1 \right]$$

$$\delta y = x^m \left[1 + m \left(\frac{\delta x}{x} \right) + \frac{m(m-1)}{2!} \cdot \left(\frac{\delta x}{x} \right)^2 + \dots - 1 \right]$$

$$\delta y = x^m \cdot \frac{\delta x}{x} \left[m + \frac{m(m-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

Dividing both sides by δx .

$$\frac{\delta y}{\delta x} = \frac{x^{m-1} \delta x}{\delta x} \left[m + \frac{m(m-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

$$\frac{\delta y}{\delta x} = x^{m-1} \left[m + \frac{m(m-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

Taking limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} x^{m-1} \left[m + \frac{m(m-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

$$\frac{dy}{dx} = mx^{m-1}$$

$$\frac{d}{dx}(x^m) = mx^{m-1}$$

Ans.

(xiii) $\frac{1}{x^m}$, $m \in \mathbb{N}$

Let $y = \frac{1}{x^m}$

$$y = x^{-m}$$

$$y + \delta y = (x + \delta x)^{-m}$$

$$\delta y = \left[x \left(1 + \frac{\delta x}{x} \right) \right]^{-m} - y$$

$$\delta y = x^{-m} \left(1 + \frac{\delta x}{x} \right)^{-m} - x^{-m} \quad \because y = x^{-m}$$

$$\begin{aligned}\delta y &= x^{-m} \left[\left(1 + \frac{\delta x}{x} \right)^{-m} - 1 \right] \\ \delta y &= x^{-m} \left[1 + (-m) \left(\frac{\delta x}{x} \right) + \frac{(-m)(-m-1)}{2!} \cdot \left(\frac{\delta x}{x} \right)^2 + \dots - 1 \right] \\ &= x^m \cdot \frac{\delta x}{x} \left[-m + \frac{(-m)(-m-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]\end{aligned}$$

Dividing both sides by δx .

$$\begin{aligned}\frac{\delta y}{\delta x} &= \frac{x^{-m-1} \delta x}{\delta x} \left[-m + \frac{(-m)(-m-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right] \\ \frac{\delta y}{\delta x} &= x^{-m-1} \left[-m + \frac{(-m)(-m-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]\end{aligned}$$

Taking limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} x^{-m-1} \left[-m + \frac{(-m)(-m-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

$$\frac{dy}{dx} = -mx^{-m-1}$$

$$\boxed{\frac{dy}{dx} \left(\frac{1}{x^m} \right) = -mx^{-m-1}}$$

Ans.

(xiv) x^{40}

Let

$$y = x^{40}$$

$$y + \delta y = (x + \delta x)^{40}$$

$$\delta y = \left[x \left(1 + \frac{\delta x}{x} \right) \right]^{40} - y$$

$$\delta y = x^{40} \left(1 + \frac{\delta x}{x} \right)^{40} - x^{40} \quad \therefore y = x^{40}$$

$$\delta y = x^{40} \left[\left(1 + \frac{\delta x}{x} \right)^{40} - 1 \right]$$

$$\delta y = x^{40} \left[1 + 40 \left(\frac{\delta x}{x} \right) + \frac{40(40-1)}{2!} \cdot \left(\frac{\delta x}{x} \right)^2 + \dots - 1 \right]$$

$$\delta y = x^{40} \cdot \frac{\delta x}{x} \left[40 + \frac{40(40-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

Dividing both sides by δx .

$$\frac{\delta y}{\delta x} = \frac{x^{40-1} \delta x}{\delta x} \left[40 + \frac{40(40-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

$$\frac{\delta y}{\delta x} = x^{39} \left[40 + \frac{40(40-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

Taking limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} x^{39} \left[40 + \frac{40(40-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

$$\frac{dy}{dx} = 40x^{39}$$

$$\frac{dy}{dx}(x^{40}) = 40x^{39}$$

Ans.

(xv) x^{-100}

$$\text{Let } y = x^{-100}$$

$$y + \delta y = (x + \delta x)^{-100}$$

$$\delta y = \left[x \left(1 + \frac{\delta x}{x} \right) \right]^{-100} - y$$

$$\delta y = x^{-100} \left(1 + \frac{\delta x}{x} \right)^{-100} - x^{-100} \quad \because y = x^{-100}$$

$$\delta y = x^{-100} \left[\left(1 + \frac{\delta x}{x} \right)^{-100} - 1 \right]$$

$$\delta y = x^{-100} \left[1 + (-100) \left(\frac{\delta x}{x} \right) + \frac{(-100)(-100-1)}{2!} \cdot \left(\frac{\delta x}{x} \right)^2 + \dots - 1 \right]$$

$$\delta y = x^{-100} \cdot \frac{\delta x}{x} \left[-100 + \frac{(-100)(-100-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

Dividing both sides by δx .

$$\frac{\delta y}{\delta x} = \frac{\delta x}{x^{1+100}} \left[-100 + \frac{(-100)(-100-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

$$\frac{\delta y}{\delta x} = \frac{1}{x^{101}} \left[-100 + \frac{(-100)(-100-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

Taking limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} \frac{1}{x^{101}} \left[-100 + \frac{(-100)(-100-1)}{2!} \cdot \frac{\delta x}{x} + \dots \right]$$

$$\frac{dy}{dx} = \frac{-100}{x^{101}}$$

$$\frac{d}{dx}(x^{-100}) = \frac{-100}{x^{101}}$$

Ans.

Q.2 Find $\frac{dy}{dx}$ from first principles if

$$\text{(i)} \quad \sqrt{x+2} \quad \text{(ii)} \quad \frac{1}{\sqrt{x+a}} \quad (\text{L.B 2004, 2010})$$

Solution:

(i) $\sqrt{x+2}$

$$\begin{aligned} \text{Let } y &= \sqrt{x+2} \\ y &= (x+2)^{1/2} \\ y + \delta y &= (x + \delta x + 2)^{1/2} \\ \delta y &= (x + 2 + \delta x)^{1/2} - y \\ \delta y &= \left[(x+2) \left(1 + \frac{\delta x}{x+2} \right) \right]^{1/2} - (x+2)^{1/2} \quad \therefore y = \sqrt{x+2} \\ \delta y &= (x+2)^{1/2} \left(1 + \frac{\delta x}{x+2} \right)^{1/2} - (x+2)^{1/2} \end{aligned}$$

$$\begin{aligned} \delta y &= (x+2)^{1/2} \left[\left(1 + \frac{\delta x}{x+2} \right)^{1/2} - 1 \right] \\ \delta y &= (x+2)^{1/2} \left[1 + \frac{1}{2} \left(\frac{\delta x}{x+2} \right) + \frac{1/2(1/2-1)}{2!} \cdot \left(\frac{\delta x}{x+2} \right)^2 + \dots - 1 \right] \\ \delta y &= (x+2)^{1/2} \cdot \frac{\delta x}{x+2} \left[\frac{1}{2} + \frac{1/2(1/2-1)}{2!} \cdot \frac{\delta x}{x+2} + \dots \right] \end{aligned}$$

Dividing both sides by δx .

$$\begin{aligned} \frac{\delta y}{\delta x} &= \frac{\delta x}{\delta x (x+2) x^{1-\frac{1}{2}}} \left[\frac{1}{2} + \frac{1/2(1/2-1)}{2!} \cdot \frac{\delta x}{x+2} + \dots \right] \\ \frac{\delta y}{\delta x} &= \frac{1}{(x+2)^{1/2}} \left[\frac{1}{2} + \frac{1/2(1/2-1)}{2!} \cdot \frac{\delta x}{x+2} + \dots \right] \end{aligned}$$

Taking limit $\delta x \rightarrow 0$

$$\begin{aligned} \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} &= \lim_{\delta x \rightarrow 0} \frac{1}{\sqrt{x+2}} \left[\frac{1}{2} + \frac{1/2(1/2-1)}{2!} \cdot \frac{\delta x}{x+2} + \dots \right] \\ \frac{\delta y}{\delta x} &= \frac{1}{2\sqrt{x+2}} \end{aligned}$$

$$\frac{d}{dx} (\sqrt{x+2}) = \frac{1}{2\sqrt{x+2}}$$

Ans.

$$\text{(ii)} \quad \frac{1}{\sqrt{x+a}}$$

Let $y = \frac{1}{\sqrt{x+a}}$

$$y = (x+a)^{-1/2}$$

$$y + \delta y = (x + \delta x + a)^{-1/2}$$

$$\delta y = (x + a + \delta x)^{-1/2} - y$$

$$\delta y = \left[(x+a) \left(1 + \frac{\delta x}{x+a} \right) \right]^{-1/2} - (x+a)^{-1/2} \quad \because y = (x+a)^{-1/2}$$

$$\delta y = (x+a)^{-1/2} \left(1 + \frac{\delta x}{x+a} \right)^{-1/2} - (x+a)^{-1/2}$$

$$\delta y = (x+a)^{-1/2} \left[\left(1 + \frac{\delta x}{x+a} \right)^{-1/2} - 1 \right]$$

$$\delta y = (x+a)^{-1/2} \left[1 + \left(\frac{1}{2} \right) \left(\frac{\delta x}{x+a} \right) + \frac{(-1/2)(-1/2-1)}{2!} \cdot \left(\frac{\delta x}{x+2} \right)^2 + \dots - 1 \right]$$

$$\delta y = (x+a)^{-1/2} \cdot \frac{\delta x}{x+a} \left[\frac{-1}{2} + \frac{(-1/2)(-1/2-1)}{2!} \cdot \frac{\delta x}{x+a} + \dots \right]$$

Dividing both sides by δx .

$$\frac{\delta y}{\delta x} = \frac{\delta x}{\delta x (x+a) x^{1/2}} \left[\frac{-1}{2} + \frac{-1/2 (-1/2-1)}{2!} \cdot \frac{\delta x}{x+a} + \dots \right]$$

$$\frac{\delta y}{\delta x} = \frac{1}{(x+a)^{3/2}} \left[\frac{-1}{2} + \frac{-1/2 (-1/2-1)}{2!} \cdot \frac{\delta x}{x+a} + \dots \right]$$

Taking limit $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} \frac{1}{(x+a)^{3/2}} \left[\frac{-1}{2} + \frac{-1/2 (-1/2-1)}{2!} \cdot \frac{\delta x}{x+a} + \dots \right]$$

$$\frac{dy}{dx} = \frac{-1}{2(x+a)^{3/2}}$$

$$\frac{d}{dx} \left(\frac{1}{\sqrt{x+a}} \right) = \frac{-1}{2(x+a)^{3/2}}$$

Ans.