

**Solution:**

$$\begin{aligned}\tan^{-1} y &= \tan(P \tan^{-1} x) \\ \tan^{-1} y &= P \tan^{-1} x\end{aligned}$$

Diff. w.r.t. 'x'

$$\frac{1}{1+y^2} \cdot y_1 = P \frac{1}{1+x^2}$$

$$(1+x^2) y_1 = P(1+y^2)$$

$$\boxed{(1+x^2) y_1 - P(1+y^2) = 0} \quad \text{Hence proved.}$$

**EXERCISE 2 . 6****Q.1: Find  $f'(x)$  if**

- |  |   |
|--|---|
| (i) $f(x) = e^{\sqrt{x}-1}$                    | (ii) $f(x) = x^3 e^{\frac{1}{x}} (x \neq 0)$            |
| (iii) $f(x) = ex(1+\ell nx)$                   | (iv) $f(x) = \frac{e^x}{e^{-x}+1}$                      |
| (v) $f(x) = \ell n(e^x + e^{-x})$              | (vi) $f(x) = \frac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}}$ |
| (vii) $f(x) = \sqrt{\ell n(e^{2x} + e^{-2x})}$ | (viii) $f(x) = \ell n(\sqrt{e^{2x} + e^{-2x}})$         |

**Solution:**

(i)  $f(x) = e^{\sqrt{x}-1}$

Diff. w.r.t. 'x'

$$f'(x) = e^{\sqrt{x}-1} \cdot \frac{d}{dx} (\sqrt{x}-1)$$

$$f'(x) = e^{\sqrt{x}-1} \cdot \frac{1}{2} x^{-\frac{1}{2}}$$

$$\boxed{f'(x) = \frac{e^{\sqrt{x}-1}}{2\sqrt{x}}} \quad \text{Ans.}$$

(ii)  $f(x) = x^3 e^{\frac{1}{x}} (x \neq 0)$

Diff. w.r.t. 'x'

$$f'(x) = x^3 \frac{d}{dx} \left( e^{\frac{1}{x}} \right) + e^{\frac{1}{x}} \frac{d}{dx} (x^3)$$

$$f'(x) = x^3 e^{\frac{1}{x}} \cdot \frac{d}{dx} \left( \frac{1}{x} \right) + e^{\frac{1}{x}} 3x^2$$

$$f'(x) = x^3 e^{\frac{1}{x}} \times \frac{-1}{x^2} + 3x^2 e^{\frac{1}{x}}$$

$$f'(x) = -x e^{\frac{1}{x}} + 3x^2 e^{\frac{1}{x}}$$

$f'(x) = x e^{\frac{1}{x}} (3x - 1)$

Ans.

(iii)  $f(x) = e^x (1 + \ln x)$

Diff. w.r.t. 'x'

$$f'(x) = e^x \frac{d}{dx} (1 + \ln x) + (1 + \ln x) \frac{d}{dx} (e^x)$$

$$f'(x) = e^x \cdot \frac{1}{x} + (1 + \ln x) e^x$$

$$f'(x) = e^x \left[ \frac{1}{x} + (1 + \ln x) \right]$$

$f'(x) = \frac{e^x [ 1 + x (1 + \ln x) ]}{x}$

Ans

(iv)  $f(x) = \frac{e^x}{e^{-x} + 1}$

Diff. w.r.t. 'x'

$$f'(x) = \frac{(e^{-x} + 1) \frac{d}{dx}(e^x) - e^x \frac{d}{dx}(e^{-x} + 1)}{(e^{-x} + 1)^2}$$

$$f'(x) = \frac{(e^{-x} + 1)e^x - e^x \cdot e^{-x}(-1)}{(e^{-x} + 1)^2}$$

$$f'(x) = \frac{e^{-x} \cdot e^x + e^x + e^{x-x}}{(e^{-x} + 1)^2}$$

$$f'(x) = \frac{e^{-x+x} + e^x + e^0}{(e^{-x} + 1)^2}$$

$$f'(x) = \frac{e^0 + e^x + 1}{(e^{-x} + 1)^2}$$

$$f'(x) = \frac{1 + e^x + 1}{(e^{-x} + 1)^2}$$

$$\boxed{f'(x) = \frac{e^x + 2}{(e^{-x} + 1)^2}}$$

Ans.

(v)  $f(x) = \ln(e^x + e^{-x})$

Diff. w.r.t. 'x'

$$f'(x) = \frac{1}{e^x + e^{-x}} \frac{d}{dx} (e^x + e^{-x})$$

$$f'(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$f'(x) = \frac{\frac{e^x - 1}{e^x}}{\frac{1}{e^x + e^{-x}}}$$

$$f'(x) = \frac{\frac{e^{2x} - 1}{e^x}}{\frac{e^{2x} + 1}{e^x}}$$

$$\boxed{f'(x) = \frac{e^{2x} - 1}{e^{2x} + 1}}$$



Ans.

(vi)  $f(x) = \frac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}}$

Diff. w.r.t. 'x'

$$f'(x) = \frac{(e^{ax} + e^{-ax}) \frac{d}{dx}(e^{ax} - e^{-ax}) - (e^{ax} - e^{-ax}) \frac{d}{dx}(e^{ax} + e^{-ax})}{(e^{ax} + e^{-ax})^2}$$

$$f'(x) = \frac{(e^{ax} + e^{-ax})(e^{ax} \cdot a - e^{-ax}(-a)) - (e^{ax} - e^{-ax})(e^{ax} \cdot a + e^{-ax} \times -a)}{(e^{ax} + e^{-ax})^2}$$

$$f'(x) = \frac{a(e^{ax} + e^{-ax})(e^{ax} + e^{-ax}) - a(e^{ax} - e^{-ax})(e^{ax} - e^{-ax})}{(e^{ax} + e^{-ax})^2}$$

$$f'(x) = \frac{a[(e^{ax} + e^{-ax})^2 - (e^{ax} - e^{-ax})^2]}{(e^{ax} + e^{-ax})^2}$$

$$f'(x) = \frac{a[e^{2ax} + e^{-2ax} + 2e^{ax}e^{-ax} - (e^{2ax} + e^{-2ax} - 2e^{ax} \cdot e^{-ax})]}{(e^{ax} + e^{-ax})^2}$$

$$f'(x) = \frac{a(e^{2ax} + e^{-2ax} + 2 - e^{2ax} - e^{-2ax} + 2)}{(e^{ax} + e^{-ax})^2}$$

$$\boxed{f'(x) = \frac{4a}{(e^{ax} + e^{-ax})^2}} \quad \text{Ans.}$$

(vii)  $f(x) = \sqrt{\ln(e^{2x} + e^{-2x})}$

Diff. w.r.t. 'x'

$$f'(x) = \frac{1}{2} [\ln(e^{2x} + e^{-2x})]^{-1/2} \cdot \frac{d}{dx} [\ln(e^{2x} + e^{-2x})]$$

$$f'(x) = \frac{1}{2\sqrt{\ln(e^{2x} + e^{-2x})}} \cdot \frac{1}{e^{2x} + e^{-2x}} \cdot \frac{d}{dx} (e^{2x} + e^{-2x})$$

$$f'(x) = \frac{1}{2\sqrt{\ln(e^{2x} + e^{-2x})}} \cdot \frac{1}{e^{2x} + e^{-2x}} \cdot (e^{2x} \cdot 2 + e^{-2x}(-2))$$

$$f'(x) = \frac{1}{2\sqrt{\ln(e^{2x} + e^{-2x})}} \cdot \frac{1}{e^{2x} + e^{-2x}} \cdot 2(e^{2x} - e^{-2x})$$

$$\boxed{f'(x) = \frac{e^{2x} - e^{-2x}}{(e^{2x} - e^{-2x})\sqrt{\ln(e^{2x} + e^{-2x})}}} \quad \text{Ans.}$$

(viii)  $f(x) = \ln(\sqrt{e^{2x} + e^{-2x}}) \quad (\text{L.B 2009 (s)})$

Diff. w.r.t. 'x'

$$f'(x) = \frac{1}{\sqrt{e^{2x} + e^{-2x}}} \cdot \frac{d}{dx} (\sqrt{e^{2x} + e^{-2x}})$$

$$f'(x) = \frac{1}{\sqrt{e^{2x} + e^{-2x}}} \cdot \frac{1}{2} (e^{2x} + e^{-2x})^{\frac{-1}{2}} \cdot \frac{d}{dx} (e^{2x} + e^{-2x})$$

$$f'(x) = \frac{1}{2\sqrt{e^{2x} + e^{-2x}}\sqrt{e^{2x} + e^{-2x}}} (e^{2x} \cdot 2 + e^{-2x} - 2)$$

$$f'(x) = \frac{2(e^{2x} - e^{-2x})}{2(e^{2x} + e^{-2x})} = \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$$

$$\boxed{f'(x) = \tan h2x} \quad \text{Ans.}$$

**Q.2:** Find  $\frac{dy}{dx}$  if

(i)  $y = x^2 \ln \sqrt{x}$

(ii)  $y = x \sqrt{\ln x}$

(iii)  $y = \frac{x}{\ell nx}$

(iv)  $y = x^2 \ln \frac{1}{x}$

- (v)  $y = \ln \sqrt{\frac{x^2 - 1}{x^2 + 1}}$  (vi)  $y = \ln(x + \sqrt{x^2 + 1})$
- (vii)  $y = \ln(9 - x^2)$  (L.B 2009) (viii)  $y = e^{-2x} \sin 2x$  (L.B 2009 (s))
- (ix)  $y = e^{-x}(x^3 + 2x^2 + 1)$  (L.B 2009) (x)  $y = xe^{\sin x}$
- (xi)  $y = 5e^{3x-4}$  (xii)  $y = (x+1)^x$
- (xiii)  $y = (\ln x)^{ex}$  (xiv)  $y = \frac{\sqrt{x^2 - 1}(x+1)}{(x^3 + 1)^{\frac{3}{2}}}$

**Solution:**

(i)  $y = x^2 \ln \sqrt{x}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = x^2 \frac{d}{dx}(\ln \sqrt{x}) + \ln \sqrt{x} \frac{d}{dx}(x^2)$$

$$\frac{dy}{dx} = x^2 \cdot \frac{1}{\sqrt{x}} \cdot \frac{d}{dx}(\sqrt{x}) + \ln \sqrt{x} \cdot 2x$$

$$\frac{dy}{dx} = \frac{x^2}{\sqrt{x}} \cdot \frac{1}{2} x^{\frac{-1}{2}} + 2x \ln \sqrt{x}$$

$$\frac{dy}{dx} = \frac{x^2}{2\sqrt{x} \cdot \sqrt{x}} + 2x \ln \sqrt{x}$$

$$\frac{dy}{dx} = \frac{x^2}{2x} + 2x \ln \sqrt{x}$$

$$\boxed{\frac{dy}{dx} = \frac{x}{2} + 2x \ln \sqrt{x}}$$

Ans.

(ii)  $y = x \sqrt{\ln x}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = x \frac{d}{dx}(\sqrt{\ln x}) + \sqrt{\ln x} \frac{d}{dx}(x)$$

$$\frac{dy}{dx} = x \cdot \frac{1}{2} (\ln x)^{\frac{-1}{2}} \cdot \frac{d}{dx}(\ln x) + \sqrt{\ln x} \cdot 1$$

$$\frac{dy}{dx} = \frac{x}{2\sqrt{\ln x}} \cdot \frac{1}{x} + \sqrt{\ln x}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\ln x}} + \sqrt{\ln x}$$

$$\boxed{\frac{dy}{dx} = \frac{1 + 2\ln x}{2\sqrt{\ln x}}} \quad \text{Ans}$$

(iii)  $y = \frac{x}{\ln x}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{(\ln x) \frac{d}{dx}(x) - x \frac{d}{dx}(\ln x)}{(\ln x)^2}$$

$$\frac{dy}{dx} = \frac{\ln x \cdot 1 - x \cdot \frac{1}{x}}{(\ln x)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{\ln x - 1}{(\ln x)^2}} \quad \text{Ans.}$$

(iv)  $y = x^2 \ln \frac{1}{x}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = x^2 \frac{d}{dx}(\ln \frac{1}{x}) + \ln \frac{1}{x} \frac{d}{dx}(x^2)$$

$$\frac{dy}{dx} = x^2 \cdot \frac{1}{x} \cdot \frac{d}{dx}\left(\frac{1}{x}\right) + \ln \frac{1}{x} \cdot 2x$$

$$\frac{dy}{dx} = x^3 \cdot \frac{-1}{x^2} + 2x \ln \frac{1}{x}$$

$$\frac{dy}{dx} = -x + 2x \ln \frac{1}{x}$$

$$\boxed{\frac{dy}{dx} = x [2 \ln \frac{1}{x} - 1]} \quad \text{Ans.}$$

(v)  $y = \ln \sqrt{\frac{x^2 - 1}{x^2 + 1}}$

$$y = \frac{1}{2} \ln \left( \frac{x^2 - 1}{x^2 + 1} \right)$$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{\frac{x^2-1}{x^2+1}} \cdot \frac{d}{dx} \left( \frac{x^2-1}{x^2+1} \right)$$

$$\frac{dy}{dx} = \frac{x^2+1}{2(x^2-1)} \cdot \left[ \frac{(x^2+1)\frac{d}{dx}(x^2-1) - (x^2-1)\frac{d}{dx}(x^2+1)}{(x^2+1)^2} \right]$$

$$\frac{dy}{dx} = \frac{(x^2+1)2x - (x^2-1)2x}{2(x^2-1)(x^2+1)}$$

$$\frac{dy}{dx} = \frac{2x(x^2+1-x^2+1)}{2(x^4-1)}$$

$$\frac{dy}{dx} = \frac{2x}{x^4-1}$$

Ans.

(vi)  $y = \ln(x + \sqrt{x^2 + 1})$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + 1}} \cdot \frac{d}{dx}(x + \sqrt{x^2 + 1})$$

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + 1}} \cdot \left[ 1 + \frac{1}{2}(x^2 + 1)^{-\frac{1}{2}} \cdot \frac{d}{dx}(x^2 + 1) \right]$$

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + 1}} \cdot \left[ 1 + \frac{1}{2\sqrt{x^2 + 1}} \cdot 2x \right]$$

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + 1}} \cdot \left[ \frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1}} \right]$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x^2 + 1}}$$

Ans.

(vii)  $y = \ln(9 - x^2)$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{1}{9 - x^2} \cdot \frac{d}{dx}(9 - x^2)$$

$$\frac{dy}{dx} = \frac{-2x}{9 - x^2}$$

Ans.

(viii)  $y = e^{-2x} \sin 2x$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = e^{-2x} \frac{d}{dx} (\sin 2x) + \sin 2x \frac{d}{dx} (e^{-2x})$$

$$\frac{dy}{dx} = e^{-2x} \cdot \cos 2x (2) + \sin 2x \cdot e^{-2x} \cdot (-2)$$

$$\frac{dy}{dx} = 2e^{-2x}(\cos 2x - \sin 2x)$$

Ans.

(ix)  $y = e^{-x}(x^3 + 2x^2 + 1)$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = e^{-x} \frac{d}{dx} (x^3 + 2x^2 + 1) + (x^3 + 2x^2 + 1) \frac{d}{dx} (e^{-x})$$

$$\frac{dy}{dx} = e^{-x} (3x^2 + 4x) + (x^3 + 2x^2 + 1) \cdot e^{-x} (-1)$$

$$\frac{dy}{dx} = e^{-x} (3x^2 + 4x - x^3 - 2x^2 - 1)$$

$$\frac{dy}{dx} = e^{-x} (-x^3 + x^2 + 4x - 1)$$

$$\frac{dy}{dx} = -e^{-x} (x^3 - x^2 - 4x + 1)$$

Ans.

(x)  $y = xe^{\sin x}$  (G.B 2007)

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = x \frac{d}{dx} (e^{\sin x}) + e^{\sin x} \frac{d}{dx} (x)$$

$$\frac{dy}{dx} = xe^{\sin x} \frac{d}{dx} (\sin x) + e^{\sin x} \cdot 1$$

$$\frac{dy}{dx} = xe^{\sin x} \cos x + e^{\sin x}$$

$$\frac{dy}{dx} = e^{\sin x} (x \cos x + 1)$$

Ans.

(xi)  $y = 5e^{3x-4}$  (G.B 2006)

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = 5e^{3x-4} \cdot \frac{d}{dx} (3x - 4)$$

$$\frac{dy}{dx} = 5e^{3x-4} \cdot 3$$

$$\boxed{\frac{dy}{dx} = 15 e^{3x-4}} \quad \text{Ans.}$$

(xii)  $y = (x+1)^x$

Taking 'ln' on both sides

$$\ln y = \ln(x+1)^x$$

$$\ln y = x \ln(x+1)$$

Diff. w.r.t. 'x'

$$\frac{1}{y} \cdot \frac{dy}{dx} = x \frac{d}{dx} [\ln(x+1)] + \ln(x+1) \frac{d}{dx} (x)$$

$$\frac{dy}{dx} = y \left[ x \cdot \frac{1}{x+1} + \ln(x+1) \right]$$

$$\boxed{\frac{dy}{dx} = (x+1)^x \left[ \frac{x + (x+1) \ln(x+1)}{x+1} \right]} \quad \text{Ans.}$$

(xiii)  $y = (\ln x)^{\ln x}$

Taking 'ln' on both sides

$$\ln y = \ln(\ln x)^{\ln x}$$

$$\ln y = \ln x \ln(\ln x)$$

Diff. w.r.t. 'x'

$$\frac{1}{y} \frac{dy}{dx} = \ln x \frac{d}{dx} [\ln(\ln x)] + \ln(\ln x) \frac{d}{dx} (\ln x)$$

$$\frac{dy}{dx} = y \left[ \ln x \cdot \frac{1}{\ln x} \cdot \frac{d}{dx} (\ln x) + \ln(\ln x) \cdot \frac{1}{x} \right]$$

$$\frac{dy}{dx} = (\ln x)^{\ln x} \left[ \frac{1}{x} + \frac{\ln(\ln x)}{x} \right]$$

$$\boxed{\frac{dy}{dx} = (\ln x)^{\ln x} \left[ 1 + \frac{\ln(\ln x)}{x} \right]} \quad \text{Ans.}$$

(xiv)  $y = \frac{\sqrt{x^2 - 1} (x+1)}{(x^3 + 1)^{\frac{3}{2}}}$

$$y = \frac{\sqrt{(x+1)(x-1)} (x+1)}{[(x+1)(x^2 - x + 1)]^{\frac{3}{2}}}$$



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$$y = \frac{\sqrt{x+1} \cdot \sqrt{x-1} (x+1)^{\frac{3}{2}}}{(x+1)^2 (x^2 - x + 1)^{\frac{3}{2}}}$$

$$y = \frac{(x-1)^{\frac{1}{2}}}{(x^2 - x + 1)^{\frac{3}{2}}}$$

Taking 'ln' on both sides

$$\ln y = \ln \left[ \frac{(x-1)^{\frac{1}{2}}}{(x^2 - x + 1)^{\frac{3}{2}}} \right]$$

$$\ln y = \ln (x-1)^{\frac{1}{2}} - \ln (x^2 - x + 1)^{\frac{3}{2}}$$

$$\ln y = \frac{1}{2} \ln (x-1) - \frac{3}{2} \ln (x^2 - x + 1)$$

Diff. w.r.t. 'x'

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{x-1} \cdot \frac{d}{dx}(x-1) - \frac{3}{2} \cdot \frac{1}{x^2 - x + 1} \frac{d}{dx}(x^2 - x + 1)$$

$$\frac{dy}{dx} = y \left[ \frac{1}{2(x-1)} - \frac{3(2x-1)}{2(x^2 - x + 1)} \right]$$

$$\frac{dy}{dx} = \frac{(x-1)^{\frac{1}{2}}}{(x^2 - x + 1)^{\frac{3}{2}}} \left[ \frac{x^2 - x + 1 - 3(2x-1)(x-1)}{2(x-1)(x^2 - x + 1)} \right]$$

$$\frac{dy}{dx} = \frac{x^2 - x + 1 - 3(2x^2 - 3x + 1)}{2\sqrt{x-1} (x^2 - x + 1)^{\frac{3}{2} + 1}}$$

$$\frac{dy}{dx} = \frac{x^2 - x + 1 - 6x^2 + 9x - 3}{2\sqrt{x-1} (x^2 - x + 1)^{\frac{5}{2}}}$$

$$\frac{dy}{dx} = \frac{-5x^2 + 8x - 2}{2\sqrt{x-1} (x^2 - x + 1)^{\frac{5}{2}}}$$

$$\boxed{\frac{dy}{dx} = -\frac{5x^2 - 8x + 2}{2\sqrt{x-1} (x^2 - x + 1)^{\frac{5}{2}}}}$$

Ans.

Q.3: Find  $\frac{dy}{dx}$  if

(i)  $y = \cosh 2x$

(ii)  $y = \sinh 3x$

(iii)  $y = \tanh^{-1}(\sin x) - \frac{\pi}{2} < x < \frac{\pi}{2}$

(iv)  $y = \sinh^{-1}(x^3)$

(v)  $y = (\ln \tan hx)$

(vi)  $y = \sinh^{-1}\left(\frac{x}{2}\right)$

**Solution:**

(i)  $y = \cosh 2x$  (L.B 2008)

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \sinh 2x \cdot \frac{d}{dx}(2x)$$

$$\boxed{\frac{dy}{dx} = 2 \sinh 2x} \quad \text{Ans.}$$

(ii)  $y = \sinh 3x$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \cosh 3x \cdot \frac{d}{dx}(3x)$$

$$\boxed{\frac{dy}{dx} = 3 \cosh 3x} \quad \text{Ans.}$$

(iii)  $y = \tanh^{-1}(\sin x)$

$$\frac{-\pi}{2} < x < \frac{\pi}{2}$$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{1}{1 - \sin^2 x} \cdot \frac{d}{dx}(\sin x)$$

$$\boxed{\frac{dy}{dx} = \frac{\cos x}{\cos^2 x} = \frac{1}{\cos x} = \sec x} \quad \text{Ans.}$$

(iv)  $y = \sinh^{-1}(x^3)$  (L.B 2008), (G.B 2008)

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 + (x^3)^2}} \cdot \frac{d}{dx}(x^3)$$

$$\boxed{\frac{dy}{dx} = \frac{3x^2}{\sqrt{1 + x^6}}} \quad \text{Ans.}$$

(v)  $y = (\ln \tan hx)$  (L.B 2009 (s))

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{1}{\tanh x} \cdot \frac{d}{dx} (\tanh x)$$

$$\frac{dy}{dx} = \frac{1}{\tanh x} \cdot \sec h^2 x$$

$$\frac{dy}{dx} = \frac{1}{\cosh^2 x}$$

$$\frac{\sinh x}{\cosh x}$$

$$\frac{dy}{dx} = \frac{1}{\sinh x \cosh x}$$

$$\frac{dy}{dx} = \frac{2}{2 \sinh x \cosh x} = \frac{2}{\sinh 2x}$$

$$\frac{dy}{dx} = 2 \operatorname{cosech} 2x$$

Ans.

(vi)  $y = \sinh^{-1}\left(\frac{x}{2}\right)$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 + \left(\frac{x}{2}\right)^2}} \cdot \frac{d}{dx} \left(\frac{x}{2}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 + \frac{x^2}{4}}} \cdot \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{\frac{4+x^2}{4}}} \cdot \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{4+x^2}} \cdot \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{4+x^2}}$$

Ans.

### EXERCISE 2.7

**Q.1:** Find  $y_2$  if