Chapter 3

INTEGRATION

EXERCISE 3.1

Q.1 Find δy and dy in the following cases.

(i) $y = x^2 - 1$ when x changes from 3 to 3.02 **(Lhr.Board 2008, 2011, Guj.Board 2008)**

(ii) $y = x^2 + 2x$ when x changes from 2 to 1.8

(iii) $y = \sqrt{x}$ when x changes from 4 to 4.41 **(Lhr. Board 2005)**

Solution:

(i) $y = x^2 - 1$ when x changes from 3 to 3.02

$x = 3, \quad \delta x = 3.02 - 3 = 0.02$

$y + \delta y = (x + \delta x)^2 - 1 - y$

$= (x + \delta x)^2 - 1 - (x^2 - 1)$

$= (x + \delta x)^2 - 1 - x^2 + 1$

$= (x + \delta x)^2 - x^2$

$= (3 + 0.02)^2 - (3)^2$

$= (3.02)^2 - 9$

$= 9.1204 - 9$

$= 0.1204$ Ans.

$y = x^2 - 1$

Taking differential on both sides

$dy = d(x^2) - d(1)$

$dy = 2xdx \qquad (\because dx = \delta x)$

$= 2(3)(0.02)$

$dy = 0.12$ Ans.

(ii) $\mathbf{y = x^2 + 2x}$ **when x changes from 2 to 1.8**

$x = 2, \quad \delta x = 1.8 - 2 = -0.2$

$$y + \delta y = (x + \delta x)^2 + 2(x + \delta x)$$
$$\delta y = (x + \delta x)^2 + 2x + 2\delta x - y$$
$$\delta y = (x + \delta x)^2 + 2x + 2\delta x - (x^2 + 2x)$$
$$= (x + \delta x)^2 + 2x + 2\delta x - x^2 - 2x$$
$$= (x + \delta x)^2 + 2\delta x - x^2$$
$$= (2 - 0.2)^2 + 2(-0.2) - (2)^2$$
$$= (1.8)^2 - 0.4 - 4$$
$$= 3.24 - 4.4$$
$$\delta y = -1.16 \qquad \text{Ans.}$$
$$y = x^2 + 2x$$

Taking differential on both sides

$$dy = d(x^2) + 2dx$$
$$dy = 2xdx + 2dx$$
$$= 2(2)(-0.2) + 2(-0.2) \qquad (\because \delta x = dx)$$
$$= -0.8 - 0.4$$
$$dy = -1.2 \qquad \text{Ans}$$

(iii) $\mathbf{y = \sqrt{x}}$ **when x changes from 4 to 4.41**

$x = 4, \delta x = 4.41 - 4 = 0.41$

$$y + \delta y = \sqrt{x + \delta x}$$
$$\delta y = \sqrt{x + \delta x} - y$$
$$\delta y = \sqrt{x + \delta x} - \sqrt{x}$$
$$= \sqrt{4 + 0.41} - \sqrt{4}$$
$$= \sqrt{4.41} - 2$$
$$= 2.1 - 2 = 0.1 \qquad \text{Ans.}$$
$$y = \sqrt{x}$$

Taking differential on both sides

$$dy = d(\sqrt{x})$$

$$dy = \frac{1}{2}x^{-½}dx$$

$$dy = \frac{1}{2\sqrt{x}}dx$$

$$dy = \frac{0.41}{2\sqrt{4}} \qquad (\because \delta x = dx)$$

$$dy = \frac{0.41}{4} = 0.1025 \qquad \text{Ans.}$$

Q.2 Using differentials find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ in the following equations.

(i) $xy + x = 4$ (ii) $x^2 + 2y^2 = 16$ (iii) $x^4 + y^2 = xy^2$

(iv) $xy - \ln x = c$

Solution:

(i) $xy + x = 4$ (Guj. Board 2008)

Taking differential on both sides

$$d(xy) + dx = d(4)$$

$$xdy + ydx + dx = 0$$

$$xdy = -ydx - dx$$

$$dy = -\frac{(y+1)dx}{x}$$

$$\frac{dy}{dx} = -\frac{y+1}{x} \qquad \text{Ans.}$$

$$\frac{dx}{dy} = -\frac{x}{y+1} \qquad \text{Ans.}$$

(ii) $x^2 + 2y^2 = 16$ (Lhr. Board 2008)

Taking differential on both sides

$$d(x^2) + 2d(y^2) = d(16)$$

$$2xdx + 4ydy = 0$$

$$4y\,dy = -2x\,dx$$

$$\frac{dy}{dx} = \frac{-2x}{4y}$$

$$\frac{dy}{dx} = -\frac{x}{2y} \quad \text{Ans.}$$

$$\frac{dx}{dy} = -\frac{2y}{x} \quad \text{Ans.}$$

(iii) $\mathbf{x^4 + y^2 = xy^2}$

Taking differential on both sides

$$d(x^4) + d(y^2) = d(xy^2)$$

$$4x^3dx + 2ydy = xd(y^2) + y^2dx$$

$$4x^3dx + 2ydy = 2xy\,dy + y^2dx$$

$$2ydy - 2xydy = y^2dx - 4x^3dx$$

$$2y\,(1 - x)\,dy = (y^2 - 4x^3)\,dx$$

$$\frac{dy}{dx} = \frac{y^2 - 4x^3}{2y\,(1 - x)} \quad \text{Ans.}$$

$$\frac{dx}{dy} = \frac{2y\,(1 - x)}{y^2 - 4x^3}$$

$$\frac{dx}{dy} = \frac{2y\,(x - 1)}{4x^3 - y^2} \quad \text{Ans.}$$

(iv) $\mathbf{xy - \ln x = c}$

Taking differential on both sides

$$d(xy) - d(\ln x) = d\,(c)$$

$$xdy + ydx - \frac{1}{x}dx = 0$$

$$xdy = \frac{1}{x}\,dx - ydx$$

$$xdy = \left(\frac{1}{x} - y\right)dx$$

$$xdy = \left(\frac{1 - xy}{x}\right)dx$$

$$\frac{dy}{dx} = \frac{1 - xy}{x^2}$$

$$\frac{dx}{dy} = \frac{x^2}{1 - xy} \qquad \text{Ans.}$$

Q.3 Use differentials to approximate the values of

(i) $\sqrt[4]{17}$ **(ii)** $(31)^{1/5}$ **(iii)** $\cos 29^{o}$ **(iv)** $\sin 61^{o}$

Solution:

(i) $\sqrt[4]{17}$

Let

$$y = f(x) = \sqrt[4]{x} \text{ with } x = 16, \delta x = dx = 1$$

$$dy = f'(x)\,dx$$

$$= \frac{1}{4} x^{\frac{-3}{4}} dx$$

$$= \frac{dx}{4x^{3/4}} = \frac{1}{4(16)^{3/4}}$$

$$= \frac{1}{4(2^4)^{3/4}}$$

$$= \frac{1}{4(8)} = \frac{1}{32}$$

$$dy = 0.03125$$

$$f(x) = \sqrt[4]{x}$$

$$f(16) = \sqrt[4]{16}$$

$$= (2^4)^{1/4}$$

$$= 2$$

Using

$$f(x + \delta x) \approx f(x) + dy$$

$$f(16 + 1) \approx f(16) + dy$$

$$f(17) \approx 2 + 0.03125$$

$\sqrt[4]{17} \approx 2.03125$ Ans.

(ii) $(31)^{1/5}$

Let

$y = f(x) = x^{1/5}$ with $x = 32$, $\delta x = dx = -1$

$$dy = f'(x)\,dx$$
$$= \frac{1}{5}x^{-4/5}dx$$
$$= \frac{dx}{5x^{4/5}}$$
$$= \frac{-1}{5(32)^{4/5}}$$
$$= \frac{-1}{5(2^5)^{4/5}}$$
$$= \frac{-1}{5(16)}$$
$$= \frac{-1}{80}$$
$$dy = -0.0125$$
$$f(x) = x^{1/5}$$
$$f(32) = (32)^{1/5}$$
$$= (2^5)^{1/5}$$
$$= 2$$

Using $f(x + \delta x) \approx f(x) + dy$

$$f(32 - 1) \approx f(32) + dy$$
$$f(31) \approx 2 - 0.0125$$
$$(31)^{1/5} \approx 1.9875$$ Ans.

(iii) **cos 29º**

Let

$y = f(x) = \text{Cos}x$ with $x = 30^\circ$, $\delta x = dx = -1^\circ$

$$= -1 \times \frac{\pi}{180} = -0.0174$$

$$\begin{aligned}
dy &= f'(x)\,dx \\
&= -\sin x\,dx \\
&= -\sin 30^\circ \times (-0.0174) \\
dy &= 0.0087 \\
f(x) &= \cos x \\
f(30^\circ) &= \cos 30^\circ \\
&= 0.8660
\end{aligned}$$

Using

$$\begin{aligned}
f(x + \delta x) &\approx f(x) + dy \\
f(30^\circ - 1^\circ) &\approx f(30^\circ) + dy \\
f(29^\circ) &\approx 0.8660 + 0.0087 \\
\cos 29^\circ &\approx 0.8747 \quad \text{Ans.}
\end{aligned}$$

(iv) sin 61º

Let

$$y = f(x) = \sin x \text{ with } x = 60^\circ,\ dx = \delta x = 1^\circ$$

$$= 1 \times \frac{\pi}{180} = 0.0174$$

$$\begin{aligned}
dy &= f'(x)\,dx \\
&= \cos x\,dx \\
&= \cos 60^\circ \times 0.0174 \\
dy &= 0.0087 \\
f(x) &= \sin x \\
f(60^\circ) &= \sin 60^\circ \\
&= 0.8660
\end{aligned}$$

Using

$$\begin{aligned}
f(x + \delta x) &\approx f(x) + dy \\
f(60^\circ + 1^\circ) &\approx f(60^\circ) + dy
\end{aligned}$$

$f(61^o) \approx 0.8660 + 0.0087$

$\text{Sin}61^o \approx 0.8747$ Ans.

Q.4 Find the approximate increase in the volume of a cube if the length of its each edge changes from 5 to 5.02

Solution:

Let x be the each edge of cube. Since the length of each edge changes from 5 to 5.02

$\therefore \quad x = 5,\ dx = \delta x = 5.02 - 5$
$= 0.02$

Volume of cube $= V = x \times x \times x$
$V = x^3$
$dv = 3x^2\, dx$
$dv = 3(5)^2\,(0.02)$
$dv = 1.5$ cubic unit

Increase in volume = dv = 1.5 cubic unit

Q.5 Find the approximate increase in the area of a circular disc if its diameter is increased from 44cm to 44.4 cm

Solution:

Let r be the radius of circular disc. Since diameter increased from 44 cm to 44.4 cm so radius is

$\frac{44}{2}$ cm to $\frac{44.4}{2}$ cm

22cm to 22.2 cm

$r = 22\text{cm}, \quad dr = \delta r = 22.2 - 22$
$= 0.2$ cm

$dA = \pi\, d\,(r^2)$
$dA = 2\pi r dr$
$= 2\pi(22)\,(0.2)$
$= 27.467\ \text{cm}^2$

$\therefore$ Increase in area $= dA = 27.467\text{cm}^2$ Ans.

EXERCISE 3.2

Q.1 Evaluate the following indefinite integrals