

$$f(61^\circ) \approx 0.8660 + 0.0087$$

$$\sin 61^\circ \approx 0.8747 \quad \text{Ans.}$$

Q.4 Find the approximate increase in the volume of a cube if the length of its each edge changes from 5 to 5.02

Solution:

Let x be the each edge of cube. Since the length of each edge changes from 5 to 5.02

$$\begin{aligned} \therefore x &= 5, dx = \delta x = 5.02 - 5 \\ &= 0.02 \end{aligned}$$

$$\begin{aligned} \text{Volume of cube} &= V = x \times x \times x \\ &V = x^3 \end{aligned}$$

$$dv = 3x^2 dx$$

$$dv = 3(5)^2 (0.02)$$

$$dv = 1.5 \text{ cubic unit}$$

$$\text{Increase in volume} = dv = 1.5 \text{ cubic unit}$$

Q.5 Find the approximate increase in the area of a circular disc if its diameter is increased from 44cm to 44.4 cm

Solution:

Let r be the radius of circular disc. Since diameter increased from 44 cm to 44.4 cm so radius is

$$\frac{44}{2} \text{ cm to } \frac{44.4}{2} \text{ cm}$$

$$22 \text{ cm to } 22.2 \text{ cm}$$

$$\begin{aligned} r &= 22 \text{ cm}, \quad dr = \delta r = 22.2 - 22 \\ &= 0.2 \text{ cm} \end{aligned}$$

$$dA = \pi d(r^2)$$

$$dA = 2\pi r dr$$

$$= 2\pi(22)(0.2)$$

$$= 27.467 \text{ cm}^2$$

$$\therefore \text{Increase in area} = dA = 27.467 \text{ cm}^2 \quad \text{Ans.}$$

EXERCISE 3.2

Q.1 Evaluate the following indefinite integrals

(i) $\int (3x^2 - 2x + 1) dx$	(ii) $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx \quad (x > 0)$
(iii) $\int x (\sqrt{x} + 1) dx, \quad (x > 0)$	(iv) $\int (2x + 3)^{1/2} dx$
(v) $\int (\sqrt{x} + 1)^2 dx \quad (x > 0)$	(vi) $\int \left(\sqrt{x} \cdot \frac{1}{\sqrt{x}} \right)^2 dx \quad (x > 0)$
(vii) $\int \frac{3x+2}{\sqrt{x}} dx \quad (x > 0)$	(viii) $\int \frac{\sqrt{y}(y+1)}{y} dy, \quad (y > 0)$
(ix) $\int \frac{(\sqrt{\theta}-1)^2}{\sqrt{\theta}} d\theta \quad (\theta > 0)$	(x) $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx, \quad (x > 0)$
(xi) $\int \frac{e^{2x} + e^x}{e^x} dx$	

Solution:

$$(i) \int (3x^2 - 2x + 1) dx$$

$$\begin{aligned} &= 3 \int x^2 dx - 2 \int x dx + \int dx \\ &= \frac{3x^3}{3} - \frac{2x^2}{2} + x + c \\ &= x^3 - x^2 + x + c \end{aligned}$$



Ans.

$$(ii) \int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx \quad (x > 0)$$

$$\begin{aligned} &= \int \sqrt{x} dx + \int \frac{dx}{\sqrt{x}} \\ &= \int x^{1/2} dx + \int x^{-1/2} dx \\ &= \frac{\frac{3}{2}x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{\frac{1}{2}x^{\frac{1}{2}}}{\frac{1}{2}} + c \\ &= \frac{2}{3}x^{3/2} + 2\sqrt{x} + c \end{aligned}$$

Ans.

$$(iii) \int x (\sqrt{x} + 1) dx \quad (x > 0)$$

$$\begin{aligned} &= \int (x\sqrt{x} + x) dx \\ &= \int x \cdot x^{1/2} dx + \int x dx \end{aligned}$$

$$\begin{aligned}
 &= \int x^{\frac{3}{2}} dx + \int x dx \\
 &= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^2}{2} + c \\
 &= \frac{2}{5} x^{\frac{5}{2}} + \frac{x^2}{2} + c \quad \text{Ans.}
 \end{aligned}$$

(iv) $\int (2x + 3)^{\frac{1}{2}} dx$

$$\begin{aligned}
 &= \frac{1}{2} \int (2x + 3)^{\frac{1}{2}} 2dx \\
 &= \frac{1}{2} \cdot \frac{(2x + 3)^{\frac{3}{2}}}{\frac{3}{2}} + c \\
 &= \frac{(2x + 3)^{\frac{3}{2}}}{3} + c
 \end{aligned}$$

(v) $\int (\sqrt{x} + 1)^2 dx \quad (x > 0)$

$$\begin{aligned}
 &= \int [(\sqrt{x})^2 + 2(\sqrt{x})(1) + (1)^2] dx \\
 &= \int x dx + 2 \int x^{\frac{1}{2}} dx + \int dx \\
 &= \frac{x^2}{2} + \frac{2x^{\frac{3}{2}}}{\frac{3}{2}} + x + c \\
 &= \frac{x^2}{2} + \frac{2}{3} x^{\frac{3}{2}} + x + c \quad \text{Ans.}
 \end{aligned}$$

(vi) $\int \left(\sqrt{x} - \frac{1}{x}\right)^2 dx \quad (x > 0)$

$$\begin{aligned}
 &= \int \left[(\sqrt{x})^2 - 2(\sqrt{x})\left(\frac{1}{\sqrt{x}}\right) + \left(\frac{1}{\sqrt{x}}\right)^2 \right] dx \\
 &= \int (x - 2 + \frac{1}{x}) dx \\
 &= \int x dx - 2 \int dx + \int \frac{dx}{x}
 \end{aligned}$$

$$= \frac{x^2}{2} - 2x + \ell \ln x + c \quad \text{Ans.}$$

(vii) $\int \frac{3x+2}{\sqrt{x}} dx \quad (x > 0)$

$$\begin{aligned} &= \int \left(\frac{3x}{\sqrt{x}} + \frac{2}{\sqrt{x}} \right) dx \\ &= \int (3x^{1-\frac{1}{2}} + 2x^{-\frac{1}{2}}) dx \\ &= 3 \int x^{\frac{1}{2}} dx + 2 \int x^{-\frac{1}{2}} dx \end{aligned}$$

$$= \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$\begin{aligned} &= 2x^{\frac{3}{2}} + 4\sqrt{x} + c \quad \text{Ans.} \\ (\text{viii}) \quad &\int \frac{\sqrt{y}(y+1)}{y} (y > 0) \\ &= \int \frac{\sqrt{y}(y+1)}{\sqrt{y} \cdot \sqrt{y}} dy \\ &= \int \left(\frac{y}{\sqrt{y}} + \frac{1}{\sqrt{y}} \right) dy \\ &= \int (y^{1-\frac{1}{2}} + y^{-\frac{1}{2}}) dy \\ &= \int y^{\frac{1}{2}} dy + \int y^{-\frac{1}{2}} dy \\ &= \frac{y^{\frac{3}{2}}}{\frac{3}{2}} + \frac{y^{\frac{1}{2}}}{\frac{1}{2}} + c \\ &= \frac{2}{3}y^{\frac{3}{2}} + 2y^{\frac{1}{2}} + c \quad \text{Ans.} \end{aligned}$$

(ix) $\int \frac{(\sqrt{\theta}-1)^2}{\sqrt{\theta}} d\theta \quad (\theta > 0) \quad (\text{Guj. Board 2006})$



$$\begin{aligned}
 &= \int \frac{(\sqrt{\theta})^2 - 2(\sqrt{\theta})(1) + (1)^2}{\sqrt{\theta}} d\theta \\
 &= \int \frac{\theta - 2\sqrt{\theta} + 1}{\sqrt{\theta}} d\theta \\
 &= \int \left(\frac{\theta}{\sqrt{\theta}} - \frac{2\sqrt{\theta}}{\sqrt{\theta}} + \frac{1}{\sqrt{\theta}} \right) d\theta \\
 &= \int (\theta^{1-\frac{1}{2}} - 2 + \theta^{-\frac{1}{2}}) d\theta \\
 &= \int \theta^{\frac{1}{2}} d\theta - 2 \int d\theta + \int \theta^{-\frac{1}{2}} d\theta \\
 &= \frac{\theta^{\frac{3}{2}}}{\frac{3}{2}} - 2\theta + \frac{\theta^{\frac{1}{2}}}{\frac{1}{2}} + c \\
 &= \frac{2}{3}\theta^{\frac{3}{2}} - 2\theta + 2\theta^{\frac{1}{2}} + c
 \end{aligned}$$

Ans.

(x) $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx \quad (x > 0) \quad (\text{Lhr. Board 2006})$

$$\begin{aligned}
 &= \int \frac{(1)^2 - 2(1)(\sqrt{x}) + (\sqrt{x})^2}{\sqrt{x}} dx \\
 &= \int \frac{1 - 2\sqrt{x} + x}{\sqrt{x}} dx \\
 &= \int \left(\frac{1}{\sqrt{x}} - \frac{2\sqrt{x}}{\sqrt{x}} + \frac{x}{\sqrt{x}} \right) dx \\
 &= \int (x^{-\frac{1}{2}} - 2 + x^{1-\frac{1}{2}}) dx \\
 &= \int x^{\frac{-1}{2}} dx - 2 \int dx + \int x^{\frac{1}{2}} dx \\
 &= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - 2x + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c \\
 &= 2x^{\frac{1}{2}} - 2x + \frac{2}{3}x^{\frac{3}{2}} + c \quad \text{Ans.}
 \end{aligned}$$

(xi) $\int \frac{e^{2x} + e^x}{e^x} dx$

$$\begin{aligned}
 &= \int \frac{e^x(e^x + 1)}{e^x} dx \\
 &= \int (e^x + 1) dx \\
 &= \int e^x dx + \int dx \\
 &= e^x + x + c \quad \text{Ans.}
 \end{aligned}$$

Q.2 Evaluate

(i) $\int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}} \quad (x+a>0, x+b>0)$

(iii) $\int \frac{dx}{\sqrt{x+a} + \sqrt{x}} \quad (x>0, a>0)$

(v) $\int \frac{(1+e^x)^3}{e^x} dx$

(vii) $\int \sqrt{1-\cos 2x} dx \quad (1-\cos 2x>0)$

(ix) $\int \sin^2 x dx$

(xi) $\int \frac{ax+b}{ax^2+2bx+c} dx$

(xiii) $\int \frac{\cos 2x - 1}{1 + \cos 2x} dx, \quad (1 + \cos 2x \neq 0)$

(ii) $\int \frac{1-x^2}{1+x^2} dx$

(iv) $\int (a-2x)^{\frac{3}{2}} dx$

(vi) $\int \sin(a+b)x dx$

(viii) $\int (\ln x) \times \frac{1}{x} dx \quad (x>0)$

(x) $\int \frac{1}{1+\cos x} dx \quad \left(\frac{-\pi}{2} < x < \pi/2\right)$

(xii) $\int \cos 3x \sin 2x dx$

(xiv) $\int \tan^2 x dx \quad (\text{Lhr. Board 2011})$

Solution:

$$\begin{aligned}
 \text{(i)} \quad & \int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}} \quad (x+a>0, x+b>0) \\
 &= \int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}} \times \frac{\sqrt{x+a} - \sqrt{x+b}}{\sqrt{x+a} - \sqrt{x+b}} \\
 &= \int \frac{\sqrt{x+a} - \sqrt{x+b}}{\sqrt{(x+a)^2} - \sqrt{(x+b)^2}} dx \\
 &= \int \frac{\sqrt{x+a} - \sqrt{x+b}}{x+a - (x+b)} dx \\
 &= \int \frac{(x+a)^{\frac{1}{2}} - (x+b)^{\frac{1}{2}}}{x+a - x-b} dx
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{a-b} \int [(x+a)^{1/2} - (x-b)^{1/2}] dx \\
 &= \frac{1}{a-b} \left[\int (x+a)^{1/2} dx - \int (x-b)^{1/2} dx \right] \\
 &= \frac{1}{a-b} \left[\frac{(x+a)^{3/2}}{\frac{3}{2}} - \frac{(x-b)^{3/2}}{\frac{3}{2}} \right] + c \\
 &= \frac{2}{3(a-b)} [(x+b)^{3/2} - (x-b)^{3/2}] + c \quad \text{Ans.}
 \end{aligned}$$

(ii) $\int \frac{1-x^2}{1+x^2} dx$ (Lhr. Board 2008)

$$\begin{aligned}
 &= \int \left(-1 + \frac{2}{1+x^2} \right) dx \\
 &= - \int dx + 2 \int \frac{dx}{1+x^2} \\
 &= -x + 2 \tan^{-1} x + c \quad \text{Ans.}
 \end{aligned}$$

(iii) $\int \frac{dx}{\sqrt{x+a} + \sqrt{x}}$ ($x > 0, a > 0$)

$$\begin{aligned}
 &= \int \frac{dx}{\sqrt{x+a} + \sqrt{x}} \times \frac{\sqrt{x+a} - \sqrt{x}}{\sqrt{x+a} - \sqrt{x}} \\
 &= \int \frac{\sqrt{x+a} - \sqrt{x}}{(\sqrt{x+a})^2 - (\sqrt{x})^2} dx \\
 &= \int \frac{(x+a)^{1/2} - x^{1/2}}{x+a-x} dx \\
 &= \frac{1}{a} \int [(x+a)^{1/2} - x^{1/2}] dx \\
 &= \frac{1}{a} \left[\int (x+a)^{1/2} dx - \int x^{1/2} dx \right]
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{a} \left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right] + c \\
 &= \frac{2}{3a} [(x+a)^{\frac{3}{2}} - x^{\frac{3}{2}}] + c \quad \text{Ans.}
 \end{aligned}$$

(iv) $\int (a - 2x)^{3/2} dx$

$$\begin{aligned}
 &= \frac{-1}{2} \int (a - 2x)^{3/2} - 2 dx \\
 &= \frac{-1}{2} \frac{(a - 2x)^{5/2}}{5/2} + c \\
 &= \frac{-1}{5} (a - 2x)^{5/2} + c \quad \text{Ans.}
 \end{aligned}$$

(v) $\int \frac{(1+e^x)^3}{e^x} dx$

$$\begin{aligned}
 &= \int \frac{(1)^3 + 3(1)^2(e^x) + 3(1)(e^x)^2 + (e^x)^3}{e^x} dx \\
 &= \int \frac{1 + 3e^x + 3e^{2x} + e^{3x}}{e^x} dx \\
 &= \int \left(\frac{1}{e^x} + \frac{3e^x}{e^x} + \frac{3e^{2x}}{e^x} + \frac{e^{3x}}{e^x} \right) dx \\
 &= \int (e^{-x} + 3 + 3e^x + e^{2x}) dx \\
 &= - \int e^{-x} - dx + 3 \int dx + 3 \int e^x dx + \frac{1}{2} \int e^{2x} \cdot 2dx + \frac{1}{2} \int e^{2x} \cdot 2dx \\
 &= -e^{-x} + 3x + 3e^x + \frac{1}{2} e^{2x} + c \quad \text{Ans.}
 \end{aligned}$$

(vi) $\int \sin(a+b)x dx$

$$= \frac{1}{a+b} \int \sin(a+b)x \cdot (a+b) dx$$

$$\begin{aligned}
 &= \frac{1}{a+b} \cdot -\cos(a+b)x + c \\
 &= -\frac{1}{a+b} \cos(a+b)x + c \quad \text{Ans.}
 \end{aligned}$$

(vii) $\int \sqrt{1 - \cos 2x} dx \quad (1 - \cos 2x > 0) \quad (\text{Lhr. Board 2008, 2009})$

$$\begin{aligned}
 &= \int \sqrt{2\sin^2 x} \quad \left(\because \cos 2x = 1 - 2\sin^2 x \right. \\
 &\quad \left. 2\sin^2 x = 1 - \cos 2x \right) \\
 &= \sqrt{2} \int \sin x dx \\
 &= \sqrt{2} (-\cos x) + c \\
 &= -\sqrt{2} \cos x + c \quad \text{Ans.}
 \end{aligned}$$

(viii) $\int (\ln x) \times \frac{1}{x} dx \quad (x > 0)$

$$= \frac{(\ln x)^2}{2} + c$$

(ix) $\int \sin^2 x dx$

$$\begin{aligned}
 &= \int \frac{1 - \cos 2x}{2} dx \quad \left(\because \cos 2x = 1 - 2\sin^2 x \right. \\
 &\quad \left. 2\sin^2 x = 1 - \cos 2x \right. \\
 &\quad \left. \sin^2 x = \frac{1 - \cos 2x}{2} \right)
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} \int (1 - \cos 2x) dx \\
 &= \frac{1}{2} \int dx - \frac{1}{2} \int \cos 2x \cdot 2 dx \\
 &= \frac{1}{2} x - \frac{1}{4} \sin 2x + c \quad \text{Ans.}
 \end{aligned}$$

(x) $\int \frac{1}{1 + \cos x} dx \quad \left(\frac{-\pi}{2} < x < \frac{\pi}{2} \right)$



$$\begin{aligned} &= \int \frac{1}{2\cos^2 \frac{x}{2}} dx \quad \left(\begin{array}{l} \because \cos 2x = 2 \cos^2 x - 1 \\ 2\cos^2 x = 1 + \cos 2x \\ 2\cos^2 \frac{x}{2} = 1 + \cos x \end{array} \right) \\ &= \int \sec^2 \frac{x}{2} \cdot \frac{1}{2} dx \\ &= \tan \frac{x}{2} + c \quad \text{Ans.} \end{aligned}$$



$$\begin{aligned}
 \text{(xi)} \quad & \int \frac{ax + b}{ax^2 + 2bx + c} dx \\
 &= \frac{1}{2} \int \frac{2ax + 2b}{ax^2 + 2bx + c} dx \\
 &= \frac{1}{2} \ln |(ax^2 + 2bx + c)| + c_1 \quad \text{Ans.}
 \end{aligned}$$

$$\begin{aligned}
 \text{(xii)} \quad & \int \cos 3x \sin 2x dx \\
 &= \frac{1}{2} \int 2\cos 3x \sin 2x dx \\
 &= \frac{1}{2} \int [\sin(3x + 2x) - \sin(3x - 2x)] dx
 \end{aligned}$$

$$\therefore 2\cos \alpha \sin \beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$$

$$\begin{aligned}
 &= \frac{1}{2} \int (\sin 5x - \sin x) dx \\
 &= \frac{1}{2} \left[\int \sin 5x dx - \int \sin x dx \right] \\
 &= \frac{1}{2} \left[\frac{1}{5} \int \sin 5x . 5dx - \int \sin x dx \right] \\
 &= \frac{1}{2} \left(\frac{-\cos 5x}{5} + \cos x \right) + c \\
 &= \frac{-1}{2} \left(\frac{\cos 5x}{5} - \cos x \right) + c \quad \text{Ans.}
 \end{aligned}$$

$$\text{(xiii)} \quad \int \frac{\cos 2x - 1}{1 + \cos 2x} dx, \quad (1 + \cos 2x \neq 0)$$

$$\begin{aligned}
 &= - \int \frac{1 - \cos 2x}{1 + \cos 2x} dx \\
 &= - \int \frac{2\sin^2 x}{2\cos^2 x} dx \\
 &= - \int \tan^2 x dx
 \end{aligned}$$

$$\begin{aligned}
 &= - \int (\sec^2 x - 1) dx \\
 &= - \int \sec^2 x dx + \int dx \\
 &= x - \tan x + c \quad \text{Ans.}
 \end{aligned}$$

(xiv) $\int \tan^2 x dx$ (Guj. Board 2005, 2007) (Lhr. Board 2011)

$$\begin{aligned}
 &= \int (\sec^2 x - 1) dx \\
 &= \int \sec^2 x dx - \int dx \\
 &= \tan x - x + c \quad \text{Ans.}
 \end{aligned}$$

EXERCISE 3.3

Evaluate the following integrals.

Q.1 $\int \frac{-2x}{\sqrt{4-x^2}} dx$



Solution:

$$\begin{aligned}
 &\int \frac{-2x}{\sqrt{4-x^2}} dx \\
 &= \int (4-x^2)^{-1/2} - 2x dx \\
 &= \frac{(4-x^2)^{1/2}}{\frac{1}{2}} + c \\
 &= 2\sqrt{4-x^2} + c \quad \text{Ans.}
 \end{aligned}$$

Q.2 $\int \frac{dx}{x^2 + 4x + 13}$

Solution:

$$\int \frac{dx}{x^2 + 4x + 13}$$