$$\boxed{A = 1}$$

To find B

Put $2 + t = 0$

$t = -2$ in equation (2)

$$1 = B(1-2)$$

$$-B = 1$$

$$\boxed{B = -1}$$

$\therefore$ From equation (1)

$$\frac{1}{(1+t)(2+t)} = \frac{1}{1+t} + \frac{-1}{2+t}$$

Integrate from 0 to 1

$$\int_0^1 \frac{dt}{(1+t)(2+t)} = \int_0^1 \frac{dt}{1+t} - \int_0^1 \frac{dt}{2+t}$$

$$= [ln|1+t|]_0^1 - [ln|2+t|]_0^1$$

$$= (ln2 - ln1) - (ln\,3 - ln2)$$

$$= ln\,2 - ln3 + ln2$$

$$= ln\frac{2\times 2}{3} = ln\frac{4}{3} \quad \text{Ans}$$

EXERCISE 3.7

Q.1 Find the area between the x-axis and the curve $y = x^2 + 1$ from $x = 1$ to $x = 2$ (Lhr. Board 2005, 2008)

Solution:

$y = x^2 + 1$ from $x = 1$ to $x = 2$

$$\text{Required area} = \int_a^b y\,dx$$

$$= \int_1^2 (x^2+1)dx$$

$$= \int_1^2 x^2\,dx + \int_1^2 dx$$

$$= \left[\frac{x^3}{3}\right]_1^2 + [x]_1^2$$

$$= \frac{1}{3}(8-1) + (2-1) = \frac{7}{3} + 1 = \frac{7+3}{3} = \frac{10}{3} \text{ sq. units} \qquad \text{Ans.}$$

Q.2 Find the area, above the x-axis and under the curve $y = 5 - x^2$ from $x=-1$ to $x = 2$. (Lhr. Board 2011)

Solution:

$$y = 5 - x^2 \quad \text{from} \quad x = -1 \quad \text{to} \quad x = 2$$

$$\text{Required area} = \int_a^b y\,dx$$

$$= \int_{-1}^{2} (5 - x^2)\,dx$$

$$= 5\int_{-1}^{2} dx - \int_{-1}^{2} x^2\,dx$$

$$= 5[x]_{-1}^{2} - \left[\frac{x^3}{3}\right]_{-1}^{2}$$

$$= 5(2+1) - \frac{1}{3}(8+1) = 5(3) - \frac{1}{3}(9) = 15 - 3 = 12 \text{ sq. units}$$

Q.3 Find the area below the curve $y = 3\sqrt{x}$ and above the x-axis between $x = 1$ and $x = 4$.

Solution:

$$y = 3\sqrt{x}$$

$$\text{Required area} = \int_a^b y\,dx$$

$$= \int_1^4 3\sqrt{x}\,dx$$

$$= 3\int_1^4 x^{½}\,dx$$

$$= 3\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$$

$$= 2\left[4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right]$$

$$= 2\left[(2^2)^{\frac{3}{2}} - 1\right] = 2(8-1) = 2(7) = 14 \text{ Sq. units}$$

Q.4 Find the area bounded by cos function from $x = \frac{-\pi}{2}$ to $x = \frac{\pi}{2}$. (Guj. Board 2008)

Solution:

$$y = \cos x$$

$$\text{Required area} = \int_a^b y\,dx = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx$$

$$= [\sin x]_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$$

$$= \sin\frac{\pi}{2} - \sin\left(\frac{-\pi}{2}\right)$$

$$= 1 + 1$$

$$= 2 \text{ Sq. units} \qquad \text{Ans.}$$

Q.5 Find the area between the x-axis and the curve $y = 4x - x^2$
(Lhr. Board 2009, Guj Board 2005, 2008)

Solution:

$$y = 4x - x^2$$

To find the limits

Put $y = 0$

$$4x - x^2 = 0$$

$$x(4 - x) = 0$$

Either

$$x = 0 \quad \text{or} \quad 4 - x = 0$$

$$x = 4$$

The curve cuts the x-axis at (0, 0) and (4, 0)

$$y \geq 0 \quad \text{for} \quad 0 \leq x \leq 4$$

That is, the area in the interval [0, 4] is above the x-axis.

$$\text{Required Area} = \int_a^b y\,dx$$

$$= \int_0^4 (4x - x^2)\,dx$$

$$= 4\int_0^4 x\,dx - \int_0^4 x^2\,dx$$

$$= 4\left[\frac{x^2}{2}\right]_0^4 - \left[\frac{x^3}{3}\right]_0^4$$

$$= 2\,(16-0) - \frac{1}{3}\,(64-0)$$

$$= 32 - \frac{64}{3} \quad = \frac{96-64}{3}$$

$$= \frac{32}{3} \text{ Sq. units} \qquad \text{Ans.}$$

Q.6 Determine the area bounded by the parabola $y = x^2 + 2x - 3$ and the x-axis

Solution:

$y = x^2 + 2x - 3$

To find the limits

Put

$y = 0$

$x^2 + 2x - 3 = 0$

$x^2 + 3x - x - 3 = 0$

$x(x+3) - 1(x+3) = 0$

$(x+3)(x-1) = 0$

Either

$x + 3 = 0$ or $x - 1 = 0$

$x = -3$ $\quad$ $x = 1$

The curve cuts the x-axis at $(-3, 0)$ and $(1, 0)$

$y \le 0$ for $-3 \le x \le 1$

That is, the area in the interval $[-3, 1]$ is below the x-axis

$$\text{Required Area} = -\int_a^b y\,dx$$

$$= -\int_{-3}^{1} (x^2 + 2x - 3)\,dx$$

$$= -\int_{-3}^{1} x^2\,dx - 2\int_{-3}^{1} x\,dx + 3\int_{-3}^{1} dx$$

$$= -\left[\frac{x^3}{3}\right]_{-3}^{1} - 2\left[\frac{x^2}{2}\right]_{-3}^{1} + 3\,[x]_{-3}^{1}$$

$$= \frac{-1}{3}\,(1+27) - (1-9) + 3\,(1+3)$$

$$= \frac{-28}{3} - (-8) + 3(4)$$

$$= \frac{-28}{3} + 8 + 12 \qquad = \frac{-28}{3} + 20$$

$$= \frac{-28 + 60}{3} \qquad = \frac{32}{3} \text{ Sq. units} \qquad \text{Ans.}$$

Q.7 Find the area bounded by the curve $y = x^3 + 1$, the x-axis and line $x = 2$.

Solution:

$y = x^3 + 1$

To find the limits

Put

$y = 0$

$x^3 + 1 = 0$

$(x)^3 + (1)^3 = 0$

$(x + 1)(x^2 - x + 1) = 0$

Either

$x + 1 = 0$ or $x^2 - x + 1 = 0$

$x = -1$ Neglecting because it has imaginary roots,

$$\text{Required Area} = \int_a^b y\,dx$$

$$= \int_{-1}^{2} (x^3 + 1)\,dx$$

$$= \int_{-1}^{2} x^3 dx + \int_{-1}^{2} dx$$

$$= \left[\frac{x^4}{4}\right]_{-1}^{2} + [x]_{-1}^{2}$$

$$= \frac{1}{4}(16 - 1) + (2 + 1)$$

$$= \frac{15}{4} + 3$$

$$= \frac{15 + 12}{4} = \frac{27}{4} \text{ Sq. units} \qquad \text{Ans.}$$

Q.8 Find the area bounded by the curve $y = x^3 - 4x$ and the x-axis.

Solution:

$y = x^3 - 4x$

To find the limits

Put

$y = 0$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x(x + 2)(x - 2) = 0$

Either

$x = 0$ or $x + 2 = 0$ or $x - 2 = 0$

$x = -2$ $x = 2$

The curve cuts the x-axis at $(-2, 0)$, $(0, 0)$ and $(2, 0)$

$y \geq 0$ for $-2 \leq x \leq 0$

That is, the area in the interval $[-2, 0]$ is above the x-axis.

$y \leq 0$ for $0 \leq x \leq 2$

That is, the area in the interval $[0, 2]$ lies below the x-axis

$$\text{Required Area} = \int_{-2}^{0} y\,dx - \int_{0}^{2} y\,dx$$

$$= \int_{-2}^{0} (x^3 - 4x)\,dx - \int_{0}^{2} (x^3 - 4x)\,dx$$

$$= \int_{-2}^{0} x^3\,dx - 4\int_{-2}^{0} x\,dx - \int_{0}^{2} x^3\,dx + 4\int_{0}^{2} x\,dx$$

$$= \left[\frac{x^4}{4}\right]_{-2}^{0} - 4\left[\frac{x^2}{2}\right]_{-2}^{0} - \left[\frac{x^4}{4}\right]_{0}^{2} + 4\left[\frac{x^2}{2}\right]_{0}^{2}$$

$$= \frac{1}{4}(0 - 16) - 2(0 - 4) - \frac{1}{4}(16 - 0) + 2(4 - 0)$$

$$= \frac{-16}{4} - 2(-4) - \frac{1}{4}(16) + 8$$

$$= -4 + 8 - 4 + 8$$

$$= 8 \text{ Sq. units Ans.}$$

Q.9 Find the area between the curve $y = x(x - 1)(x + 1)$ and the x-axis.

Solution:

$y = x(x - 1)(x + 1)$

To find the limits

Put

$y = 0$

$x(x - 1)(x + 1) = 0$

Either

$$x = 0 \quad \text{or} \quad x - 1 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = 1 \qquad\qquad x = -1$$

The curve cuts the x-axis at (-1,0), (0, 0) and (1, 0)

$$y \geq 0 \quad \text{for} \quad -1 \leq x \leq 0$$

That is, the area in the interval [-1,0] lies above the x-axis.

$$y \leq 0 \quad \text{for} \quad 0 \leq x \leq 1$$

That is, the area in the interval [0, 1] lies below the x-axis.

$$\text{Required Area} = \int_{-1}^{0} y\,dx - \int_{0}^{1} y\,dx$$
$$= \int_{-1}^{0} x(x-1)(x+1)\,dx - \int_{0}^{1} x(x-1)(x+1)\,dx$$
$$= \int_{-1}^{0} x(x^2-1)\,dx - \int_{0}^{1} x(x^2-1)\,dx$$
$$= \int_{-1}^{0} (x^3-x)\,dx - \int_{0}^{1} (x^3-x)\,dx$$
$$= \int_{-1}^{0} x^3\,dx - \int_{-1}^{0} x\,dx - \int_{0}^{1} x^3\,dx + \int_{0}^{1} x\,dx$$
$$= \left[\frac{x^4}{4}\right]_{-1}^{0} - \left[\frac{x^2}{2}\right]_{-1}^{0} - \left[\frac{x^4}{4}\right]_{0}^{1} + \left[\frac{x^2}{2}\right]_{0}^{1}$$
$$= \frac{1}{4}(0-1) - \frac{1}{2}(0-1) - \frac{1}{4}(1-0) + \frac{1}{2}(1-0)$$
$$= \frac{-1}{4} + \frac{1}{2} - \frac{1}{4} + \frac{1}{2}$$
$$= \frac{-1+2-1+2}{4} = \frac{2}{4} = \frac{1}{2}\text{ Sq. units} \qquad \text{Ans.}$$

Q.10 Find the area above the x-axis bounded by the curve $y^2 = 3 - x$ from $x = -1$ to $x=2$.

Solution:

$$y^2 = 3 - x \quad \text{from} \quad x = -1 \text{ to } x = 2$$
$$y = \sqrt{3 - x}$$

$$\text{Required Area} = \int_{a}^{b} y\,dx$$

$$= \int_{-1}^{2} \sqrt{3-x}\, dx \qquad = -\int_{-1}^{2} (3-x)^{½} \,.\, -dx$$

$$= -\left[\frac{(3-x)^{\frac{3}{2}}}{\frac{3}{2}}\right]_{-1}^{2}$$

$$= \frac{-2}{3}\left[(3-2)^{\frac{3}{2}} - (3+1)^{\frac{3}{2}}\right]$$

$$= \frac{-2}{3}\left[(1)^{\frac{3}{2}} - (4)^{\frac{3}{2}}\right]$$

$$= \frac{-2}{3}\left[1 - (2^2)^{\frac{3}{2}}\right]$$

$$= \frac{-2}{3}\ (1-8)$$

$$= \frac{-2}{3}(-7) = \frac{14}{3} \text{ Sq. units} \qquad \text{Ans.}$$

Q.11 Find the area between the x-axis and the curve $y = \cos\frac{1}{2}x$ from $x = -\pi$ to π.

Solution:

$$y = \cos\frac{1}{2}x \qquad \text{from} \quad x = -\pi \text{ to } \quad x = \pi$$

$$\text{Required Area} = \int_{a}^{b} y dx$$

$$= \int_{-\pi}^{\pi} \cos\frac{1}{2}x\, dx$$

$$= \left[\frac{\sin\frac{x}{2}}{\frac{1}{2}}\right]_{-\pi}^{\pi}$$

$$= 2\left[\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)\right]$$

$$= 2\,(1+1)$$

$$= 2\,(2)$$

$$= 4 \text{ Sq. units} \qquad \text{Ans.}$$

Q.12 Find the area between the x-axis and the curve y = sin2x from x = 0 to x = $\boldsymbol{\frac{\pi}{3}}$.

Solution:

$y = \sin 2x$ from $x = 0$ to $x = \frac{\pi}{3}$

$$\text{Required Area} = \int_a^b y\,dx$$

$$= \int_0^{\frac{\pi}{3}} \text{Sin } 2x\,dx$$

$$= \left[\frac{-\cos 2x}{2}\right]_0^{\frac{\pi}{3}}$$

$$= \frac{-1}{2}\left[\cos 2\left(\frac{\pi}{3}\right) - \cos 2(0)\right]$$

$$= \frac{-1}{2}\left[\frac{-1}{2} - 1\right]$$

$$= \frac{-1}{2}\left(\frac{-1-2}{2}\right)$$

$$= \frac{-1}{2}\left(\frac{-3}{2}\right)$$

$$= \frac{3}{4} \text{ Sq. units} \qquad \text{Ans.}$$

Q.13 Find the area between the x-axis and the curve y = $\boldsymbol{\sqrt{2ax - x^2}}$ when a > 0.

Solution:

$$y = \sqrt{2ax - x^2}$$

To find the limits

Put

$$y = 0$$

$$\sqrt{2ax - x^2} = 0$$

$$2ax - x^2 = 0$$

$$x(2a - x) = 0$$

Either

$x = 0$ or $2a - x = 0$

$x = 2a$

$$\text{Required Area} = \int_a^b y\,dx$$

$$= \int_0^{2a} \sqrt{2ax - x^2}\; dx$$

$$= \int_0^{2a} \sqrt{a^2 - a^2 + 2ax - x^2}\; dx$$

$$= \int_0^{2a} \sqrt{a^2 - (a^2 - 2ax + x^2)}\; dx$$

$$= \int_0^{2a} \sqrt{a^2 - (a - x)^2}\; dx$$

Put $a - x = a\sin\theta$

$-dx = a\cos\theta\, d\theta$

$dx = -a\cos\theta\, d\theta$

When $x = 0$, $a - 0 = a\sin\theta$

$\sin\theta = \dfrac{a}{a} = 1$

$\theta = \dfrac{\pi}{2}$

When $x = 2a$, $a - 2a = a\sin\theta$

$-a = a\sin\theta$

$\sin\theta = \dfrac{-a}{a} = -1$

$\theta = \dfrac{-\pi}{2}$

$$= \int_{\frac{\pi}{2}}^{\frac{-\pi}{2}} \sqrt{a^2 - a^2\sin^2\theta}\; (-a\cos\theta)\, d\theta$$

$$= -a\int_{\frac{\pi}{2}}^{\frac{-\pi}{2}} \sqrt{a^2(1 - \sin^2\theta)}\; \cos\theta\, d\theta$$

$$= a\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} a\sqrt{\cos^2\theta}\; .\; \cos\theta d\theta$$

$$= a^2\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos\theta \,.\, \cos\theta \; d\theta$$

$$= a^2 \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta \; d\theta$$

$$= a^2 \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1 + \cos 2\theta}{2}\right) d\theta$$

$$= \frac{a^2}{2} \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (1 + \cos 2\theta) \; d\theta$$

$$= \frac{a^2}{2} \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} d\theta + \frac{a^2}{2} \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos 2\theta \; d\theta$$

$$= \frac{a^2}{2} [\theta]_{\frac{-\pi}{2}}^{\frac{\pi}{2}} + \frac{a^2}{2} \left[\frac{\sin 2\theta}{2}\right]_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{a^2}{2}\left[\frac{\pi}{2} + \frac{\pi}{2}\right] + \frac{a^2}{4}\left[\sin 2\left(\frac{\pi}{2}\right) - \sin 2\left(\frac{-\pi}{2}\right)\right]$$

$$= \frac{a^2}{2}\left(\frac{\pi + \pi}{2}\right) + \frac{a^2}{4}(0 + 0)$$

$$= \frac{a^2}{2}\left(\frac{2\pi}{2}\right)$$

$$= \frac{a^2\pi}{2} \text{ Sq. units} \qquad \text{Ans.}$$

EXERCISE 3.8

Q.1 **Check that each of the following equations written against the differential equation in its solution.**

(i) $x\dfrac{dy}{dx} = 1 + y \qquad\qquad y = cx - 1$