

$$\begin{aligned}
 &= a^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta \cdot \cos\theta d\theta \\
 &= a^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta d\theta \\
 &= a^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1 + \cos 2\theta}{2} \right) d\theta \\
 &= \frac{a^2}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + \cos 2\theta) d\theta \\
 &= \frac{a^2}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta + \frac{a^2}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos 2\theta d\theta \\
 &= \frac{a^2}{2} [\theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{a^2}{2} \left[\frac{\sin 2\theta}{2} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
 &= \frac{a^2}{2} \left[\frac{\pi}{2} + \frac{\pi}{2} \right] + \frac{a^2}{4} \left[\sin 2\left(\frac{\pi}{2}\right) - \sin 2\left(-\frac{\pi}{2}\right) \right] \\
 &= \frac{a^2}{2} \left(\frac{\pi + \pi}{2} \right) + \frac{a^2}{4} (0 + 0) \\
 &= \frac{a^2}{2} \left(\frac{2\pi}{2} \right) \\
 &= \frac{a^2\pi}{2} \text{ Sq. units} \quad \text{Ans.}
 \end{aligned}$$

EXERCISE 3.8

Q.1 Check that each of the following equations written against the differential equation in its solution.

(i) $x \frac{dy}{dx} = 1 + y$ $y = cx - 1$

$$(ii) \quad x^2(2y+1)\frac{dy}{dx} - 1 = 0 \quad y^2 + y = c - \frac{1}{x}$$

$$(iii) \quad y\frac{dy}{dx} - e^{2x} = 1 \quad y^2 = e^{2x} + 2x + c$$

$$(iv) \quad \frac{1}{x}\frac{dy}{dx} - 2y = 0 \quad y = ce^{x^2}$$

$$(v) \quad \frac{dy}{dx} = \frac{y^2 + 1}{e^{-x}} \quad y = \tan(e^x + c)$$

Solution:

$$(i) \quad x\frac{dy}{dx} = 1 + y \quad \text{---} \quad (1) \quad (\text{Lhr. Board 2007})$$

$$y = cx - 1$$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = c$$

Put in equation (1)

$$xc = 1 + e^x - 1$$

$$xc = cx$$

Which is true

$$\therefore y = cx - 1 \text{ is the solution of } x\frac{dy}{dx} = 1 + y$$

$$(ii) \quad x^2(2y+1)\frac{dy}{dx} - 1 = 0 \quad \text{---} \quad (1)$$

$$y^2 + y = c - \frac{1}{x}$$

Diff. w.r.t. 'x'

$$2y\frac{dy}{dx} + \frac{dy}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx}(2y+1) = \frac{1}{x^2}$$

$$\frac{dy}{dx} = \frac{1}{x^2(2y+1)}$$

Put in equation (1)

$$x^2(2y+1)\frac{1}{x^2(2y+1)} - 1 = 0$$

$$1 - 1 = 0$$

$$0 = 0$$

Which is true

$$\therefore y^2 + y = c - \frac{1}{x} \text{ is the solution of } x^2 (2y + 1) \frac{dy}{dx} - 1 = 0$$

(iii) $y \frac{dy}{dx} - e^{2x} = 1 \quad \text{---} \quad (1)$

$$y^2 = e^{2x} + 2x + c$$

Diff. W.r.t. 'x'

$$2y \frac{dy}{dx} = 2e^{2x} + 2$$

$$\frac{dy}{dx} = \frac{2e^{2x} + 2}{2y} = \frac{2(e^{2x} + 1)}{2y} = \frac{e^{2x} + 1}{y}$$

Put in equation (1)

$$y \left(\frac{e^{2x} + 1}{y} \right) - e^{2x} = 1$$

$$e^{2x} + 1 - e^{2x} = 1$$

$$1 = 1$$

Which is true

$$\therefore y^2 = e^{2x} + 2x + c \text{ is the solution of } y \frac{dy}{dx} - e^{2x} = 1$$

(iv) $\frac{1}{x} \frac{dy}{dx} - 2y = 0 \quad \text{---} \quad (1)$

$$y = ce^{x^2}$$

diff. w.r.t. 'x'

$$\frac{dy}{dx} = cxe^{x^2} \cdot 2x$$

$$= 2cxe^{x^2}$$

Put in equation (1)

$$\frac{1}{x} \cdot 2cxe^{x^2} - 2(ce^{x^2}) = 0$$

$$2ce^{x^2} - 2ce^{x^2} = 0$$

$$0 = 0$$

Which is true

$$\therefore y = ce^{x^2} \text{ is the solution of } \frac{1}{x} \frac{dy}{dx} - 2y = 0$$

(v) $\frac{dy}{dx} = \frac{y^2 + 1}{e^{-x}} \quad \text{---} \quad (1)$

$$y = \tan(e^x + c)$$

Diff. w.r.t. 'x'

$$\begin{aligned}\frac{dy}{dx} &= \sec^2(e^x + c) \cdot e^x \\ &= \frac{1 + \tan^2(e^x + c)}{e^{-x}} \\ &= \frac{1 + y^2}{e^{-x}}\end{aligned}$$

∴ From equation (1)

$$\frac{y^2 + 1}{e^{-x}} = \frac{y^2 + 1}{e^{-x}}$$

Which is true

∴ $y = \tan(e^x + c)$ is the solution of $\frac{dy}{dx} = \frac{y^2 + 1}{e^{-x}}$

Q.2 $\frac{dy}{dx} = -y$

Solution:

$$\frac{dy}{dx} = -y$$

Separate variables

$$\frac{dy}{y} = -dx$$

Integrate



$$\int \frac{dy}{y} = - \int dx$$

$$\ln y = -x + \ln c_1$$

$$e^{\ln y} = e^{-x+c_1}$$

$$y = e^{-x} \cdot e^{c_1}$$

$$\therefore y = e^{\ln y}$$

$$y = ce^{-x}$$

$$\therefore e^{c_1} = c \quad \text{Ans.}$$

Q.3 $y dx + x dy = 0$ (Guj. Board 2006)

Solution:

$$y dx + x dy = 0$$

Separate variables

$$y dx = -x dy$$

$$\frac{dy}{y} = -\frac{dx}{x}$$

Integrate

$$\int \frac{dy}{y} = - \int \frac{dx}{x}$$

$$\ln y = -\ln x + \ln c$$

$$\ln y = \ln \frac{c}{x}$$

$$y = \frac{c}{x}$$

$$\boxed{xy = c} \quad \text{Ans}$$

$$\text{Q.4} \quad \frac{dy}{dx} = \frac{1-x}{y} \quad (\text{Lhr. Board 2008})$$

Solution:

$$\frac{dy}{dx} = \frac{1-x}{y}$$

Separate variables

$$ydy = (1-x)dx$$

Integrate

$$\int ydy = \int dx - \int xdx$$

$$\frac{y^2}{2} = x - \frac{x^2}{2} + c$$

$$y^2 = 2(x - \frac{x^2}{2}) + 2c$$

$$y^2 = 2x - x^2 + c$$

$$y^2 = x(2-x) + c \quad \text{Ans.}$$

$$\therefore 2c_1 =$$

$$\text{Q.5} \quad \frac{dy}{dx} = \frac{y}{x^2}, (y > 0) \quad (\text{Guj. Board 2008})$$

Solution:

$$\frac{dy}{dx} = \frac{y}{x^2}, (y > 0)$$

Separate variables

$$\frac{dy}{y} = \frac{dx}{x^2}$$

Integrate

$$\int \frac{dy}{y} = \int \frac{dx}{x^2}$$



$$\ln y = \frac{x^{-1}}{-1} + \ln c$$

$$\ln y = \frac{-1}{x} + \ln c$$

$$e^{\ln y} = e^{\frac{-1}{x} + \ln c}$$

$$y = e^{\frac{-1}{x}} e^{\ln c}$$

$$y = ce^{\frac{-1}{x}}$$

$$\therefore y = e^{\ln y}$$

Ans.

Q.6 $\sin y \csc x \frac{dy}{dx} = 1$ (Guj. Board 2005, 2008)

Solution:

$$\sin y \csc x \frac{dy}{dx} = 1$$

Separate variables

$$\sin y dy = \frac{dx}{\csc x}$$

$$\sin y dy = \sin x dx$$

Integrate

$$\int \sin y dy = \int \sin x dx$$

$$-\cos y = -\cos x + c$$

$$\cos y = \cos x + c$$

Ans.

Q.7 $x dy + y(x-1) dx = 0$ (Lhr. Board 2007)

Solution:

$$x dy + y(x-1) dx = 0$$

Separate variables

$$x dy = -y(x-1) dx$$

$$\frac{dy}{y} = \frac{-x+1}{x} dx$$

$$\frac{dy}{y} = \left(\frac{-x}{x} + \frac{1}{x} \right) dx$$

Integrate

$$\int \frac{dy}{y} = \int dx + \int \frac{dx}{x}$$

$$\ln y = -x + \ln x + \ln c$$

$$e^{\ln y} = e^{-x + \ln x + \ln c}$$

$$\therefore y = e^{\ln y}$$



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$$y = e^{-x} \cdot e^{\ln x} \cdot e^{\ln c}$$

$y = cx e^{-x}$

Ans.

Q.8 $\frac{x^2 + 1}{y + 1} = \frac{x}{y} \cdot \frac{dy}{dx}$, ($x, y > 0$) (Guj. Board 2006)

Solution:

$$\frac{x^2 + 1}{y + 1} = \frac{x}{y} \cdot \frac{dy}{dx}$$

Separate variables

$$\frac{x^2 + 1}{x} dx = \frac{y + 1}{y} dy$$

$$\left(\frac{y}{y} + \frac{1}{y}\right) dy = \left(\frac{x^2}{x} + \frac{1}{x}\right) dx$$

$$dy + \frac{dy}{y} = x dx + \frac{dx}{x}$$

Integrate

$$\int dy + \int \frac{dy}{y} = \int x dx + \int \frac{dx}{x}$$

$$y + \ln y = \frac{x^2}{2} + \ln x + \ln c$$

$$e^{y+\ln y} = e^{\frac{-x^2}{2}} + \ln x + \ln c$$

$$e^y \cdot e^{\ln y} = e^{\frac{x^2}{2}} \cdot e^{\ln x} \cdot e^{\ln c}$$

$$ye^y = cxe^{\frac{x^2}{2}}$$

Ans.

$\therefore y = e^{\ln y}$

Q.9 $\frac{1}{x} \frac{dy}{dx} = \frac{1}{2} (1 + y^2)$ (Lhr. Board 2008)

Solution:

$$\frac{1}{x} \frac{dy}{dx} = \frac{1}{2} (1 + y^2)$$

Separate variables

$$\frac{dy}{1 + y^2} = \frac{x}{2} dx$$

Integrate

$$\int \frac{dy}{1 + y^2} = \frac{1}{2} \int x dx$$

$$\tan^{-1} y = \frac{1}{2} \cdot \frac{x^2}{2} + c$$

$$\tan^{-1}y = \frac{x^2}{4} + c \quad \text{Ans.}$$

Q.10 $2x^2y \frac{dy}{dx} = x^2 - 1$

Solution:

$$2x^2y \frac{dy}{dx} = x^2 - 1$$

Separate variables

$$2ydy = \frac{x^2 - 1}{x^2} dx$$

$$2ydy = \left(\frac{x^2}{x^2} - \frac{1}{x^2}\right) dx$$

$$2ydy = dx - x^{-2} dx$$

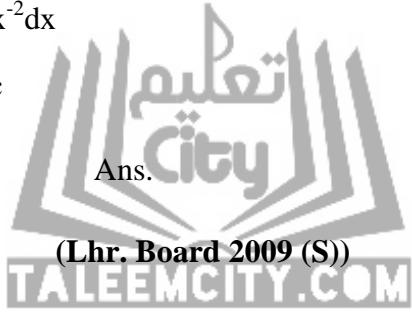
Integrate

$$2 \int ydy = \int dx - \int x^{-2} dx$$

$$\frac{2y^2}{2} = x - \frac{x^{-1}}{-1} + c$$

$$y^2 = x + \frac{1}{x} + c$$

Q.11 $\frac{dy}{dx} + \frac{2xy}{2y+1} = x$



(Lhr. Board 2009 (S))

Solution:

$$\frac{dy}{dx} + \frac{2xy}{2y+1} = x$$

Separate variables

$$\frac{dy}{dx} = x - \frac{2xy}{2y+1}$$

$$\frac{dy}{dx} = x \left(1 - \frac{2y}{2y+1}\right)$$

$$\frac{dy}{dx} = x \left(\frac{2y+1-2y}{2y+1}\right)$$

$$\frac{dy}{dx} = \frac{x}{2y+1}$$

$$(2y+1) dy = x dx$$

Integrate

$$2 \int ydy + \int dy = \int x dx$$

$$\begin{aligned}\frac{2y^2}{2} + y &= \frac{x^2}{2} + c \\ y^2 + y &= \frac{x^2}{2} + c \\ y(y+1) &= \frac{x^2}{2} + c \quad \text{Ans.}\end{aligned}$$

Q.12 $(x^2 - yx^2) \frac{dy}{dx} + y^2 + xy^2 = 0$ (Lhr. Board 2006, 2011)

Solution:

$$(x^2 - yx^2) \frac{dy}{dx} + y^2 + xy^2 = 0$$

Separate variables

$$x^2(1-y) \frac{dy}{dx} = -y^2 - xy^2$$

$$x^2(1-y) \frac{dy}{dx} = -y^2(1+x)$$

$$\left(\frac{1-y}{-y^2}\right) dy = \left(\frac{1+x}{x^2}\right) dx$$

$$\left(\frac{-1}{y^2} + \frac{1}{y^2}\right) dy = \left(\frac{1}{x^2} + \frac{x}{x^2}\right) dx$$

$$-y^{-2} dy + \frac{dy}{y} = x^{-2} dx + \frac{dx}{x}$$



Integrate

$$-\int y^{-2} dy + \int \frac{dy}{y} = \int x^{-2} dx + \int \frac{dx}{x}$$

$$-\frac{y^{-1}}{-1} + \ln y = \frac{x^{-1}}{-1} + \ln x + c$$

$$\frac{1}{y} + \ln y = \frac{-1}{x} + \ln x + c$$

$$\ln y + \frac{1}{y} = \ln x - \frac{1}{x} + c \quad \text{Ans}$$

Q.13 $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$ (Lhr. Board 2005)

Solution:

$$\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$$

$$\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$$

Separate variables

$$\sec^2 y \tan x dy = -\sec^2 x \tan y dx$$

$$\frac{\sec^2 y}{\tan y} dy = \frac{-\sec^2 x}{\tan x} dx$$

Integrate

$$\int \frac{\sec^2 y}{\tan y} dy = - \int \frac{\sec^2 x}{\tan x} dx$$

$$\ln(\tan y) = -\ln(\tan x) + \ln c$$

$$\ln(\tan y) = \ln \frac{c}{\tan x}$$

$$\tan y = \frac{c}{\tan x}$$

$$\tan x \tan y = c \quad \text{Ans.}$$

$$\text{Q.14} \quad \left(y - x \frac{dy}{dx} \right) = 2 \left(y^2 + \frac{dy}{dx} \right)$$

Solution:

$$\left(y - x \frac{dy}{dx} \right) = 2 \left(y^2 + \frac{dy}{dx} \right)$$

Separate variables

$$y - x \frac{dy}{dx} = 2y^2 + 2 \frac{dy}{dx}$$

$$y - 2y^2 = 2 \frac{dy}{dx} + x \frac{dy}{dx}$$

$$\frac{dy}{dx}(2+x) = y(1-2y)$$

$$\frac{dy}{y(1-2y)} = \frac{dx}{2+x}$$

Integrate

$$\int \frac{dy}{y(1-2y)} = \int \frac{dx}{2+x}$$

$$I = \ln(2+x) + c_1 \quad (1)$$

$$I = \int \frac{dy}{y(1-2y)}$$

Let

$$\frac{1}{y(1-2y)} = \frac{A}{y} + \frac{B}{1-2y} \quad (2)$$

Multiplying $y(1-2y)$ on both sides in eq. (2)

$$1 = A(1-2y) + By \quad (3)$$

To find A

$$\text{Put } y = 0 \text{ in eq. (3)}$$

$$1 = A$$

To find B

Put

$$1 - 2y = 0$$

$$2y = 1$$

$$y = \frac{1}{2} \text{ in eq. (3)}$$

$$1 = B \left(\frac{1}{2} \right)$$

$$B = 2$$

\therefore From equation (2)

$$\frac{1}{y(1-2y)} = \frac{1}{y} + \frac{2}{1-2y}$$

Integrate

$$\begin{aligned} \int \frac{dy}{y(1-2y)} &= \int \frac{dy}{y} + \int \frac{2}{1-2y} dy \\ I &= \ln y - \int \frac{2}{2y-1} dy \\ &= \ln y - \ln(2y-1) + c_2 \end{aligned}$$

Put in eq. (1)

$$\ln y - \ln(2y-1) + c_2 = \ln(2+x) + c_1$$

$$\ln \frac{y}{2y-1} = \ln(2+x) + c_1 - c_2$$

$$\ln \frac{y}{2y-1} = \ln(2+x) + \ln c, \text{ where } c_1 - c_2 = \ln c$$

$$\ln \frac{y}{2y-1} = \ln c(2+x)$$

$$\frac{y}{2y-1} = c(x+2) \quad \text{Ans.}$$

$$\text{Q.15 } 1 + \cos x \tan y \frac{dy}{dx} = 0$$

Solution:

$$1 + \cos x \tan y \frac{dy}{dx} = 0$$

Separate variables

$$\cos x \tan y \frac{dy}{dx} = -1$$

$$\tan y dy = \frac{-1}{\cos x} dx$$

$$\frac{-\sin y}{\cos y} dy = \sec x dx$$

Integrate

$$\int \frac{-\sin y}{\cos y} dy = \int \sec x dx$$

$$\ln(\cos y) = \ln(\sec x + \tan x) + \ln c$$

$$\ln(\cos y) = \ln c (\sec x + \tan x)$$

$$\cos y = c(\sec x + \tan x) \quad \text{Ans.}$$

$$\text{Q.16} \quad y - x \frac{dy}{dx} = 3(1 + x \frac{dy}{dx})$$

Solution:

$$y - x \frac{dy}{dx} = 3(1 + x \frac{dy}{dx})$$

Separate variables

$$y - x \frac{dy}{dx} = 3 + 3x \frac{dy}{dx}$$

$$y - 3 = 3x \frac{dy}{dx} + x \frac{dy}{dx}$$

$$y - 3 = (3x + x) \frac{dy}{dx}$$

$$\frac{dx}{4x} = \frac{dy}{y - 3}$$

Integrate

$$\int \frac{dy}{y - 3} = \frac{1}{4} \int \frac{dx}{x}$$

$$\ln(y - 3) = \frac{1}{4} \ln x + \ln c$$

$$\ln(y - 3) = \frac{1}{4} \ln x^{\frac{1}{4}} + \ln c$$

$$\ln(y - 3) = \ln c x^{\frac{1}{4}}$$

$$y - 3 = c x^{\frac{1}{4}} \quad y = 3 + c x^{\frac{1}{4}} \quad \text{Ans.}$$



Q.17 $\sec x + \tan y \frac{dy}{dx} = 0$

Solution:

$$\sec x + \tan y \frac{dy}{dx} = 0$$

Separate variables

$$\tan y \frac{dy}{dx} = -\sec x$$

$$-\tan y dy = \sec x dx$$

$$\frac{-\sin y}{\cos y} dy = \sec x dx$$

Integrate

$$\int \frac{-\sin y}{\cos y} dy = \int \sec x dx$$

$$\ln(\cos y) = \ln(\sec x + \tan x) + \ln C$$

$$\ln(\cos y) = \ln C (\sec x + \tan x)$$

$$\cos y = C(\sec x + \tan x) \quad \text{Ans.}$$

Q.18 $(e^x + e^{-x}) \frac{dy}{dx} = e^x - e^{-x}$ (Lhr. Board 2011)

Solution:

$$(e^x + e^{-x}) \frac{dy}{dx} = e^x - e^{-x}$$

Separate variables

$$dy = \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

Integrate

$$\int dy = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

$$y = \ln(e^x + e^{-x}) + C \quad \text{Ans.}$$

Q.19 Find the general solution of the equation $\frac{dy}{dx} - x = xy^2$. Also find the particular solution if $y = 1$ when $x = 0$.

Solution:

$$\frac{dy}{dx} - x = xy^2$$

To find General Solution

$$\frac{dy}{dx} - x = xy^2$$

Separate variables

$$\frac{dy}{dx} = xy^2 + x$$

$$\frac{dy}{dx} = x(y^2 + 1)$$

$$\frac{dy}{y^2 + 1} = x dx$$

Integrate

$$\int \frac{dy}{y^2 + 1} = \int x dx$$

$$\tan^{-1} y = \frac{x^2}{2} + c \quad \text{--- (1)}$$

To find particular solution

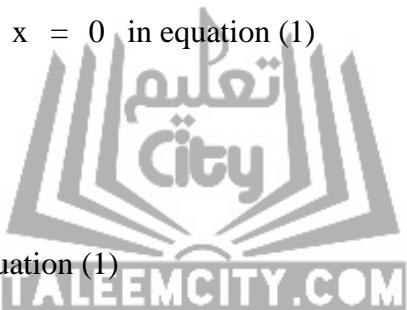
$$\text{Put } y = 1, \quad x = 0 \text{ in equation (1)}$$

$$\tan^{-1}(1) = \frac{(0)^2}{2} + c$$

$$c = \frac{\pi}{4}$$

$$\text{Put } c = \frac{\pi}{4} \text{ in equation (1)}$$

$$\tan^{-1} y = \frac{x^2}{2} + \frac{\pi}{4} \quad \text{Ans.}$$



Q.20 Solve the differential equation $\frac{dx}{dt} = 2x$ given $x = 4$ when $t = 0$.

Solution:

$$\frac{dx}{dt} = 2x$$

Separate variable

$$\frac{dx}{x} = 2dt$$

Integrate

$$\int \frac{dx}{x} = 2 \int dt$$

$$\ln x = 2t + \ln c$$

$$e^{\ln x} = e^{2t + \ln c}$$

$$\begin{aligned}
 e^{\ln x} &= e^{2t} \cdot e^{\ln c} \\
 x &= ce^{2t} \quad \text{--- (1)} \\
 \text{Put } x &= 4, \quad t = 0 \text{ in equation (1)} \\
 4 &= ce^{2(0)} \\
 c &= 4 \\
 \text{Put } c &= 4 \text{ in equation (1)} \\
 x &= 4e^{2t} \quad \text{Ans.}
 \end{aligned}$$

Q.21 Solve the differential equation $\frac{ds}{dt} + 2st = 0$. Also find the particular solution if $s = 4e$, when $t = 0$.

Solution:

$$\frac{ds}{dt} + 2st = 0$$

Separate variables

$$\frac{ds}{dt} = -2st$$

$$\frac{ds}{s} = -2tdt$$

Integrate

$$\int \frac{ds}{s} = -2 \int t dt$$

$$\ln s = -\frac{2t^2}{2} + \ln c$$

$$e^{\ln s} = e^{-t^2} + \ln c$$

$$s = e^{-t^2} \cdot e^{\ln c}$$

$$s = ce^{-t^2} \quad \text{--- (1)}$$



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To find particular solution

$$\text{Put } s = 4e, \quad t = 0 \text{ in equation (1)}$$

$$4e = ce^0$$

$$c = 4e$$

$$\text{Put } c = 4e \text{ in equation (1)}$$

$$s = 4e \cdot e^{-t^2}$$

$$s = 4e^{1-t^2} \quad \text{Ans.}$$

Q.22 In a culture bacteria increases at the rate proportional to the number of bacteria present. If bacteria are 200 initially and are doubled in 2 hours, find the number of bacteria present four hours later.

Solution:

Let P be the number of bacteria present at time t .

$$\frac{dp}{dt} = kP \quad (k > 0)$$

Separate variable

$$\frac{dp}{p} = kdt$$

Integrate

$$\int \frac{dp}{p} = k \int dt$$

$$\ln p = kt + c_1 \quad (1)$$

Put $p = 200$, $t = 0$ in equation (1)

$$\ln 200 = 0 + c_1$$

$$c_1 = \ln 200$$

Put $c_1 = \ln 200$ in equation (1)

$$\ln p = kt + \ln 200 \quad (2)$$

Put $P = 400$, $t = 2$ hour in equation (2)

$$\ln 400 = 2k + \ln 200$$

$$\ln 400 - \ln 200 = 2k$$

$$k = \frac{1}{2} \ln \left(\frac{400}{200} \right)$$

$$k = \frac{1}{2} \ln 2$$

Put $k = \frac{1}{2} \ln 2$ in equation (2)

$$\ln P = \frac{1}{2} \ln 2t + \ln 200 \quad (3)$$

To find the number of bacteria presents four hour later

Put $t = 4$ hour in equation (3)

$$\ln P = \frac{1}{2} \ln 2 \times 4 + \ln 200$$

$$\ln P = 2 \ln 2 + \ln 200$$

$$\ln P = \ln 2^2 + \ln 200$$

$$\ln P = \ln (4 \times 200)$$

P = 800	Ans.
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Q.23 A ball is thrown vertically upward with a velocity of 2450 cm/sec. Neglecting air resistance, find

- (i) velocity of ball at any time t .
- (ii) distance travelled in any time t .
- (iii) maximum height attained by the ball.

Solution:

- (i) Let 'v' be the velocity at any time 't' then by Newton's law of motion

$$\frac{dv}{dt} = -g$$

Separate variables

$$dv = -g dt$$

Integrate $\int dv = -g \int dt$

$$v = -gt + c_1 \quad (1)$$

Put $v = 2450$ cm/sec and $t = 0$ in eq. (1)

$$2450 = 0 + c_1$$

$$c_1 = 2450$$

Put $c_1 = 2450$ in equation (1)

$$v = -980t + 2450 \quad (2) \quad (\text{taking } g = 980)$$

- (ii) Let 'h' be the height attained at any time 't'.

$$\frac{dh}{dt} = 2450 - 980 t$$

Separate variables

$$dh = 2450 dt - 980 t dt$$

Integrate $\int dh = 2450 \int dt - 980 \int t dt$

$$h = 2450t - 980 \frac{t^2}{2} + c_2$$

$$h = 2450t - 490t^2 + c_2 \quad (3)$$

Put $h = 0$, $t = 0$ in equation (3)

$$0 = 0 - 0 + c_2$$

$$c_2 = 0$$

Put $c_2 = 0$ in equation (3)

$$h = 2450t - 490t^2 \quad (4)$$

- (iii) The maximum height attained by the ball when $v = 0$

$$-980t + 2450 = 0$$

$$980t = 2450$$

$$t = \frac{2450}{980} = 2.5 \text{ Sec}$$

Put $t = 2.5$ sec in equation (4)

$$h = 2450(2.5) - 490(2.5)^2$$

$$h = 6125 - 490(6.25) = 6125 - 3062.5$$

$$h = 3062.5 \text{ cm} \quad \text{Ans.}$$