Chapter 4	INTRODUCTION TO ANALYTIC GEOMETRY

Chapter 4: INTRODUCTION TO ANALYTIC GEOMETRY

EXERCISE 4.1

Q.1 Describe the location in the plane of the point P(x, y) for which

(i) $x > 0$	**(ii)** $x > 0$ and $y > 0$	**(iii)** $x = 0$
(iv) $y = 0$	**(v)** $x < 0$ and $y \geq 0$	**(vi)** $x = y$
(vii) $\|x\| = \|y\|$	**(viii)** $\|x\| \geq 3$	**(ix)** $x > 2$ and $y = 2$
(x) x and y have opposite signs		

Solution:

(i) $x > 0$

Right half plane

(ii) $x > 0$ and $y > 0$

The 1st quadrant

(iii) $x = 0$

y – axis

(iv) $y = 0$

x – axis

(v) $x < 0$ and $y \geq 0$

2nd quadrant and – ve x – axis

(vi) $x = y$

It is a line bisecting 1st and 3 rd quadrant

(vii) $|x| = |y|$

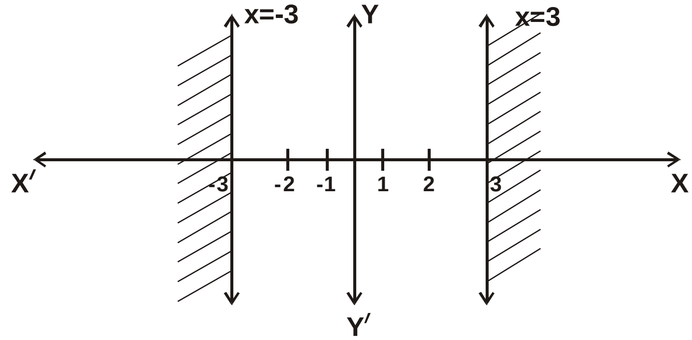
Points in the first and 3rd quadrants having both the coordinates equal or point in the second and fourth quadrant having both the coordinates equal but opposite in signs.

OR

(1) It is a line bisecting 1st and 3rd quadrant.

(2) It is a line bisecting 2nd and 4th quadrant.

(viii) $|x| \geq 3$



$\Rightarrow \quad \pm x \geq 3$

$\Rightarrow \quad x \geq 3 \quad$ or $\quad -x \geq 3$

$\quad\quad x \geq 3 \quad$ or $\quad x \leq -3$

Which is the set of points lying on right side of x = 3 and the points lying on left side of x = – 3

(ix) **$x > 2$ and $y = 2$**

The points lies in the first quadrant with $x > 2$ and $y = 2$

(x) **x and y have opposite signs.**

The point lies in 2^{nd} and 4^{th} quadrant.

Q.2 Find in each of the following.

(i) The distance between the two given points.

(ii) Midpoint of the line segment joining the two points.

(a) **A(3, 1) ; B(– 2, – 4)** **(b)** **A(– 8, 3) ; B(2, – 1)**

(c) $A\left(-\sqrt{5}, \frac{-1}{3}\right)$; $B(-3\sqrt{5}, 5)$

Solution:

(a) A(3, 1) ; B(–2, – 4) **(Lhr. Board 2007)**

(i) $|AB| = \sqrt{(-2-3)^2 + (-4-1)^2}$

$= \sqrt{(-5)^2 + (-5)^2}$

$= \sqrt{25 + 25}$

$= \sqrt{50} = 5\sqrt{2}$

(ii) Mid point $= \left(\frac{3-2}{2}, \frac{1-4}{2}\right) = \left(\frac{1}{2}, \frac{-3}{2}\right)$

(b) **A(– 8, 3) ; B(2, – 1)**

(i) $|AB| = \sqrt{(2+8)^2 + (-1-3)^2}$

$= \sqrt{(10)^2 + (-4)^2}$

$= \sqrt{100 + 16}$

$= \sqrt{116}$

$= \sqrt{4 \times 29} = 2\sqrt{29}$

(ii) Mid point $= \left(\dfrac{-8+2}{2}\ ,\ \dfrac{3-1}{2}\right)$

$= \left(\dfrac{-6}{2}\ ,\ \dfrac{2}{2}\right) \quad = \quad (-3, 1)$ Ans

(c) A $\left(-\sqrt{5}\ ,\ \dfrac{-1}{3}\right)$; B$(-3\sqrt{5}\ , 5)$ (Lhr. Board 2006, Guj. Board 2008)

(i) $|AB| = \sqrt{(-3\sqrt{5} + \sqrt{5})^2 + (5 + \frac{1}{3})^2}$

$= \sqrt{(-2\sqrt{5})^2 + \left(\dfrac{15+1}{3}\right)^2}$

$= \sqrt{20 + \left(\dfrac{16}{3}\right)^2}$

$= \sqrt{20 + \dfrac{256}{9}}$

$= \sqrt{\dfrac{180 + 256}{9}}$

$= \sqrt{\dfrac{436}{9}}$

$= \sqrt{\dfrac{4 \times 109}{9}}$

$= \dfrac{2\sqrt{109}}{3}$ Ans.

(ii) Mid point $= \left(\dfrac{-\sqrt{5} - 3\sqrt{5}}{2},\ \dfrac{-\frac{1}{3} + 5}{2}\right)$

$= \left(\dfrac{-4\sqrt{5}}{2},\ \dfrac{\frac{-1+15}{3}}{2}\right) = \left(-2\sqrt{5}\ ,\ \dfrac{14}{6}\right)$

$= \left(-2\sqrt{5}\ ,\ \dfrac{7}{3}\right)$ Ans

Q.3 Which of the following points are at a distance of 15 units from origin?

(a) $(\sqrt{176}\ , 7)$ **(b)** $(10, -10)$

(c) $(1, 15)$ **(d)** $\left(\frac{15}{2}, \frac{15}{2}\right)$

Solution:

(a) **Let A $(\sqrt{176}\ ,\ 7)$, O(0, 0)**

$$|AO| = \sqrt{(0-\sqrt{176})^2 + (0-7)^2}$$
$$= \sqrt{176 + 49}$$
$$= \sqrt{225}$$
$$= 15$$

$\therefore$ A $(\sqrt{176}\ ,\ 7)$ is at a distance of 15 units from origin.

(b) **Let A(10, – 10) , O(0, 0)**

$$|AO| = \sqrt{(0-10)^2 + (0+10)^2}$$
$$= \sqrt{100 + 100}$$
$$= \sqrt{200}$$
$$= 10\sqrt{2}$$

$\therefore$ A(10, – 10) is not a distance of 15 units from origin.

(c) **Let A(1, 15) , O(0, 0)**

$$|AO| = \sqrt{(0-1)^2 + (0-15)^2}$$
$$= \sqrt{1 + 225} = \sqrt{226}$$

$\therefore$ A (1, 15) is not a distance of 15 units from origin.

(d) **Let A $\left(\frac{15}{2}\ ,\ \frac{15}{2}\right)$, O(0, 0)**

$$|AO| = \sqrt{\left(0-\frac{15}{2}\right)^2 + \left(0-\frac{15}{2}\right)^2}$$
$$|AO| = \sqrt{\frac{225}{4} + \frac{225}{4}}$$
$$|AO| = \sqrt{\frac{225+225}{4}}$$
$$|AO| = \sqrt{\frac{450}{4}}$$
$$|AO| = \sqrt{\frac{225}{2}}$$

$|AO| = \dfrac{15}{\sqrt{2}}$

$\therefore$ $A\left(\dfrac{15}{2}, \dfrac{15}{2}\right)$ is not a distance of 15 units from origin.

Q.4 Show that

(i) the points A(0, 2), B $(\sqrt{3}, -1)$ and C(0, – 2) are vertices of a right triangle.

(ii) the points A(3, 1), B(–2, –3) and C(2, 2) are vertices of an isosceles triangle.

(iii) the points A(5, 2), B(– 2, 3), C(–3, –4) and D(4, –5) are vertices of a parallelogram. Is the parallelogram a square?

Solution:

(i) A(0, 2) , B $(\sqrt{3}, -1)$, C(0, – 2)

$|AB| = \sqrt{(\sqrt{3}-0)^2 + (-1-2)^2}$

$|AB| = \sqrt{3 + (-3)^2} = \sqrt{3+9} = \sqrt{12}$

$|AB|^2 = 12$

$|BC| = \sqrt{(0-\sqrt{3})^2 + (-2+1)^2}$

$|BC| = \sqrt{3 + (-1)^2}$

$|BC| = \sqrt{3+1}$

$|BC| = \sqrt{4}$

$|BC|^2 = 4$

$|AC| = \sqrt{(0-0)^2 + (-2-2)^2}$

$= \sqrt{0 + (-4)^2}$

$|AC| = \sqrt{16}$

$|AC|^2 = 16$

Since

$|AC|^2 = |AB|^2 + |BC|^2$

Shows the given vertices A(0, 2), B$(\sqrt{3}, -1)$ and C(0, – 2) form a right angle triangle.

(ii) A(3, 1) , B(– 2 , – 3) , C (2, 2)

$|AB| = \sqrt{(-2-3)^2 + (-3-1)^2}$

$|AB| = \sqrt{(-5)^2 + (-4)^2}$

$|AB| = \sqrt{25+16}$

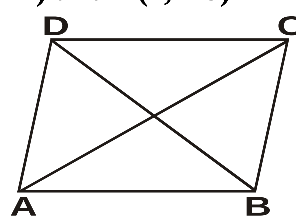
$|AB| = \sqrt{41}$

$$|BC| = \sqrt{(2+2)^2 + (2+3)^2}$$
$$|BC| = \sqrt{(4)^2 + (5)^2}$$
$$|BC| = \sqrt{16 + 25}$$
$$|BC| = \sqrt{41}$$
$$|AC| = \sqrt{(2-3)^2 + (2-1)^2}$$
$$|AC| = \sqrt{(-1)^2 + (1)^2}$$
$$|AC| = \sqrt{1 + 1}$$
$$|AC| = \sqrt{2}$$

Since $|AB| = |BC|$

Show A (3, 1), B (–2, – 3) and C (2, 2) are vertices of an isosceles triangle.

(iii) **A(5, 2), B(–2, 3), C(–3, –4) and D(4, – 5)**



$$|AB| = \sqrt{(-2-5)^2 + (3-2)^2}$$
$$|AB| = \sqrt{(-7)^2 + (1)^2}$$
$$|AB| = \sqrt{49 + 1}$$
$$|AB| = \sqrt{50}$$
$$|BC| = \sqrt{(-3+2)^2 + (-4-3)^2}$$
$$|BC| = \sqrt{(-1)^2 + (-7)^2}$$
$$|BC| = \sqrt{1 + 49}$$
$$|BC| = \sqrt{50}$$
$$|DC| = \sqrt{(-3-4)^2 + (-4+5)^2}$$
$$|DC| = \sqrt{(-7)^2 + (1)^2}$$
$$|DC| = \sqrt{49 + 1}$$
$$|DC| = \sqrt{50}$$
$$|AD| = \sqrt{(4-5)^2 + (-5-2)^2}$$
$$|AD| = \sqrt{(-1)^2 + (-7)^2}$$
$$|AD| = \sqrt{1 + 49} = \sqrt{50}$$

$\therefore$ $|AB| = |DC|$

and $|AD| = |BC|$

Shows that the points A (5, 2), B (–2, 3), C (– 3, – 4) and D (4, – 5) are the vertices of a parallelogram.

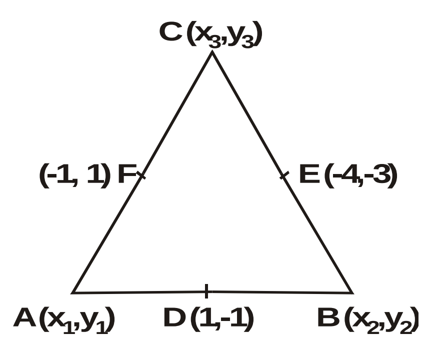
Let $\overline{AC}$ and $\overline{BD}$ be the diagonals of a parallelogram ABCD.

$$|AC| = \sqrt{(-3-5)^2 + (-4-2)^2}$$
$$|AC| = \sqrt{(-8)^2 + (-6)^2}$$
$$|AC| = \sqrt{64 + 36}$$
$$|AC| = \sqrt{100}$$
$$|AC| = 10$$
$$|BD| = \sqrt{(4+2)^2 + (-5-3)^2}$$
$$|BD| = \sqrt{(6)^2 + (-8)^2}$$
$$|BD| = \sqrt{36 + 64}$$
$$|BD| = \sqrt{100}$$
$$|BD| = 10$$
$$\therefore \quad |AC| = |BD|$$

As all sides of parallelogram are equal and diagonals are also equal so this parallelogram forms a square.

Q.5 The midpoints of the sides of a triangle are (1, –1), (–4, –3) and (–1, 1). Find the coordinates of the vertices of a triangle.

Solution:



Let D (1, – 1), E (– 4, – 3) and F (– 1, 1) be the mid points of sides $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively.

Let A (x_1, y_1), B (x_2, y_2) and C(x_3, y_3) be the three vertices of a ΔABC.

Since D is the mid point of $\overline{AB}$.

$\therefore$ By ratio formula

$$\frac{x_1 + x_2}{2} = 1 \qquad \frac{y_1 + y_2}{2} = -1$$

$x_1 + x_2 = 2 \quad ...(1) \qquad y_1 + y_2 = -2 \quad ....(2)$

Since F be the mid point of $\overline{AC}$.

$\therefore$ By ratio formula

$$\frac{x_2 + x_3}{2} = -4 \quad , \quad \frac{y_2 + y_3}{2} = -3$$

$x_2 + x_3 = -8 \quad ..... (3)$

$y_2 + y_3 = -6 \quad ..... (4)$

$\therefore$ By ratio formula

$$\frac{x_1 + x_3}{2} = -1 \qquad \frac{y_1 + y_3}{2} = 1$$

$x_1 + x_3 = -2 \quad ...(5) \qquad y_1 + y_3 = 2 \quad ....(6)$

Equation (1) – equation (3), we get

$$\begin{array}{rcrcl} x_1 & + & x_2 & = & 2 \\ -x_2 & \pm & x_3 & = & \mp 8 \\ \hline x_1 & - & x_3 & = & 10 \end{array} \quad ... (7)$$

Equation (5) + Equation (7), we get

$$\begin{array}{rcrcl} x_1 & + & x_3 & = & -2 \\ x_1 & - & x_3 & = & 10 \\ \hline \end{array}$$

$$2x_1 = 8$$

$$x_1 = \frac{8}{2} = 4$$

Put $x_1 = 4$ in Equation (7)

$4 - x_3 = 10$

$4 - 10 = x_3$

$x_3 = -6$

Put $x_3 = -6$ in equation (3)

$x_2 - 6 = -8$

$x_2 = -8 + 6$

$x_2 = -2$

Equation (2) – Equation (4), we get

$$\begin{array}{rcrcl} y_1 & + & y_2 & = & -2 \\ -y_2 & \pm & y_3 & = & \mp 6 \\ \hline y_1 & - & y_3 & = & 4 \end{array} \quad ... (8)$$

Equation (6) + Equation (8), we get

$$\begin{array}{rcrcl} y_1 & + & y_3 & = & 2 \\ y_1 & - & y_3 & = & 4 \\ \hline & & 2y_1 & = & 6 \end{array}$$

$$y_1 = \frac{6}{2} = 3$$

Put $y_1 = 3$ in equation (8)

$3 - y_3 = 4$

$3 - 4 = y_3$

$y_3 = -1$

Put $y_3 = -1$ in equation (4)

$y_2 - 1 = -6$

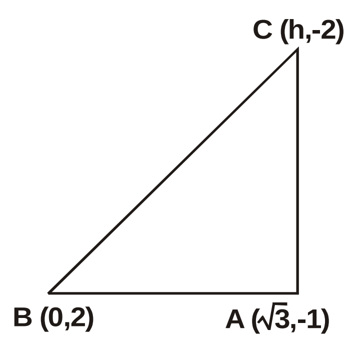
$y_2 = -6 + 1$

$y_2 = -5$

$\therefore$ A(4, 3), B(– 2, – 5), C(– 6, – 1) Ans

Q.6: Find h such that the points A ($\sqrt{3}$, – 1), B (0, 2) and C (h, – 2) are vertices of right triangle with right angle at vertex A.

Solution:



A ($\sqrt{3}$, – 1), B (0, 2) C (h, – 2)

$|AB| = \sqrt{(0-\sqrt{3})^2 + (2+1)^2}$

$|AB| = \sqrt{3 + (3)^2}$

$|AB| = \sqrt{3 + 9}$

$|AB| = \sqrt{12}$

$|BC| = \sqrt{(h-0)^2 + (-2-2)^2}$

$|BC| = \sqrt{h^2 + (-4)^2}$

$|BC| = \sqrt{h^2 + 16}$

$|AC| = \sqrt{(h-\sqrt{3})^2 + (-2+1)^2}$

$|AC| = \sqrt{h^2 + 3 - 2h\sqrt{3} + (-1)^2}$

$|AC| = \sqrt{h^2 + 3 - 2\sqrt{3}h + 1}$

$|AC| = \sqrt{h^2 - 2\sqrt{3}h + 4}$

Since ABC is a right triangle with right angle at vertex A.

$\therefore$ By Pythagrous theorem

$|BC|^2 = |AB|^2 + |AC|^2$

$\left(\sqrt{h^2+16}\right)^2 = \left(\sqrt{12}\right)^2 + \left(\sqrt{h^2 - 2\sqrt{3}h + 4}\right)^2$

$h^2 + 16 = 12 + h^2 - 2\sqrt{3}h + 4$

$h^2 - h^2 + 2\sqrt{3}h = 16 - 16$

$2\sqrt{3}h = 0$

$\boxed{h = 0}$ Ans

Q.7 Find h such that A (– 1, h), B (3, 2) and C (7, 3) are collinear. (Lhr. Board 2009 (S))

Solution:

A (– 1, h), B (3, 2) and C (7, 3)

Since A, B and C are collinear.

$$\therefore \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} -1 & h & 1 \\ 3 & 2 & 1 \\ 7 & 3 & 1 \end{vmatrix} = 0$$

$$-1\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} - h\begin{vmatrix} 3 & 1 \\ 7 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 2 \\ 7 & 3 \end{vmatrix} = 0$$

$-1(2-3) - h(3-7) + 1(9-14) = 0$

$-1(-1) - h(-4) + 1(-5) = 0$

$1 + 4h - 5 = 0$

$4h - 4 = 0$

$4h = 4$

$$h = \frac{4}{4}$$

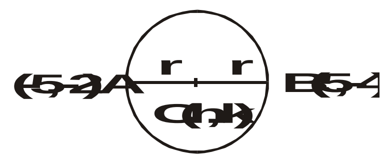
$\boxed{h = 1}$ Ans

Q.8 The points A(–5, – 2) and B(5, – 4) are ends of a diameter of a circle. Find the centre and radius of the circle.

Solution:

A(– 5, – 2) , B(5, – 4)

Let C (h, k) be the center of the circle having radius r. Since C (h, k) be the mid point of $\overline{AB}$.



$\therefore$ By ratio formula

$$h = \frac{-5+5}{2} \quad , \quad k = \frac{-2-4}{2}$$

$$h = \frac{0}{2} \quad , \quad k = \frac{-6}{2} = -3$$

$$h = 0$$

$$\therefore \quad C(h, k) = C(0, -3)$$

$$r = |AC| = \sqrt{(0+5)^2 + (-3+2)^2}$$

$$= \sqrt{(5)^2 + (-1)^2}$$

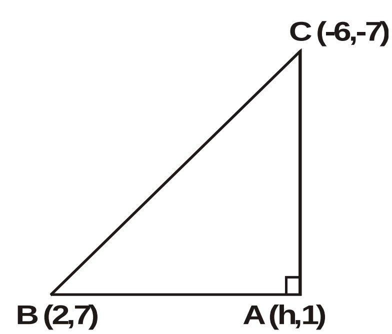
$$= \sqrt{25+1}$$

$$= \sqrt{26} \quad \text{Ans}$$

Q.9 Find h such that the points A (h, 1), B (2, 7) and C (– 6, – 7) are vertices of a right triangle with right angle at the vertex A.

Solution:

A (h, 1), B (2, 7), C (– 6, – 7)



$$|AB|^2 = (2-h)^2 + (7-1)^2$$

$$= 4 + h^2 - 4h + 36$$

$$= h^2 - 4h + 40$$

$$|BC|^2 = (-6-2)^2 + (-7-7)^2$$

$$= (-8)^2 + (-14)^2$$

$$= 64 + 196$$

$$= 260$$

$$|AC|^2 = (-6-h)^2 + (-7-1)^2$$

$$= 36 + h^2 + 12h + 64 = h^2 + 12h + 100$$

Since ABC is a right triangle with right angle at vertex A.

$\therefore$ By Pythagoras theorem

$$|BC|^2 = |AB|^2 + |AC|^2$$
$$260 = h^2 - 4h + 40 + h^2 + 12h + 100$$
$$0 = 2h^2 + 8h + 140 - 260$$
$$2h^2 + 8h - 120 = 0$$
$$2(h^2 + 4h - 60) = 0$$
$$h^2 + 4h - 60 = 0$$
$$h^2 + 10h - 6h - 60 = 0$$
$$h(h + 10) - 6(h + 10) = 0$$
$$(h + 10)(h - 6) = 0$$

Either

$h + 10 = 0$ or $h - 6 = 0$

$h = -10$ $\quad$ $h = 6$

$\therefore$ $h = 6, -10$ Ans

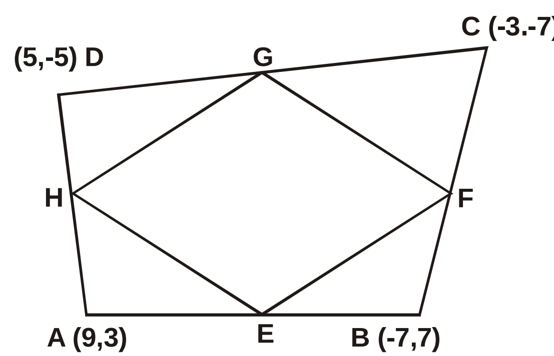
Q.10 A quadrilateral has the points A(9, 3) , B(– 7, 7), C(–3, – 7) and D(5, – 5) as its vertices. Find the midpoints of its sides. Show that the figure formed by joining the midpoints consecutively is a parallelogram.

Solution:

A (9, 3) , B(– 7, 7)

C (– 3, – 7) , D(5, – 5)

Let E, F, G and H be the mid points of sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{AD}$ respectively.



$$\text{Mid point E of } \overline{AB} = \left(\frac{9-7}{2}, \frac{3+7}{2}\right)$$

$$\text{Mid point E of } \overline{AB} = \left(\frac{2}{2}, \frac{10}{2}\right)$$

$$\text{Mid point E of } \overline{AB} = (1, 5)$$

$$\text{Mid point F of } \overline{BC} = \left(\frac{-3-7}{2}, \frac{-7+7}{2}\right)$$

$$\text{Mid point F of } \overline{BC} = \left(\frac{-10}{2}, \frac{0}{2}\right)$$

Mid point F of $\overline{BC}$ = $(-5, 0)$

Mid point G of $\overline{CD}$ = $\left(\dfrac{5-3}{2}, \dfrac{-5-7}{2}\right)$

Mid point G of $\overline{CD}$ = $\left(\dfrac{2}{2}, \dfrac{-12}{2}\right)$

Mid point G of $\overline{CD}$ = $(1, -6)$

Mid point H of $\overline{AD}$ = $\left(\dfrac{5+9}{2}, \dfrac{-5+3}{2}\right)$

Mid point H of $\overline{AD}$ = $\left(\dfrac{14}{2}, \dfrac{-2}{2}\right)$

Mid point H of $\overline{AD}$ = $(7, -1)$

$|EF| = \sqrt{(-5-1)^2 + (0-5)^2}$

$|EF| = \sqrt{(-6)^2 + (-5)^2}$

$|EF| = \sqrt{36 + 25}$

$|EF| = \sqrt{61}$

$|FG| = \sqrt{(1+5)^2 + (-6-0)^2}$

$|FG| = \sqrt{(6)^2 + (-6)^2}$

$|FG| = \sqrt{36 + 36}$

$|FG| = \sqrt{72}$

$|GH| = \sqrt{(7-1)^2 + (-1+6)^2}$

$|GH| = \sqrt{(6)^2 + (5)^2}$

$|GH| = \sqrt{36 + 25}$

$|GH| = \sqrt{61}$

$|HE| = \sqrt{(1-7)^2 + (5+1)^2}$

$|EF| = \sqrt{(-6)^2 + (6)^2}$

$|EF| = \sqrt{36 + 36}$

$|EF| = \sqrt{72}$

Since $|EF| = |GH|$

$|FG| = |HE|$

Shows the figure formed by joining the midpoints consecutively is a parallelogram.

Q.11: Find h such that the quadrilateral with vertices A(– 3, 0), B(1, – 2), C(5, 0) and D(1, h) is parallelogram. Is it square?

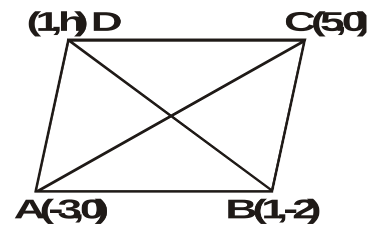
Solution:

A (– 3, 0) , B (1, – 2)

C (5, 0) , D (1, h)

Since ABCD is a parallelogram.

$\therefore$ $|AD| = |BC|$



$$\sqrt{(1+3)^2 + (h-0)^2} = \sqrt{(5-1)^2 + (0+2)^2}$$

$$\sqrt{16 + h^2} = \sqrt{16 + 4}$$

Squaring on both sides

$$16 + h^2 = 20$$

$$h^2 = 20 - 16$$

$$h^2 = 4$$

$$\boxed{h = \pm 2}$$

Now when $h = -2$ then D (1, h) = D (1, – 2) but we also have B (1, 2)

i.e. B an D represents the same point, which cannot be happened in quadrilateral so we cannot take h = – 2.

$\boxed{h = 2}$ Ans

Let $\overline{AC}$ and $\overline{BC}$ be the diagonals of parallelogram ABCD.

$$|AC| = \sqrt{(5+3)^2 + (0-0)^2}$$

$$|AC| = \sqrt{(8)^2 + 0}$$

$$|AC| = \sqrt{64 + 0}$$

$$|AC| = 8$$

$$|BD| = \sqrt{(1-1)^2 + (2+2)^2}$$

$$|BD| = \sqrt{0 + (4)^2}$$

$$|BD| = \sqrt{16}$$

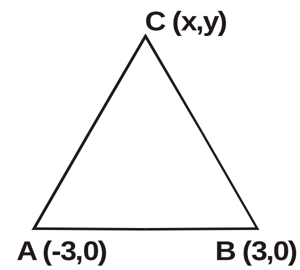
$$|BD| = 4$$

$$\therefore \quad |AC| \neq |BD|$$

So the parallelogram ABCD is not a square.

Q.12 If two vertices of an equilateral triangle are A(–3, 0) and B(3, 0), find the third vertex. How many of these triangles are possible?

Solution:



A(–3, 0) , B(3, 0)

Let C (x, y) be the required vertex.

Since ABC is an equilateral triangle

$\therefore\ |AB| = |AC| = |BC|$

$|AB| = |AC|$

$|AB|^2 = |AC|^2$

$(3+3)^2 + (0-0)^2 = (x+3)^2 + (y-0)^2$

$(6)^2 + 0 = x^2 + 9 + 6x + y^2$

$36 = x^2 + 6x + 9 + y^2$

$0 = x^2 + y^2 + 6x + 9 - 36$

$x^2 + y^2 + 6x - 27 = 0$ ...... (1)

$|AC| = |BC|$

$|AC|^2 = |BC|^2$

$(x+3)^2 + (y-0)^2 = (x-3)^2 + (y-0)^2$

$x^2 + 9 + 6x + y^2 = x^2 + 9 - 6x + y^2$

$x^2 + 6x + y^2 - x^2 + 6x - y^2 = 9 - 9$

$12x = 0$

$\boxed{x = 0}$

Put $x = 0$ in equation (1)

$(0)^2 + y^2 + 6(0) - 27 = 0$

$y^2 = 27$

$\sqrt{y^2} = \sqrt{27}$

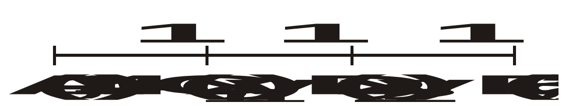
$y = \pm 3\sqrt{3}$

$\therefore$ $(0, 3\sqrt{3})$, $(0, -3\sqrt{3})$

So there are two triangles are possible.

Q.13: Find the points trisecting the join of A(–1, 4) and B(6, 2).
(Lhr. Board 2006, 2009, 2011) (Guj. Board 2008).

Solution:



A (–1, 4) , B (6, 2)

Let C (x_1, y_1) and D (x_2, y_2) be the required points.

Since C lies between A and B.

$\therefore$ By ratio formula

$$x_1 = \frac{1(6) + 2(-1)}{1 + 2} \quad , \quad y_1 = \frac{1(2) + 2(4)}{1 + 2}$$

$$x_1 = \frac{6 - 2}{3} \quad , \quad y_1 = \frac{2 + 8}{3}$$

$$x_1 = \frac{4}{3} \quad , \quad y_1 = \frac{10}{3}$$

$$\therefore \quad C\left(\frac{4}{3} \; , \; \frac{10}{3}\right)$$

Since D is the mid points of $\overline{CB}$.

$\therefore$ By ratio formula

$$x_2 = \frac{\frac{4}{3} + 6}{2} \quad , \quad y_2 = \frac{\frac{10}{3} + 2}{2}$$

$$x_2 = \frac{\frac{4 + 18}{3}}{2} \quad , \quad y_2 = \frac{\frac{10 + 6}{3}}{2}$$

$$x_2 = \frac{22}{6} \quad , \quad y_2 = \frac{16}{6}$$

$$x_2 = \frac{11}{3} \quad , \quad y_2 = \frac{8}{3}$$

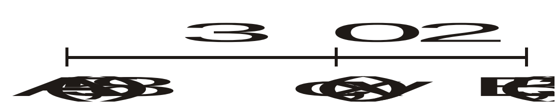
$\therefore$ $D\left(\frac{11}{3} \; , \; \frac{8}{3}\right)$ Ans

Q.14 Find the point three-fifth of the way along the line segment from A (–5, 8) to B (5, 3). (Lhr. Board 2007)

Solution:

A(– 5, 8) , B(5, 3)

Let C(x, y) be the required point.



$\therefore$ By ratio formula

$$x = \frac{2(-5) + 3(5)}{3 + 2} \quad , \quad y = \frac{2(8) + 3(3)}{3 + 2}$$

$$x = \frac{-10 + 15}{5} \quad , \quad y = \frac{16 + 9}{5}$$

$$x = \frac{5}{5} \quad , \quad y = \frac{25}{5}$$

$$x = 1 \quad , \quad y = 5$$

$\therefore$ C(x, y) = C(1, 5) Ans

Q.15 Find the point P on the join of A (1, 4) and B (5, 6) that is twice as far from A as B is from A and lies

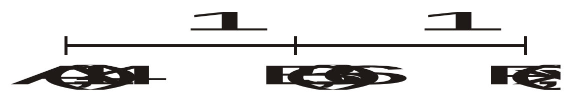
(i) On the same side of A as B does.

(ii) On the opposite side of A as B does.

Solution:

A (1, 4), B(5, 6)

Let P (x, y) be the required point.



$\therefore$ By ratio formula

$$5 = \frac{1 + x}{2} \quad , \quad 6 = \frac{4 + y}{2}$$

$$1 + x = 10 \quad , \quad 12 = 4 + y$$

$$x = 10 - 1 \quad , \quad y = 12 - 4$$

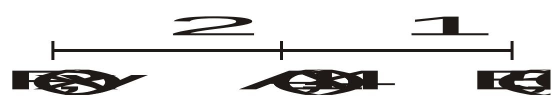
$$x = 9 \quad , \quad y = 8$$

$\therefore$ P(x, y) = P(9, 8)

(ii)

Let P (x, y) be the required point.

$\therefore$ By ratio formula.



$$1 = \frac{1(x) + 2(5)}{2 + 1} \quad , \quad 4 = \frac{1(y) + 2(6)}{2 + 1}$$

$$1 = \frac{x + 10}{3} \quad , \quad 4 = \frac{y + 12}{3}$$

$$x + 10 = 3 \quad , \quad y + 12 = 12$$

$$x = 3 - 10 \quad , \quad y = 12 - 12$$

$$x = -7 \quad , \quad y = 0$$

$\therefore$ P (x, y) = P (– 7, 0) Ans

Q.16 Find the point which is equidistant from the points A (5, 3), B (–2, 2) and C(4, 2). What is the radius of the circum circle of the ΔABC?

Solution:

A (5, 3), B (–2, 2), C(4, 2)

Let P (x, y) be the required point which is equidistant from A, B and C.

$$|PA| = |PB| = |PC|$$

$$|PA|^2 = |PB|^2 = |PC|^2$$

Taking $|PB|^2 = |PC|^2$

$$(-2 - x)^2 + (2 - y)^2 = (4 - x)^2 + (2 - y)^2$$

$$4 + x^2 + 4x + 4 + y^2 - 4y = 16 + x^2 - 8x + 4 + y^2 - 4y$$

$$x^2 + 4x + y^2 - 4y - x^2 + 8x - y^2 + 4y = 20 - 4 - 4$$

$$12x = 12$$

$$x = \frac{12}{12} = 1$$

and

$$|PA|^2 = |PB|^2$$

$$(5 - x)^2 + (3 - y)^2 = (-2 - x)^2 + (2 - y)^2$$

$$(5 - 1)^2 + 9 + y^2 - 6y = (-2 - 1)^2 + 4 + y^2 - 4y$$

$$16 + 9 + y^2 - 6y = 9 + 4 + y^2 - 4y$$

$$y^2 - 6y + 4y - y^2 = 13 - 16 - 9$$

$$-2y = -12$$

$$y = \frac{-12}{-2} = 6$$

$\therefore$ $P(x, y) = P(1, 6)$

Radius of circum-circle $= |PA|$

$$= \sqrt{(1-5)^2 + (6-3)^2}$$

$$= \sqrt{(-4)^2 + (3)^2}$$

$$= \sqrt{16 + 9}$$

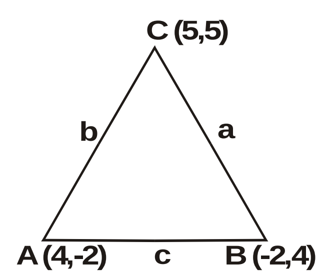
$$= \sqrt{25}$$

$$= 5 \quad \text{Ans}$$

Q.17: The points (4, – 2), (–2, 4) and (5, 5) are the vertices of a triangle. Find in-centre of the triangle.

Solution:

Let A(4, – 2), B(–2, 4) and C(5, 5) be the three given vertices of a ΔABC.



$$a = |BC| = \sqrt{(5+2)^2 + (5-4)^2}$$

$$= \sqrt{(7)^2 + (1)^2}$$

$$= \sqrt{49 + 1}$$

$$= \sqrt{50}$$

$$= 5\sqrt{2}$$

$$b = |AC| = \sqrt{(5-4)^2 + (5+2)^2}$$

$$= \sqrt{(1)^2 + (7)^2} = \sqrt{1 + 49}$$

$$= \sqrt{50} = 5\sqrt{2}$$

$$c = |AB| = \sqrt{(-2-4)^2 + (4+2)^2}$$

$$= \sqrt{(-6)^2 + (6)^2}$$
$$= \sqrt{36 + 36}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2}$$

In centre of a Δ ABC is
$$\left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right)$$

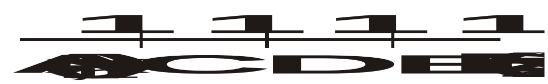
$$\Delta \text{ ABC} = \left(\frac{5\sqrt{2}(4) + 5\sqrt{2}(-2) + 6\sqrt{2}(5)}{5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}}, \frac{5\sqrt{2}(-2) + 5\sqrt{2}(4)}{5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}}\right)$$
$$= \left(\frac{\sqrt{2}(20 - 10 + 30)}{16\sqrt{2}}, \frac{\sqrt{2}(-10 + 20 + 30)}{16\sqrt{2}}\right)$$
$$= \left(\frac{40}{16}, \frac{40}{16}\right)$$
$$= \left(\frac{5}{2}, \frac{5}{2}\right) \quad \text{Ans.}$$

Q.18: Find the points that divide the line segment joining A (x_1, y_1) and B (x_2, y_2) into four equal parts.

Solution:

A (x_1, y_1) , B (x_2, y_2)

Let C, D and E be the required points.



$$\text{Coordinates of C} = \left(\frac{1(x_2) + 3(x_1)}{1 + 3}, \frac{1(y_2) + 3(y_1)}{1 + 3}\right)$$
$$= \left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right)$$
$$\text{Coordinates of D} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$\text{Coordinates of E} = \left(\frac{1(x_1) + 3(x_2)}{1 + 3}, \frac{1(y_1) + 3(y_2)}{1 + 3}\right)$$
$$= \left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right) \quad \text{Ans.}$$

EXERCISE 4.2

Q.1: The two points P and O$^{/}$ are given in xy – coordinate system. Find the XY-Coordinates of P referred to the translated axes O$^{/}$X and O$^{/}$Y.

(i) P(3, 2) ; O$^{/}$(1, 3) **(ii) P(–2, 6); O/(–3, 2) (Lhr. Board 2011)**

(iii) P(–6, –8); O$^{/}$(– 4, – 6) **(iv) P$\left(\frac{3}{2}, \frac{5}{2}\right)$; O/$\left(-\frac{1}{2}, \frac{7}{2}\right)$**