

$$\begin{aligned}
 &= \sqrt{(-6)^2 + (6)^2} \\
 &= \sqrt{36 + 36} \\
 &= \sqrt{72} \\
 &= 6\sqrt{2}
 \end{aligned}$$

In centre of a  $\Delta ABC$  is  
 $\left( \frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c} \right)$

$$\begin{aligned}
 \Delta ABC &= \left( \frac{5\sqrt{2}(4) + 5\sqrt{2}(-2) + 6\sqrt{2}(5)}{5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}}, \frac{5\sqrt{2}(-2) + 5\sqrt{2}(4)}{5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}} \right) \\
 &= \left( \frac{\sqrt{2}(20 - 10 + 30)}{16\sqrt{2}}, \frac{\sqrt{2}(-10 + 20 + 30)}{16\sqrt{2}} \right) \\
 &= \left( \frac{40}{16}, \frac{40}{16} \right) \\
 &= \left( \frac{5}{2}, \frac{5}{2} \right) \quad \text{Ans.}
 \end{aligned}$$

**Q.18:** Find the points that divide the line segment joining A ( $x_1, y_1$ ) and B ( $x_2, y_2$ ) into four equal parts.

**Solution:**

A ( $x_1, y_1$ ), B ( $x_2, y_2$ )

Let C, D and E be the required points.



$$\text{Coordinates of } C = \left( \frac{1(x_2) + 3(x_1)}{1+3}, \frac{1(y_2) + 3(y_1)}{1+3} \right)$$

$$= \left( \frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4} \right)$$

$$\text{Coordinates of } D = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\text{Coordinates of } E = \left( \frac{1(x_1) + 3(x_2)}{1+3}, \frac{1(y_1) + 3(y_2)}{1+3} \right)$$

$$= \left( \frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4} \right) \quad \text{Ans.}$$

### EXERCISE 4 . 2

**Q.1:** The two points P and O' are given in xy – coordinate system. Find the XY-Coordinates of P referred to the translated axes O'X and O'Y.

(i) P(3, 2); O'(1, 3)      (ii) P(-2, 6); O'(-3, 2) (Lhr. Board 2011)

(iii) P(-6, -8); O'(-4, -6)      (iv) P( $\frac{3}{2}, \frac{5}{2}$ ); O'( $-\frac{1}{2}, \frac{7}{2}$ )

Solution:

(i)  $P(3, 2); O'(1, 3)$

$$x = 3, y = 2, h = 1, k = 3$$

By using

$$X = x - h, Y = y - k$$

$$X = 3 - 1, Y = 2 - 3$$

$$X = 2, Y = -1$$

$$\therefore P(X, Y) = P(2, -1) \quad \text{Ans}$$

(ii)  $P(-2, 6); O'(-3, 2)$

$$x = -2, y = 6, h = -3, k = 2$$

By using

$$X = x - h, Y = y - k$$

$$X = -2 + 3, Y = 6 - 2$$

$$X = 1, Y = 4$$

$$\therefore P(X, Y) = P(1, 4) \quad \text{Ans}$$

(iii)  $P(-6, -8); O'(-4, -6)$

$$x = -6, y = -8, h = -4, k = -6$$

By using

$$X = x - h, Y = y - k$$

$$X = -6 + 4, Y = -8 + 6$$

$$X = -2, Y = -2$$

$$\therefore P(X, Y) = P(-2, -2) \quad \text{Ans}$$

(iv)  $P\left(\frac{3}{2}, \frac{5}{2}\right); O'\left(\frac{-1}{2}, \frac{7}{2}\right)$

$$x = \frac{3}{2}, y = \frac{5}{2}, h = \frac{-1}{2}, k = \frac{7}{2}$$

By using

$$X = x - h, Y = y - k$$

$$X = \frac{3}{2} + \frac{1}{2}, Y = \frac{5}{2} - \frac{7}{2}$$

$$X = \frac{3+1}{2}, Y = \frac{5-7}{2}$$

$$X = \frac{4}{2}, Y = \frac{-2}{2}$$

$$X = 2, Y = -1$$

$$\therefore P(X, Y) = P(2, -1) \quad \text{Ans}$$

**Q.2:** The xy-coordinate axes are translated through the point  $O'$  whose coordinates are given in xy-coordinate system. The coordinates of P are given in the XY-coordinate system. Find the coordinates of P in xy-coordinate system.

(i)  $P(8, 10); O'(3, 4)$  (ii)  $P(-5, -3); O'(-2, -6)$

(iii)  $P\left(\frac{-3}{4}, \frac{-7}{6}\right); O'\left(\frac{1}{4}, \frac{-1}{6}\right)$  (iv)  $P(4, -3); O'(-2, 3)$

**Solution:**

(i)  $P(8, 10); O'(3, 4)$

$$X = 8, Y = 10, h = 3, k = 4$$

By using

$$X = x - h, Y = y - k$$

$$8 = x - 3, 10 = y - 4$$

$$x = 8 + 3, y = 10 + 4$$

$$x = 11, y = 14$$

$$\therefore P(x, y) = P(11, 14) \quad \text{Ans}$$

(ii)  $P(-5, -3); O'(-2, -6)$

$$X = -5, Y = -3, h = -2, k = -6$$

By using

$$X = x - h, Y = y - k$$

$$-5 = x + 2, -3 = y + 6$$

$$x = -5 - 2, y = -3 - 6$$

$$x = -7, y = -9$$

$$\therefore P(x, y) = P(-7, -9) \quad \text{Ans}$$

(iii)  $P\left(\frac{-3}{4}, \frac{-7}{6}\right); O'\left(\frac{1}{4}, \frac{-1}{6}\right)$

$$X = \frac{-3}{4}, Y = \frac{-7}{6}, h = \frac{1}{4}, k = \frac{-1}{6}$$

By using

$$X = x - h, Y = y - k$$

$$\frac{-3}{4} = x - \frac{1}{4}, \frac{-7}{6} = y + \frac{1}{6}$$

$$x = \frac{-3}{4} + \frac{1}{4}, y = \frac{-7}{6} - \frac{1}{6}$$

$$x = \frac{-3 + 1}{4}, y = \frac{-7 - 1}{6}$$

$$x = \frac{-2}{4}, \quad y = \frac{-8}{6}$$

$$x = \frac{-1}{2}, \quad y = \frac{-4}{3}$$

$$\therefore P(x, y) = P\left(\frac{-1}{2}, \frac{-4}{3}\right) \quad \text{Ans.}$$

(iv) **P(4, -3) ; O'(-2, 3)**

$$X = 4, \quad Y = -3, \quad h = -2, \quad k = 3$$

By using

$$X = x - h, \quad Y = y - k$$

$$4 = x + 2, \quad -3 = y - 3$$

$$x = 4 - 2, \quad y = -3 + 3$$

$$x = 2, \quad y = 0$$

$$\therefore P(x, y) = P(2, 0) \quad \text{Ans}$$

**Q.3:** The xy-coordinate axes are rotated about the origin through the indicated angle. The new axes are OX and OY. Find the XY-coordinates of the point P with the given xy-coordinates.

**Solution:**

(i) **P(5, 3) ;  $\theta = 45^\circ$**

$$x = 5, \quad y = 3$$

By using

$$X = x \cos \theta + y \sin \theta, \quad Y = y \cos \theta - x \sin \theta$$

$$X = 5 \cos 45^\circ + 3 \sin 45^\circ, \quad Y = 3 \cos 45^\circ - 5 \sin 45^\circ$$

$$X = \frac{5}{\sqrt{2}} + \frac{3}{\sqrt{2}}, \quad Y = \frac{3}{\sqrt{2}} - \frac{5}{\sqrt{2}}$$

$$X = \frac{5+3}{\sqrt{2}}, \quad Y = \frac{3-5}{\sqrt{2}}$$

$$X = \frac{8}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}, \quad Y = \frac{-2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$X = \frac{8\sqrt{2}}{2}, \quad Y = \frac{-2\sqrt{2}}{2}$$

$$X = 4\sqrt{2}, \quad Y = -\sqrt{2}$$

$$\therefore P(X, Y) = P(4\sqrt{2}, -\sqrt{2}) \quad \text{Ans}$$

(ii) **P(3, -7) ;  $\theta = 30^\circ$**

$$x = 3, \quad y = -7$$

By using

$$X = x \cos \theta + y \sin \theta, \quad Y = y \cos \theta - x \sin \theta$$

$$\begin{aligned} X &= 3 \cos 30^\circ - 7 \sin 30^\circ & Y &= -7 \cos 30^\circ - 3 \sin 30^\circ \\ X &= \frac{3\sqrt{3}}{2} - \frac{7}{2}, \quad Y = \frac{-7\sqrt{3}}{2} - \frac{3}{2} \\ X &= \frac{3\sqrt{3}-7}{2}, \quad Y = \frac{-7\sqrt{3}-3}{2} \\ \therefore P(X, Y) &= P\left(\frac{3\sqrt{3}-7}{2}, \frac{-7\sqrt{3}-3}{2}\right) \text{ Ans} \end{aligned}$$

(iii) **P(11, -15);  $\theta = 60^\circ$**

$$x = 11, \quad y = -15$$

By using

$$\begin{aligned} X &= x \cos \theta + y \sin \theta, \quad Y = y \cos \theta - x \sin \theta \\ X &= 11 \cos 60^\circ - 15 \sin 60^\circ, \quad Y = -15 \cos 60^\circ - 11 \sin 60^\circ \\ X &= \frac{11}{2} - \frac{15\sqrt{3}}{2} & Y &= \frac{-15}{2} - \frac{11\sqrt{3}}{2} \\ X &= \frac{11-15\sqrt{3}}{2} & Y &= \frac{-15-11\sqrt{3}}{2} \\ \therefore P(X, Y) &= P\left(\frac{11-15\sqrt{3}}{2}, \frac{-15-11\sqrt{3}}{2}\right) \text{ Ans} \end{aligned}$$

(iv) **P(15, 10);  $\theta = \arctan \frac{1}{3}$**

$$x = 15, \quad y = 10, \quad \theta = \tan^{-1} \frac{1}{3}$$

$$\tan \theta = \frac{1}{3}$$

$$\sin \theta = \frac{1}{\sqrt{10}}, \quad \cos \theta = \frac{3}{\sqrt{10}}$$

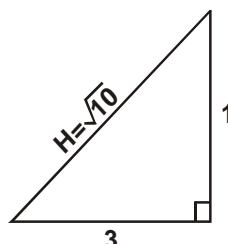
$$H^2 = (3)^2 + (1)^2$$

$$H^2 = 9 + 1$$

$$H^2 = 10$$

$$H = \sqrt{10}$$

By using



$$X = x \cos \theta + y \sin \theta, \quad Y = y \cos \theta - x \sin \theta$$

$$X = 15\left(\frac{3}{\sqrt{10}}\right) + 10\left(\frac{1}{\sqrt{10}}\right)$$

$$X = \frac{45}{\sqrt{10}} + \frac{10}{\sqrt{10}}$$

$$X = \frac{45+10}{\sqrt{10}} = \frac{55}{\sqrt{10}}$$

$$Y = 10\left(\frac{3}{\sqrt{10}}\right) - 15\left(\frac{1}{\sqrt{10}}\right)$$

$$Y = \frac{30}{\sqrt{10}} - \frac{15}{\sqrt{10}}$$

$$Y = \frac{30-15}{\sqrt{10}}$$

$$Y = \frac{15}{\sqrt{10}}$$

$$\therefore P(X, Y) = P\left(\frac{55}{\sqrt{10}}, \frac{15}{\sqrt{10}}\right) \quad \text{Ans.}$$

**Q.4:** The xy-coordinate axes are rotated about the origin through the indicated angle and the new axes are OX and OY. Find the xy-coordinates of P with the given XY-Coordinates.

$$(i) \quad P(-5, 3); \quad \theta = 30^\circ$$

$$(ii) \quad P(-7\sqrt{2}, 5\sqrt{2}); \quad \theta = 45^\circ$$

**Solution:**

$$(i) \quad P(-5, 3); \quad \theta = 30^\circ$$

$$X = -5, \quad Y = 3$$

$$\text{By using } X = x \cos \theta + y \sin \theta, \quad Y = y \cos \theta - x \sin \theta$$

$$-5 = x \cos 30^\circ + y \sin 30^\circ, \quad 3 = y \cos 30^\circ - x \sin 30^\circ$$

$$-5 = \frac{x\sqrt{3}}{2} + \frac{y}{2}, \quad 3 = \frac{y\sqrt{3}}{2} - \frac{x}{2}$$

$$3 = \frac{\sqrt{3}y - x}{2}$$

$$-5 = \frac{\sqrt{3}x + y}{2}, \quad \sqrt{3}y - x = 6 \quad \dots\dots (2)$$

$$\sqrt{3}x + y = -10 \quad \dots\dots (1)$$

Equation (1) + Equation (2)  $\times \sqrt{3}$ , we get

$$\sqrt{3}x + y = -10$$

$$3y - \sqrt{3}x = 6\sqrt{3}$$

$$\underline{4y = -10 + 6\sqrt{3}}$$

$$y = \frac{2(-5 + 3\sqrt{3})}{4}$$

$$\begin{aligned} y &= \frac{-5 + 3\sqrt{3}}{2} \\ &= \frac{3\sqrt{3} - 5}{2} \end{aligned}$$

Put  $y = \frac{3\sqrt{3} - 5}{2}$  in equation (1)

$$\sqrt{3}x + y = -10$$

$$\sqrt{3}x = -10 - y$$

$$\sqrt{3}x = -10 - \frac{3\sqrt{3} - 5}{2}$$

$$\sqrt{3}x = \frac{-20 - 3\sqrt{3} + 5}{2}$$

$$x = \frac{-15 - 3\sqrt{3}}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{3(-5 - \sqrt{3})\sqrt{3}}{2(3)}$$

$$= \frac{-5\sqrt{3} - 3}{2}$$

$$\therefore P(x, y) = P\left(\frac{-5\sqrt{3} - 3}{2}, \frac{3\sqrt{3} - 5}{2}\right) \quad \text{Ans}$$

(ii)  $P(-7\sqrt{2}, 5\sqrt{2})$ ;  $\theta = 45^\circ$

$$X = -7\sqrt{2}, Y = 5\sqrt{2}$$

By using

$$X = x \cos \theta + y \sin \theta,$$

$$-7\sqrt{2} = x \cos 45^\circ + y \sin 45^\circ$$

$$Y = y \cos \theta - x \sin \theta$$

$$5\sqrt{2} = y \cos 45^\circ - x \sin 45^\circ$$

$$-7\sqrt{2} = \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}, \quad 5\sqrt{2} = \frac{y}{\sqrt{2}} - \frac{x}{\sqrt{2}}$$

$$-7\sqrt{2} = \frac{x+y}{\sqrt{2}}, \quad 5\sqrt{2} = \frac{y-x}{\sqrt{2}}$$

$$x+y = -14 \quad \dots\dots (1) \quad y-x = 10 \quad \dots\dots (2)$$

Adding equation (1) and equation (2), we get

$$x + y = -14$$

$$\underline{y - x = 10}$$

$$2y = -4$$

$$y = \frac{-4}{2} = -2$$

Put  $y = -2$  in equation (1)

$$x - 2 = -14$$

$$x = -14 + 2$$

$$= -12$$

$$\therefore P(x, y) = P(-12, -2) \quad \text{Ans}$$

### EXERCISE 4 . 3

**Q.1 Find the slope and inclination of the line joining the points:**

- (i)  $(-2, 4)$  ;  $(5, 11)$
- (ii)  $(3, -2)$  ;  $(2, 7)$
- (iii)  $(4, 6)$  ;  $(4, 8)$

**Solution:**

- (i)  $(-2, 4)$  ;  $(5, 11)$

Let  $A(-2, 4)$  ;  $B(5, 11)$

$$\text{Slope of line } AB = m = \frac{11 - 4}{5 + 2}$$

$$= \frac{7}{7} = 1$$

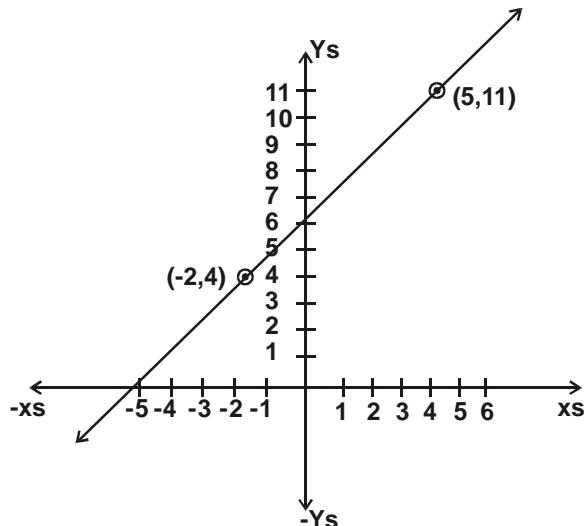
$$\tan \alpha = m$$

$$\tan \alpha = 1$$

$$\alpha = \tan^{-1}(1) = 45^\circ$$

- (ii)  $(3, -2)$  ;  $(2, 7)$

Let  $A(3, -2)$  ,  $B(2, 7)$



$$\text{Slope of line } AB = m = \frac{7 + 2}{2 - 3} = \frac{9}{-1} = -9$$

$$\tan \alpha = m = -9$$

$$\alpha = \tan^{-1}(-9) = 180^\circ - \tan^{-1} 9$$

$$= 180 - 83.66^\circ$$

$$= 96.34^\circ \quad \text{Ans.}$$