Adding equation (1) and equation (2), we get

$$x + y = -14$$
$$\underline{y - x = 10}$$
$$2y = -4$$
$$y = \frac{-4}{2} = -2$$

Put $y = -2$ in equation (1)

$$x - 2 = -14$$
$$x = -14 + 2$$
$$= -12$$

$\therefore$ $P(x, y) = P(-12, -2)$ Ans

EXERCISE 4.3

Q.1 Find the slope and inclination of the line joining the points:

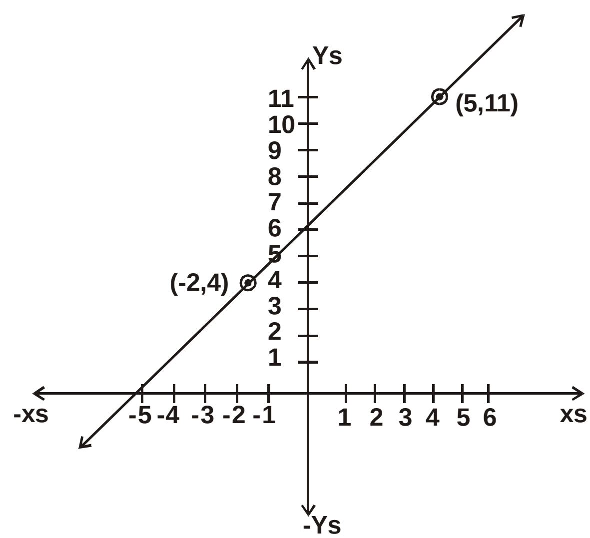
(i) (– 2, 4) ; (5, 11) (ii) (3, – 2) ; (2, 7)

(iii) (4, 6) ; (4, 8)

Solution:

(i) $(-2, 4)$; $(5, 11)$

Let $A(-2, 4)$; $B(5, 11)$



Slope of line AB = $m = \frac{11-4}{5+2}$

$$= \frac{7}{7} = 1$$

$\tan\alpha = m$

$\tan\alpha = 1$

$\alpha = \tan^{-1}(1) = 45^\circ$

(ii) $(3, -2)$; $(2, 7)$

Let $A(3, -2)$, $B(2, 7)$

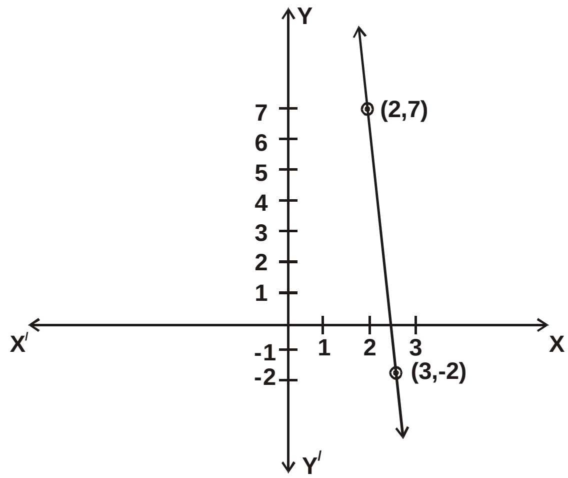
Slope of line AB $= m = \frac{7+2}{2-3} = \frac{9}{-1} = -9$

$\tan\alpha = m = -9$

$\alpha = \tan^{-1}(-9) = 180^\circ - \tan^{-1} 9$

$= 180 - 83.66^\circ$

$= 96.34^\circ$ Ans.



(iii) (4, 6) ; (4, 8)

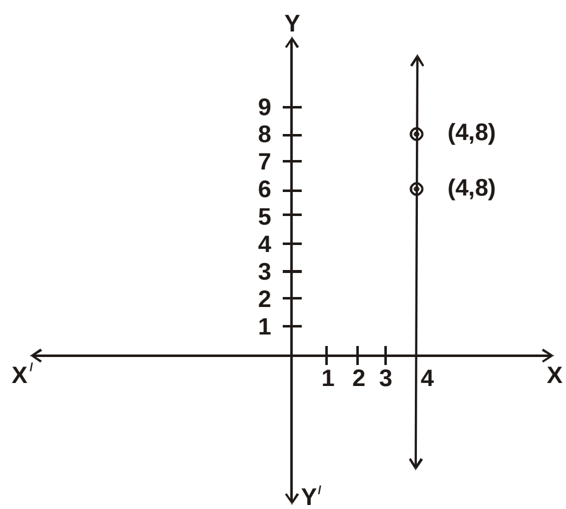
Let A(4, 6) , B(4, 8)

Slope of line AB = $\frac{8-6}{4-4} = \frac{2}{0} = \infty$ (Undefined)

$m = \tan\alpha = \infty$

$\alpha = \tan^{-1}(\infty)$

$\alpha = 90^{o}$



Q.2 In the triangle A (8, 6), B(– 4, 2), C(– 2, – 6) find the slope of

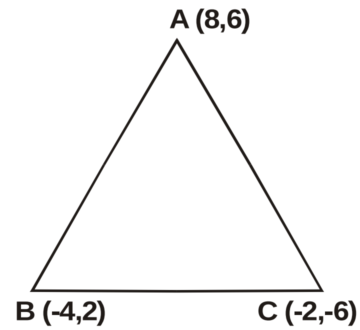
(i) each side of the triangle

(ii) each median of the triangle

(iii) each altitude of the triangle

Solution:

A (8, 6), B (– 4, 2), C (– 2, – 6)



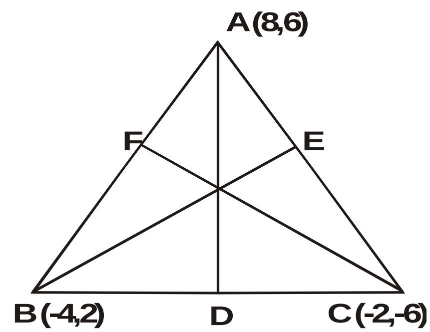
(i) Slope of side AB $= \dfrac{2-6}{-4-8}$

$= \dfrac{-4}{-12} = \dfrac{1}{3}$

Slope of side BC $= \dfrac{-6-2}{-2+4} = \dfrac{-8}{2} = -4$

Slope of side AC $= \dfrac{-6-6}{-2-8} = \dfrac{-12}{-10} = \dfrac{6}{5}$

(ii)



Let $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ be the medians of a triangle ABC and D, E, F are the mid points of sides $\overline{BC}$, $\overline{AC}$ and $\overline{AB}$ respectively.

Coordinates of D $= \left(\dfrac{-4-2}{2}, \dfrac{2-6}{2}\right)$

$= \left(\dfrac{-6}{2}, \dfrac{-4}{2}\right) = (-3, -2)$

Coordinates of E $= \left(\dfrac{8-2}{2}, \dfrac{6-6}{2}\right)$

$= \left(\dfrac{6}{2}, \dfrac{0}{2}\right) = (3, 0)$

Coordinates of F $= \left(\dfrac{8-4}{2}, \dfrac{6+2}{2}\right)$

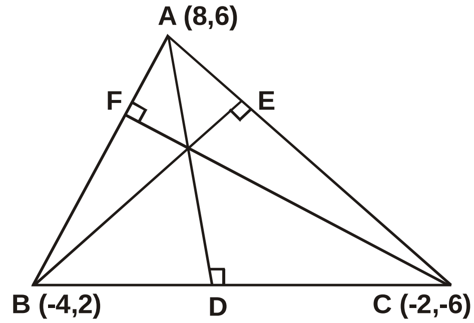
$= \left(\dfrac{4}{2}, \dfrac{8}{2}\right) = (2, 4)$

Slope of median AD $= \dfrac{-2-6}{-3-8} = \dfrac{-8}{-11} = \dfrac{8}{11}$

Slope of median BE $= \dfrac{0-2}{3+4} = \dfrac{-2}{7}$

Slope of median CF $= \dfrac{4+6}{2+2} = \dfrac{10}{4} = \dfrac{5}{2}$

(iii) Let $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ are the altitudes of ΔABC. Since altitudes are perpendicular to the sides.



$$\text{Slope of altitude } AD = \frac{-1}{\text{Slope of side BC}} = \frac{-1}{-4} = \frac{1}{4}$$

$$\text{Slope of altitude } BE = \frac{-1}{\text{Slope of side AC}} = \frac{-1}{\frac{6}{5}} = \frac{-5}{6}$$

$$\text{Slope of altitude } CF = \frac{-1}{\text{Slope of side AB}} = \frac{-1}{\frac{1}{3}} = -3$$

Q.3 By means of slopes, show that the following points lie on the same line.

(a) (– 1, – 3) ; (1, 5) ; (2, 9)
(b) (4, – 5) ; (7, 5) ; (10, 15)
(c) (– 4, 6) ; (3, 8) ; (10, 10)
(d) (a, 2b) ; (c, a + b); (2c – a, 2a)

Solution:

(a) (– 1, – 3) ; (1, 5) ; (2, 9)

Let A(–1, – 3) , B(1, 5) , C(2, 9)

$$\text{Slope of AB} = \frac{5+3}{1+1} = \frac{8}{2} = 4$$

$$\text{Slope of BC} = \frac{9-5}{2-1} = \frac{4}{1} = 4$$

$\therefore$ Slope of AB = Slope of BC

Show A, B and C lie on the same line.

(b) (4, – 5) ; (7, 5) ; (10, 15) (Guj. Board 2006)

Let A(4, – 5), B(7, 5), C(10, 15)

$$\text{Slope of AB} = \frac{5+5}{7-4} = \frac{10}{3}$$

Slope of BC $= \dfrac{15-5}{10-7} = \dfrac{10}{3}$

$\therefore$ Slope of AB = Slope of BC

Shows A, B and C lie on the same line.

(c) (– 4, 6) ; (3, 8); (10, 10)

Let A (– 4, 6) , B (3, 8), C(10, 10)

Slope of AB $= \dfrac{8-6}{3+4} = \dfrac{2}{7}$

Slope of BC $= \dfrac{10-8}{10-3} = \dfrac{2}{7}$

$\therefore$ Slope of AB = Slope of BC

Shows A, B and C lie on the same line.

(d) (a, 2b) ; (c, a + b); (2c – a, 2a)

Let A(a, 2b) , B(c, a + b), C(2c – a, 2a)

Slope of AB $= \dfrac{a+b-2b}{c-a} = \dfrac{a-b}{c-a}$

Slope of BC $= \dfrac{2a-(a+b)}{2c-a-c} = \dfrac{2a-a-b}{c-a} = \dfrac{a-b}{c-a}$

$\therefore$ Slope of AB = Slope of BC

Shows the points A, B and C lie on the same line.

Q.4 Find K so that the line joining A (7, 3); B (K, – 6) and the line joining C (– 4, 5); D(– 6, 4) are (i) Parallel (ii) Perpendicular

Solution:

A (7, 3) ; B(K, – 6)

Let m_1 be the slope of line AB.

$m_1 = \dfrac{-6-3}{K-7} = \dfrac{-9}{K-7}$

C (– 4, 5) ; D (– 6, 4)

Let m_2 be the slope of line CD

$m_2 = \dfrac{4-5}{-6+4} = \dfrac{-1}{-2} = \dfrac{1}{2}$

(i) Since the lines AB and CD are parallel

$\therefore m_1 = m_2$

$\dfrac{-9}{K-7} = \dfrac{1}{2}$

$-18 = K-7$

$-18+7 = K$

$\boxed{K = -11}$ Ans

(ii) Since the lines AB and CD are perpendicular.

$$\therefore\ m_1 \times m_2 = -1$$
$$\left(\frac{-9}{K-7}\right)\left(\frac{1}{2}\right) = -1$$
$$9 = 2(K-7)$$
$$9 = 2K - 14$$
$$9 + 14 = 2K$$
$$2K = 23$$
$$\boxed{K = \frac{23}{2}} \quad \text{Ans.}$$

Q.5: Using slopes, show that the triangle with its vertices A (6, 1), B (2, 7) and C(– 6, – 7) is a right triangle.

Solution:

A (6, 1), B (2, 7), C (– 6, – 7)

Let m_1, m_2 and m_3 be the slopes of sides AB, BC and AC respectively.

$$m_1 = \frac{7-1}{2-6} = \frac{6}{-4} = \frac{-3}{2}$$
$$m_2 = \frac{-7-7}{-6-2} = \frac{-14}{-8} = \frac{7}{4}$$
$$m_3 = \frac{-7-1}{-6-6} = \frac{-8}{-12} = \frac{2}{3}$$

Since $m_1 \times m_3 = \frac{-3}{2} \times \frac{2}{3}$

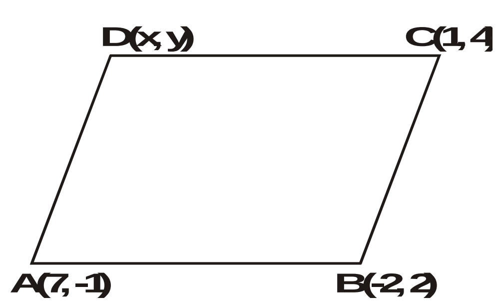
$$m_1 \times m_3 = -1$$

$\therefore$ The side AB and AC are perpendicular.

Shows the triangle ABC is a right angle triangle.

Q.6 The three points A (7, – 1), B (–2, 2) and C (1, 4) are consecutive vertices of a parallelogram. Find the fourth vertex.

Solution:



A (7, –1), B (– 2, 2), C (1, 4)

Let D (x, y) be the required vertex of a parallelogram.

$$\text{Slope of } \overline{AB} = \frac{2+1}{-2-7}$$
$$= \frac{3}{-9} = \frac{-1}{3}$$

$$\text{Slope of } \overline{CD} = \frac{y-4}{x-1}$$

$$\text{Slope of } \overline{AD} = \frac{y+1}{x-7}$$

$$\text{Slope of } \overline{BC} = \frac{4-2}{1+2} = \frac{2}{3}$$

Since ABCD is a parallelogram.

$\therefore$ Slope of $\overline{AB}$ = Slope of $\overline{CD}$

$$\frac{-1}{3} = \frac{y-4}{x-1}$$

$$-x+1 = 3y-12$$

$$1+12 = x+3y$$

$$x+3y = 13 \quad \ldots (1)$$

Slope of $\overline{AD}$ = Slope of $\overline{BC}$

$$\frac{y+1}{x-7} = \frac{2}{3}$$

$$3y+3 = 2x-14$$

$$3+14 = 2x-3y$$

$$2x-3y = 17 \quad \ldots (2)$$

Eq. (1) + Eq. (2), we get

$$\begin{aligned} x+3y &= 13 \\ 2x-3y &= 17 \\ \hline 3x &= 30 \end{aligned}$$

$$x = \frac{30}{3} = 10$$

Put $x = 10$ in equation (1)

$$10+3y = 13$$

$$3y = 13-10$$

$$y = \frac{3}{3} = 1$$

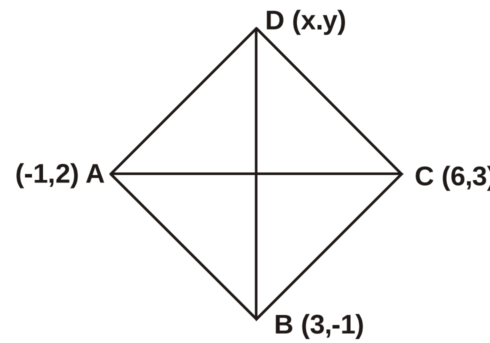
$\therefore$ D (10, 1) is the fourth vertex of parallelogram.

Q.7 The points A (– 1, 2), B (3, – 1) and C (6, 3) are consecutive vertices of a rhombus. Find the fourth vertex and show that the diagonals of the rhombus are perpendicular to each other.

Solution:

Let A (– 1, 2), B (3, – 1), C (6, 3)

Let D (x, y) by the fourth vertex of rhombus.



$$\text{Slope of side AB} = \frac{-1-2}{3+1} = \frac{-3}{4}$$

$$\text{Slope of side BC} = \frac{3+1}{6-3} = \frac{4}{3}$$

$$\text{Slope of side CD} = \frac{y-3}{x-6}$$

$$\text{Slope of side DA} = \frac{2-y}{-1-x}$$

Since ABCD is a rhombus

$\therefore$ Slope of side AB = Slope of side CD

$$\frac{-3}{4} = \frac{y-3}{x-6}$$

$$-3(x-6) = 4(y-3)$$

$$-3x + 18 = 4y - 12$$

$$18 + 12 = 3x + 4y$$

$$3x + 4y = 30 \quad \text{...... (1)}$$

Slope of side BC = Slope of side DA

$$\frac{4}{3} = \frac{2-y}{-1-x}$$

$$4(-1-x) = 3(2-y)$$

$$-4 - 4x = 6 - 3y$$

$$-4 - 6 = 4x - 3y$$

$$4x - 3y = -10 \quad \text{...... (2)}$$

Eq. (1) × 3 + Eq, (2) × 4, we get

$$9x + 12y = 90$$

$$16x - 12y = -40$$

$$25x = 50$$

$$x = \frac{50}{25} = 2$$

Put $x = 2$ in eq. (1)

$$3(2) + 4y = 30$$
$$6 + 4y = 30$$
$$4y = 30 - 6$$
$$y = \frac{24}{4} = 6$$

$\therefore$ D (2, 6) is the fourth vertex of rhombus.

$$\text{Slope of diagonal } \overline{AC} = \frac{3-2}{6+1} = \frac{1}{7}$$

$$\text{Slope of diagonal } \overline{BD} = \frac{y+1}{x-3}$$
$$= \frac{6+1}{2-3} = -7$$

Since (Slope of diagonal $\overline{AC}$) (Slope of diagonal $\overline{BD}$) $= -1$

Shows the diagonals $\overline{AC}$ and $\overline{BD}$ are perpendicular to each other.

Q.8 Two pairs of points are given. Find whether the two lines determined by these points are

(i) Parallel (ii) Perpendicular (iii) None

(a) (1, – 2), (2, 4) and (4, 1) , (– 8, 2)

(b) (– 3, 4), (6, 2) and (4, 5) , (– 2, – 7)

Solution:

(a) (1, – 2), (2, 4) and (4, 1) (– 8, 2)

Let A (1, – 2), B (2, 4) and C (4, 1), D (– 8, 2)

Let m_1 and m_2 be the slopes of lines AB and CD.

$$m_1 = \frac{4+2}{2-1} = \frac{6}{1} = 6$$

$$m_2 = \frac{2-1}{-8-4} = \frac{1}{-12} = \frac{-1}{12}$$

Since

$$m_1 \times m_2 = 6 \times \frac{-1}{12} \neq -1$$

and $m_1 \neq m_2$

So the given two lines neither parallel nor perpendicular.

(b) (– 3, 4), (6, 2) and (4, 5) , (– 2, – 7)

Let A (– 3, 4), B (6, 2) and C (4, 5), (– 2, – 7)

Let m_1 and m_2 be the slope of lines AB and CD.

$$m_1 = \frac{2-4}{6+3} = \frac{-2}{9}$$

$$m_2 = \frac{-7-5}{-2-4} = \frac{-12}{-6} = 2$$

Since $m_1 \times m_2 \neq -1$ and $m_1 \neq m_2$

So the given lines are neither parallel nor perpendicular.

Q.9: Find an equation of

(a) the horizontal line through (7, – 9)

(b) the vertical line through (– 5, 3)

(c) the line bisecting the first and 3rd quadrants.

(d) the line bisecting the second and 4th quadrant.

Solution:

(a) the horizontal line through (7, – 9)

Slope of horizontal line $= m = \tan 0^\circ = 0$

Equation of line is

$$y - y_1 = m(x - x_1)$$

$$y + 9 = 0(x - 7)$$

$$\boxed{y + 9 = 0}$$

$$\boxed{y = -9}$$ Ans

(b) the vertical line through (– 5, 3)

As line is vertical and passing through (–5, 3) and eq. of vertical line is $x = c$

here $\boxed{x = -5}$

(c) the line bisecting the first and 3rd quadrant passing through origin.

Slope $= m = \tan 45^\circ = 1$ (Guj. Board 2006)

Eq. of line is

$$y = mx$$

$$\boxed{y = x}$$ Ans

(d) The line bisecting the 2^{nd} and 4^{th} quadrant passing through origin.

Slope = m = tan 135° = – 1

Eq. of line is

$$y = mx$$
$$y = -x$$
$$\boxed{x = -y} \quad \text{Ans}$$

Q.10: Find an equation of the line

(a) through A (– 6, 5) having slope 7

(b) through (8, – 3) having slope 0

(c) through (– 8, 5) having slope undefined (Lhr. Board 2009 (S))

(d) through (–5, – 3) and (9, – 1) (Lhr. Board 2009)

(e) y – intercept = – 7 and Slope = – 5

(f) x – intercept = – 3 and y – intercept = 4

(g) x – intercept = 9 and Slope = – 4 (Lhr. Board 2008, 2011)

Solution:

(a) through A (– 6, 5) having slope 7

$$y - y_1 = m(x - x_1)$$
$$y - 5 = 7(x + 6)$$
$$y - 5 = 7x + 42$$
$$0 = 7x + 42 - y + 5$$
$$7x - y + 47 = 0 \quad \text{Ans}$$

(b) through (8, – 3) having slope 0

$$y - y_1 = m(x - x_1)$$
$$y + 3 = 0(x - 8)$$
$$y + 3 = 0$$
$$y = -3 \quad \text{Ans}$$

(c) through (– 8, 5) having slope undefined

$$y - y_1 = m(x - x_1)$$
$$y - 5 = \infty(x + 8)$$
$$y - 5 = \frac{1}{0}(x + 8)$$
$$x + 8 = 0$$
$$x = -8 \quad \text{Ans}$$

(d) through (– 5, – 3) and (9, – 1)

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y+3}{-1+3} = \frac{x+5}{9+5}$$

$$\frac{y+3}{2} = \frac{x+5}{14}$$

$$14y + 42 = 2x + 10$$

$$0 = 2x + 10 - 14y - 42$$

$$2x - 14y - 32 = 0$$

$$2(x - 7y - 16) = 0$$

$$x - 7y - 16 = 0 \qquad \text{Ans}$$

(e) **y-intercept = – 7 and Slope = – 5**

$$y = mx + c$$

$$y = -5x - 7$$

$$5x + y + 7 = 0 \qquad \text{Ans}$$

(f) **x – intercept = – 3 , y – intercept = 4**

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{-3} + \frac{y}{4} = 1$$

$$\frac{-4x + 3y}{12} = 1$$

$$-4x + 3y = 12$$

$$0 = 12 + 4x - 3y$$

$$4x - 3y + 12 = 0$$

(g)

x – intercept = – 9 , Slope = – 4

Since x – intercept = – 9 mean the curve cuts the x-axis at (– 9, 0)

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -4(x + 9)$$

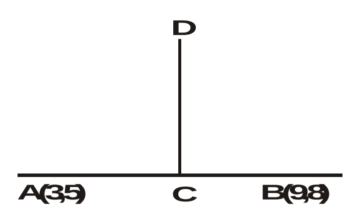
$$y = -4x - 36$$

$$4x + y + 36 = 0 \qquad \text{Ans}$$

Q.11 **Find an equation of the perpendicular bisector of the line segment joining the points A (3, 5) and B (9, 8).**

Solution:

Let $\overline{CD}$ be the perpendicular bisector and C be the mid point of $\overline{AB}$.



$$\text{Coordinates of C} = \left(\frac{3+9}{2}, \frac{5+8}{2}\right)$$

Coordinates of C $= \left(\frac{12}{2}\ ,\ \frac{13}{2}\right)$

Coordinates of C $= \left(6\ ,\ \frac{13}{2}\right)$

Slope of AB $= \frac{8-5}{9-3} = \frac{3}{6} = \frac{1}{2}$

Since $\overline{CD}$ is 1 to $\overline{AB}$

Slope of $\overline{CD}$ $= \frac{-1}{\frac{1}{2}} = -2$

Eq. of line is

$$y - y_1 = m(x - x_1)$$

$$y - \frac{13}{2} = -2\,(x-6)$$

$$\frac{2y-13}{2} = -2x + 12$$

$$2y - 13 = -4x + 24$$

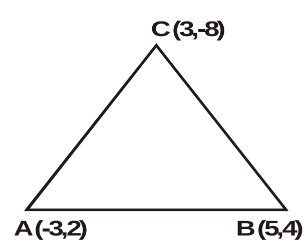
$$4x + 2y - 13 - 24 = 0$$

$$4x + 2y - 37 = 0 \quad \text{Ans}$$

Q.12: Find equations of the sides, altitudes and medians of the triangle whose vertices are A (– 3, 2), B (5, 4) and C (3, – 8). (Lhr. Board 2007, 2011)

Solution:

A (– 3, 2), B (5, 4), C (3, – 8)



<u>Equations of sides</u>

Equation of side AB

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-2}{4-2} = \frac{x+3}{5+3}$$

$$\frac{y-2}{2} = \frac{x+3}{8}$$

$$8y - 16 = 2x + 6$$

$$0 = 2x + 6 - 8y + 16$$

$$2x - 8y + 22 = 0$$

$$2(x - 4y + 11) = 0$$

$$\boxed{x - 4y + 11 = 0} \quad \text{Ans.}$$

Equation of side BC

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 4}{-8 - 4} = \frac{x - 5}{3 - 5}$$

$$\frac{y - 4}{-12} = \frac{x - 5}{-2}$$

$$-2y + 8 = -12x + 60$$

$$12x - 2y + 8 - 60 = 0$$

$$12x - 2y - 52 = 0$$

$$2(6x - y - 26) = 0$$

$$\boxed{6x - y - 26 = 0} \quad \text{Ans}$$

Equation of side AC is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 2}{-8 - 2} = \frac{x + 3}{3 + 3}$$

$$\frac{y - 2}{-10} = \frac{x + 3}{6}$$

$$6y - 12 = -10x - 30$$

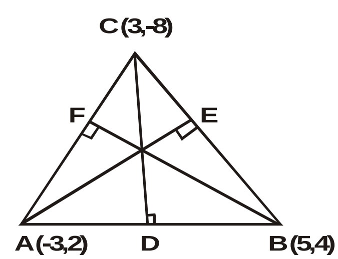
$$10x + 6y - 12 + 30 = 0$$

$$10x + 6y + 18 = 0$$

$$2(5x + 3y + 9) = 0$$

$$\boxed{5x + 3y + 9 = 0} \quad \text{Ans}$$

<u>Equations of altitudes</u>



$$\text{Slope of side AB} = \frac{4 - 2}{5 + 3}$$

$$= \frac{2}{8} = \frac{1}{4}$$

$$\text{Slope of side BC} = \frac{-8 - 4}{3 - 5}$$

$$= \frac{-12}{-2} = 6$$

$$\text{Slope of side AC} = \frac{-8 - 2}{3 + 3}$$

$$= \frac{-10}{6} = \frac{-5}{3}$$

$$\text{Slope of altitude CD} = \frac{-1}{\frac{1}{4}} = -4$$

$$\text{Slope of altitude AE} = \frac{-1}{6}$$

$$\text{Slope of altitude BF} = \frac{-1}{\frac{-5}{3}} = \frac{3}{5}$$

Equation of altitude CD passing through (3, – 8) and having slope – 4 is

$$y - y_1 = m(x - x_1)$$
$$y + 8 = -4(x - 3)$$
$$y + 8 = -4x + 12$$
$$4x + y + 8 - 12 = 0$$
$$\boxed{4x + y - 4 = 0} \quad \text{Ans}$$

Equation of altitude AE passing through (– 3, 2) and having slope $\frac{-1}{6}$ is

$$y - y_1 = m(x - x_1)$$
$$y - 2 = \frac{-1}{6}(x + 3)$$
$$6y - 12 = -x - 3$$
$$x + 6y - 12 + 3 = 0$$
$$\boxed{x + 6y - 9 = 0} \quad \text{Ans}$$

Equations of altitude BF passing through B (5, 4) and having slope $\frac{3}{5}$ is

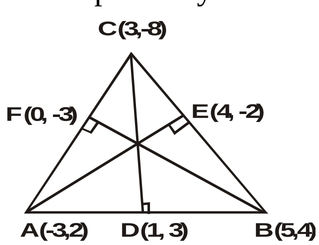
$$y - y_1 = m(x - x_1)$$
$$y - 4 = \frac{3}{5}(x - 5)$$
$$5y - 20 = 3x - 15$$
$$0 = 3x - 15 - 5y + 20$$
$$\boxed{3x - 5y + 5 = 0} \quad \text{Ans}$$

<u>Equations of Medians</u>

Since D, E and F be the mid points of sides AB, BC and AC respectively.

$$\text{Coordinates of D} = \left(\frac{-3+5}{2}, \frac{2+4}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{6}{2}\right) = (1, 3)$$



$$\text{Coordinates of E} = \left(\frac{3+5}{2}, \frac{-8+4}{2}\right)$$

$$= \left(\frac{8}{2}\ ,\ \frac{-4}{2}\right) = (4, -2)$$

Coordinates of F $= \left(\frac{-3+3}{2}\ ,\ \frac{2-8}{2}\right)$

Coordinates of F $= \left(\frac{0}{2}\ ,\ \frac{-6}{2}\right) = (0, -3)$

Equations of median CD is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y+8}{3+8} = \frac{x-3}{1-3}$$

$$\frac{y+8}{11} = \frac{x-3}{-2}$$

$$-2y - 16 = 11x - 33$$

$$0 = 11x - 33 + 2y + 16$$

$$\boxed{11x + 2y - 17 = 0} \quad \text{Ans}$$

Equation of median AE is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y-2}{-2-2} = \frac{x+3}{4+3}$$

$$\frac{y-2}{-4} = \frac{x+3}{7}$$

$$7y - 14 = -4x - 12$$

$$4x + 7y - 14 + 12 = 0$$

$$\boxed{4x + 7y - 2 = 0} \quad \text{Ans}$$

Equation of median BF is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y-4}{-3-4} = \frac{x-5}{0-5}$$

$$\frac{y-4}{-7} = \frac{x-5}{-5}$$

$$-5y + 20 = -7x + 35$$

$$7x - 5y + 20 - 35 = 0$$

$$\boxed{7x - 5y - 15 = 0} \quad \text{Ans}$$

Q.13 Find an equation of the line through (– 4, – 6) and perpendicular to a line having slope $\frac{-3}{2}$. (Lhr. Board 2006, 2007).

Solution:

Slope of given line $= \frac{-3}{2}$

Slope of required line $= \frac{-1}{\frac{-3}{2}} = \frac{2}{3}$

Equation of the line passing through (– 4, – 6) and having slope $\frac{-2}{3}$ is

$$y - y_1 = m(x - x_1)$$
$$y + 6 = \frac{2}{3}(x + 4)$$
$$3y + 18 = 2x + 8$$
$$0 = 2x + 8 - 3y - 18$$
$$2x - 3y - 10 = 0 \quad \text{Ans}$$

Q.14 Find an equation of the line through (11, – 5) and parallel to line with slope – 24.

Solution:

Slope of given line = – 24

Slope of required line = – 24

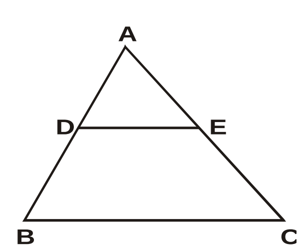
Eq. of line passing through (11, – 5) and having slope – 24 is

$$y - y_1 = m(x - x_1)$$
$$y + 5 = -24(x - 11)$$
$$y + 5 = -24x + 264$$
$$24x + y - 259 = 0 \quad \text{Ans}$$

Q.15 The points A (– 1, 2), B (6, 3) and C (2, – 4) are vertices of a triangle. Show that the line joining the midpoint D of AB and the mid point E of AC is parallel to BC and DE = $\frac{1}{2}$ BC. (Lhr. Board 2006)

Solution:

A (– 1, 2), B (6, 3), C (2, – 4)



Coordinates of D $= \left(\frac{-1+6}{2}, \frac{2+3}{2}\right)$

Coordinates of D $= \left(\frac{5}{2}, \frac{5}{2}\right)$

Coordinates of E $= \left(\frac{-1+2}{2}, \frac{2-4}{2}\right)$

Coordinates of E $= \left(\frac{1}{2}, \frac{-2}{2}\right)$

Coordinates of E $= \left(\frac{1}{2}, -1\right)$

$$\text{Slope of DE} = \frac{-1-\frac{5}{2}}{\frac{1}{2}-\frac{5}{2}} = \frac{\frac{-2-5}{2}}{\frac{1-5}{2}} = \frac{-7}{-4} = \frac{7}{4}$$

$$\text{Slope of BC} = \frac{-4-3}{2-6} = \frac{-7}{-4} = \frac{7}{4}$$

Since Slope of DE = Slope of BC

$\therefore$ DE is parallel to BC

$$|DE| = \sqrt{\left(\frac{1}{2}-\frac{5}{2}\right)^2 + \left(-1-\frac{5}{2}\right)^2}$$

$$|DE| = \sqrt{\left(\frac{1-5}{2}\right)^2 + \left(\frac{-2-5}{2}\right)^2}$$

$$|DE| = \sqrt{\left(\frac{-4}{2}\right)^2 + \left(\frac{-7}{2}\right)^2}$$

$$|DE| = \sqrt{4 + \frac{49}{4}}$$

$$|DE| = \sqrt{\frac{16+49}{4}}$$

$$|DE| = \frac{\sqrt{65}}{2}$$

$$|BC| = \sqrt{(2-6)^2 + (-4-3)^2}$$

$$|BC| = \sqrt{16+49}$$

$$|BC| = \sqrt{65}$$

$$\therefore \quad |DE| = \frac{\sqrt{65}}{2}$$

$$|DE| = \frac{1}{2}|BC|$$ Hence proved.

Q.16: A milkman can sell 560 litres of milk at Rs. 12.50 per litre and 700 litres of milk at 12.00 per litre. Assuming the graph of the sale price and the milk sold to be a straight line, find the number of litres of milk that the milkman can sell 12.25 per litre.

Solution:

Let ℓ denoted the number of litres of milk and P denotes the price of milk per litre.

The $(\ell_1, P_1) = (560, 12.50)$ and $(\ell_2, P_2) = (700, 12.00)$

Equation of line is

$$\frac{\ell - \ell_1}{\ell_2 - \ell_1} = \frac{P - P_1}{P_2 - P_1}$$

$$\frac{\ell - 560}{700 - 560} = \frac{P - 12.50}{12.00 - 12.50}$$

$$\frac{\ell - 560}{140} = \frac{P - 12.50}{-0.50}$$

$$-0.50\,\ell + 280 = 140\,P - 1750$$

$$0 = 140\,P - 1750 + 0.50\ell - 280$$

$$0.50\ell + 140\,P - 2030 = 0$$

Put $P = 12.25$

$$0.50\ell + 140(12.25) - 2030 = 0$$

$$0.50\ell + 1715 - 2030 = 0$$

$$0.50\ell = 315$$

$$\ell = \frac{315}{0.50}$$

$$\ell = 630 \qquad \text{Ans.}$$

So the milkman can sell 630 litres at Rs. 12.25 per litre.

Q.17 The population of Pakistan to the nearest million was 60 million in 1961 and 95 million in 1981. Using t as the number of years after 1961, find an equation of the line that gives the population in terms of t. Use this equation to find the population in (a) 1947 (b) 1997

Solution:

Let P be the population and t be the number of years.

$(P_1, t_1) = (60, 1961)$, $(P_2, t_2) = (95, 1981)$

Eq. of line is

$$\frac{P - P_1}{P_2 - P_1} = \frac{t - t_1}{t_2 - t_1}$$

$$\frac{P - 60}{95 - 60} = \frac{t - 1961}{1981 - 1961}$$

$$\frac{P - 60}{35} = \frac{t - 1961}{20}$$

$$P - 60 = \frac{35}{20}(t - 1961)$$

$$P = \frac{7}{4}(t - 1961) + 60$$

$$P = \frac{7t - 13727}{4} + 240$$

$$P = \frac{7t - 13487}{4} \quad \text{.......... (1)}$$

(i) Put $t = 1947$ in equation (1)

$$P = \frac{7(1947) - 13487}{4}$$

$$P = \frac{13629 - 13487}{4}$$

$P = 35.5$ million Ans.

(ii) Put $t = 1997$ in equation (1)

$$P = \frac{7(1997) - 13487}{4}$$

$$= \frac{13979 - 13487}{4} = 123 \text{ million}$$ Ans.

Q.18 A house was purchased for Rs. 1 million in 1980. It is worth Rs. 4 million in 1996. Assuming that the value increased by the same amount each year, find an equation that gives the value of the house after years of the date of purchase. What was its value in 1990?

Solution:

Let V be the value of house and t be the years.

$(V_1, t_1) = (1, 1980)$, $(V_2, t_2) = (4, 1996)$

$$\frac{V - V_1}{V_2 - V_1} = \frac{t - t_1}{t_2 - t_1}$$

$$\frac{V - 1}{4 - 1} = \frac{t - 1980}{1996 - 1980}$$

$$\frac{V - 1}{3} = \frac{t - 1980}{16}$$

$$V - 1 = \frac{3}{16}(t - 1980)$$

$$V = \frac{3t - 5940}{16} + 1$$

$$V = \frac{3t - 5940 + 16}{16}$$

$$V = \frac{3t - 5924}{16}$$

Put $t = 1990$

$$V = \frac{3(1990) - 5924}{16}$$

$$V = \frac{5970 - 5924}{16}$$

$V = 2.875$ Million Ans.

Q.19: Plot the Celsius (C) and Fahrenheit (F) temperature scales on the horizontal axis and vertical axis respectively. Draw the line joining the freezing point and the boiling point of water. Find an equation giving F temperature in terms of C.

Solution:

Freezing point of water = 0ºC = 32 F

Boiling point of water = 100ºC = 212 F

$(C_1, F_1) = (0, 32)$

$(C_2, F_2) = (100, 212)$

$$\frac{F - F_1}{F_2 - F_1} = \frac{C - C_1}{C_2 - C_1}$$

$$\frac{F - 32}{212 - 32} = \frac{C - 0}{100 - 0}$$

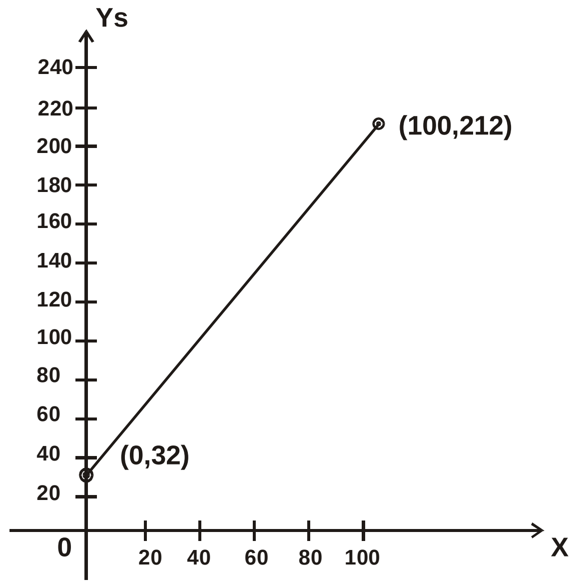
$$\frac{F - 32}{180} = \frac{C}{100}$$

$$100\,F - 3200 = 180\,C$$

$$100\,F = 180\,C + 3200$$

$$F = \frac{180}{100}\,C + \frac{3200}{100}$$

$$F = \frac{9}{5}C + 32 \quad \text{Ans.}$$



Q.20 The average entry test score of engineering candidates was 592 in the year 1998 while the score was 564 in 2002. Assuming that the relationship between time and score is linear, find the average score for 2006.

Solution:

Let S be the score and t be the time.

$(S_1, t_1) = (592, 1998), \quad (S_2, t_2) = (564, 2002)$

$$\frac{S - S_1}{S_2 - S_1} = \frac{t - t_1}{t_2 - t_1}$$

$$\frac{S - 592}{564 - 592} = \frac{t - 1998}{2002 - 1998}$$

$$\frac{S - 592}{-28} = \frac{t - 1998}{4}$$

$$\frac{S - 592}{-7} = t - 1998$$

$$S - 592 = -7t + 13986$$

$$S = -7t + 13986 + 592$$

$$S = -7t + 14578$$

Put $t = 2006$

$$S = -7(2006) + 14578$$

$$S = -14042 + 14578$$

$$\boxed{S = 536} \quad \text{Ans}$$

Q.21 Convert each of the following equation into

(i) Slope intercept form (ii) two intercepts form

(iii) Normal form

(a) $2x - 4y + 11 = 0$ (b) $4x + 7y - 2 = 0$

(c) $15y - 8x + 3 = 0$ (Lhr. Board 2006, 2009, 2009 (S))

Also find the length of perpendicular from (0, 0) to each line.

Solution:

(a) $2x - 4y + 11 = 0$

(i) <u>Slope intercept Form</u>

$$2x - 4y + 11 = 0$$
$$-4y = -2x - 11$$
$$y = \frac{-2x}{-4} - \frac{11}{-4}$$
$$y = \frac{x}{2} + \frac{11}{4} \quad \text{Ans}$$

(ii) <u>Two intercepts Form</u>

$$2x - 4y + 11 = 0$$
$$2x - 4y = -11$$

Dividing both sides by -11

$$\frac{2x}{-11} - \frac{4y}{-11} = \frac{-11}{-11}$$
$$\frac{x}{\frac{-11}{2}} + \frac{y}{\frac{11}{4}} = 1 \quad \text{Ans}$$

(iii) <u>Normal Form</u>

$$2x - 4y + 11 = 0$$
$$2x - 4y = -11$$
$$-2x + 4y = 11$$

Dividing by $\sqrt{(-2)^2 + (4)^2} = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$

$$\frac{-2x}{2\sqrt{5}} + \frac{4y}{2\sqrt{5}} = \frac{11}{2\sqrt{5}}$$
$$\frac{-x}{\sqrt{5}} + \frac{2y}{\sqrt{5}} = \frac{11}{2\sqrt{5}} \quad \text{........... (1)}$$

Compare it with

$$x \cos \alpha + y \sin \alpha = P$$

$\cos\alpha = \dfrac{-1}{\sqrt{5}}$, $\sin\alpha = \dfrac{2}{\sqrt{5}}$, $P = \dfrac{11}{2\sqrt{5}}$

α lies in 2^{nd} quadrant

$\therefore \quad \alpha = \cos^{-1}\left(\dfrac{-1}{\sqrt{5}}\right) = 116.57^o$

Put in equation (ii)

$$x\cos 116.57^o + y\sin 116.57^o = \frac{11}{2\sqrt{5}}$$

Length of perpendicular from origin to line is $P = \dfrac{11}{2\sqrt{5}}$ Ans.

(b) $4x + 7y - 2 = 0$

(i) <u>Slope intercept form</u>

$4x + 7y - 2 = 0$

$7y = -4x + 2$

$y = \dfrac{-4}{7}x + \dfrac{2}{7}$ Ans

(ii) <u>Two intercepts form</u>

$4x + 7y - 2 = 0$

$4x + 7y = 2$

Dividing both sides by 2

$\dfrac{4x}{2} + \dfrac{7y}{2} = \dfrac{2}{2}$

$2x + \dfrac{y}{2/7} = 1$

$\dfrac{x}{\frac{1}{2}} + \dfrac{y}{\frac{2}{7}} = 1$ Ans

(iii) <u>Normal Form</u>

$4x + 7y - 2 = 0$

$4x + 7y = 2$

Dividing by $\sqrt{(4)^2 + (7)^2} = \sqrt{16 + 49} = \sqrt{65}$

$\dfrac{4x}{\sqrt{65}} + \dfrac{7y}{\sqrt{65}} = \dfrac{2}{\sqrt{65}}$ .... (1)

Compare it with

$x\cos\alpha + y\sin\alpha = P$

$\cos\alpha = \frac{4}{\sqrt{65}}, \quad \sin\alpha = \frac{7}{\sqrt{65}}, \quad P = \frac{2}{\sqrt{65}}$

α lies in 1^{st} quadrant

$\therefore \quad \alpha = \cos^{-1}\left(\frac{4}{\sqrt{65}}\right) = 60.26^o$

Put in equation (1)

$x\cos 60.26^o + y\sin 60.26^o = \frac{2}{\sqrt{65}}$

Length of perpendicular from origin to line is $P = \frac{2}{\sqrt{65}}$

(c) $15y - 8x + 3 = 0$

(i) <u>Slope intercept form</u>

$15y - 8x + 3 = 0$

$15y = 8x - 3$

$y = \frac{8}{15}x - \frac{3}{15}$

$y = \frac{8}{15}x - \frac{1}{15}$ Ans

(ii) <u>Two intercept form</u>

$15y - 8x + 3 = 0$

$-8x + 15y = -3$

Dividing both sides by -3

$\frac{-8x}{-3} + \frac{15y}{-3} = \frac{-3}{-3}$

$\frac{x}{\frac{3}{8}} + \frac{y}{\frac{-3}{15}} = 1$

$\frac{x}{\frac{3}{8}} + \frac{x}{\frac{-1}{5}} = 1$ Ans

(iii) <u>Normal Form</u>

$15y - 8x + 3 = 0$

$-8x + 15y = -3$

$8x - 15y = 3$

Dividing by $\sqrt{(8)^2 + (-15)^2} = \sqrt{64 + 225} = \sqrt{289} = 17$

$\frac{8x}{17} - \frac{15y}{17} = \frac{3}{17}$ .... (1)

Compare it with

$x\cos\alpha + y\sin\alpha = P$

$\cos\alpha = \dfrac{8}{17}$, $\sin\alpha = \dfrac{-15}{17}$, $P = \dfrac{3}{17}$

α lies in 4^{th} quadrant

$\therefore \quad \alpha = \cos^{-1}\left(\dfrac{8}{17}\right) = -61.93^\circ + 360^\circ$

$= 360^\circ - 61.93^\circ$

$= 298.07^\circ$

Put in eq. (1)

$x\cos 298.07^\circ + y\sin 298.07^\circ = \dfrac{3}{17}$

Length of perpendicular from origin to line is $P = \dfrac{3}{17}$

Q.22: In each of the following check whether the two lines are

(i) Parallel (ii) Perpendicular

(iii) Neither parallel nor perpendicular

(a) $2x + y - 3 = 0$; $4x + 2y + 5 = 0$

(b) $3y = 2x + 5$; $3x + 2y - 8 = 0$

(c) $4y + 2x - 1 = 0$; $x - 2y - 7 = 0$

(d) $4x - y + 2 = 0$; $12x - 3y + 1 = 0$

(e) $12x + 35y - 7 = 0$; $105x - 36y + 11 = 0$

Solution:

(a) $2x + y - 3 = 0$; $4x + 2y + 5 = 0$

Let m_1 be the slope of 1^{st} line and m_2 be the slope of 2^{nd} line.

$m_1 = \dfrac{-2}{1} = -2$, $m_2 = \dfrac{-4}{2} = -2$

$\therefore \quad m_1 = m_2$

So the lines are parallel.

(b) $3y = 2x + 5$; $3x + 2y - 8 = 0$

$2x - 3y + 5 = 0$

Let m_1 be the slope of 1^{st} line and m_2 be the slope of 2^{nd} line.

$m_1 = \dfrac{-2}{-3} = \dfrac{2}{3}$, $m_2 = \dfrac{-3}{2}$

$\therefore \quad m_1 \times m_2 = \dfrac{2}{3} \times \dfrac{-3}{2} = -1$

So the lines are perpendicular.

(c) $\mathbf{4y + 2x - 1 = 0 \quad ; \quad x - 2y - 7 = 0}$

Let m_1 be the slope of 1st line and m_2 be the slope of 2nd line.

$$m_1 = \frac{-2}{4} = \frac{-1}{2} \quad , \quad m_2 = -\frac{1}{-2} = \frac{1}{2}$$

Since $m_1 \times m_2 \neq -1$, $m_1 \neq m_2$

So the given lines are neither parallel nor perpendicular.

(d) $\mathbf{4x - y + 2 = 0 \quad ; \quad 12x - 3y + 1 = 0}$

Let m_1 be the slope of 1st line and m_2 be the slope of 2nd line.

$$m_1 = -\frac{4}{-1} = 4 \quad , \quad m_2 = -\frac{12}{-3} = 4$$

$\therefore$ $m_1 = m_2$

So the lines are parallel.

(e) $\mathbf{12x + 35y - 7 = 0 \quad ; \quad 105x - 36y + 11 = 0}$

Let m_1 be the slope of 1st line and m_2 be the slope of 2nd line.

$$m_1 = -\frac{12}{35} \quad , \quad m_2 = \frac{-105}{-36} = \frac{35}{12}$$

Since $m_1 \times m_2 = \frac{-12}{35} \times \frac{35}{12} = -1$

So the lines are perpendicular.

Q.23: Find the distance between the given parallel lines. Sketch the lines. Also find an equation of the parallel line lying midway between them.

(a) $\mathbf{3x - 4y + 3 = 0 \quad ; \quad 3x - 4y + 7 = 0}$

(b) $\mathbf{12x + 5y - 6 = 0 \quad ; \quad 12x + 5y + 13 = 0}$

(c) $\mathbf{x + 2y - 5 = 0 \quad ; \quad 2x + 4y = 1}$

Solution:

(a) $\mathbf{3x - 4y + 3 = 0}$ **.... (1)**

$\mathbf{3x - 4y + 7 = 0}$ **.... (2)**

Put $x = 0$ in eq. (1)

$-4y + 3 = 0$

$-4y = -3 \qquad y = \frac{3}{4}$

$\therefore$ Point is $\left(0\ ,\ \frac{3}{4}\right)$

Now we find the distance from $(0, \frac{3}{4})$ to the line (2).

$$d = \frac{|\,3(0) - 4\left(\frac{3}{4}\right) + 7\,|}{\sqrt{(3)^2 + (-4)^2}}$$

$$d = \frac{|0 - 3 + 7|}{\sqrt{9 + 16}}$$

$$d = \frac{4}{\sqrt{25}}$$

$$d = \frac{4}{5} \qquad \text{Ans.}$$

Put $y = 0$ in eq. (1)

$$3x + 3 = 0$$

$$3x = -3$$

$$x = \frac{-3}{3} = -1$$

$\therefore$ Point is $(-1, 0)$

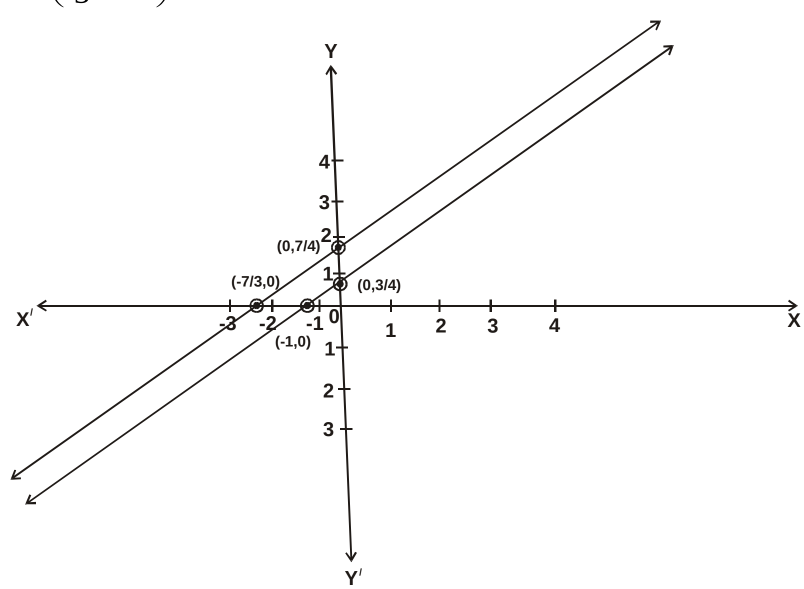
Put $y = 0$ in eq. (2)

$$3x - 4(0) + 7 = 0$$

$$3x = -7$$

$$x = \frac{-7}{3}$$

$\therefore$ Point is $\left(\frac{-7}{3}, 0\right)$



Mid point of $\left(\frac{-7}{3}, 0\right)$ and $(-1, 0)$ is

$$= \left(\frac{\frac{-7}{3}-1}{2} \;,\; \frac{0+0}{2}\right) = \left(\frac{\frac{-7-3}{3}}{2} \;,\; \frac{0}{2}\right) = \left(\frac{-10}{6} \;,\; 0\right) = \left(\frac{-5}{3} \;,\; 0\right)$$

Slope of parallel lines $= \frac{-3}{-4} = \frac{3}{4}$

Eq. of line passing through $\left(\frac{-5}{3} \;,\; 0\right)$ and having slope $\frac{3}{4}$ is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{3}{4}\left(x + \frac{5}{3}\right)$$

$$4y = 3x + 5$$

$$\boxed{3x - 4y + 5 = 0} \quad \text{Ans}$$

(b) $\mathbf{12x + 5y - 6 = 0} \quad \text{..... (1)}$

$\mathbf{12x + 5y + 13 = 0} \quad \text{..... (2)}$

Put x = 0 in eq. (1)

$$5y - 6 = 0$$

$$5y = 6$$

$$5y = \frac{6}{5}$$

$\therefore$ Point is $\left(0 \;,\; \frac{6}{5}\right)$

Now we find the distance from the point $\left(0 \;,\; \frac{6}{5}\right)$ to the line (2) is

$$d = \frac{|\,12(0) + 5\left(\frac{6}{5}\right) + 13\,|}{\sqrt{(12)^2 + (5)^2}} = \frac{|\,0 + 6 + 13\,|}{\sqrt{144 + 25}} = \frac{|\,19\,|}{\sqrt{169}} = \frac{19}{13} \quad \text{Ans}$$

Put $y = 0$ in eq. (1)

$$12x + 0 - 6 = 0$$

$$12x = 6$$

$$x = \frac{6}{12} = \frac{1}{2}$$

$\therefore$ Point is $\left(\frac{1}{2} \;,\; 0\right)$

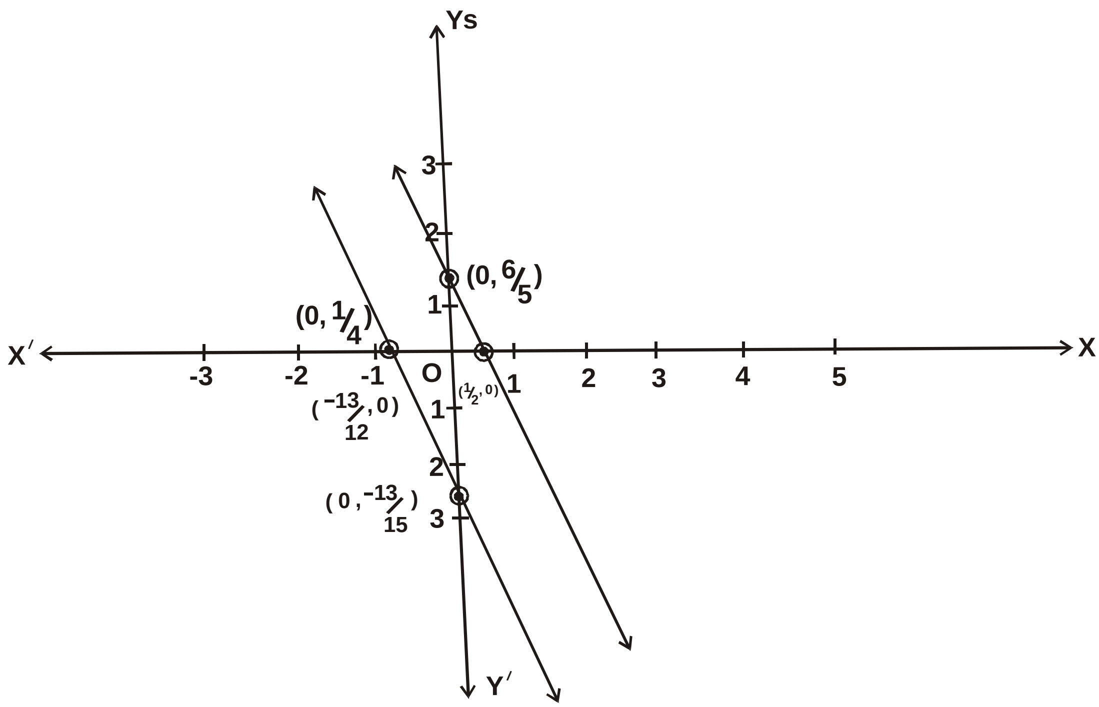
Put $y = 0$ in eq. (2)

$$12x + 0 + 13 = 0$$

$$12x = -13$$

$$x = \frac{-13}{12}$$

$\therefore$ Point is $\left(\frac{-13}{12}\ ,\ 0\right)$



Mid point of $\left(\frac{-13}{12}\ ,\ 0\right)$ and $\left(\frac{1}{2}\ ,\ 0\right)$ is

$$= \left(\frac{\frac{-13}{12}+\frac{1}{2}}{2}\ ,\ \frac{0+0}{2}\right) = \left(\frac{\frac{-13+6}{12}}{2}\ ,\ \frac{0}{2}\right)$$

$$= \left(\frac{-7}{24}\ ,\ 0\right)$$

Slope of parallel lines = $\frac{-12}{5}$

Eq. of line passing through $\left(\frac{-7}{24}\ ,\ 0\right)$ and having slope $\frac{-12}{15}$ is

$$y - y_1 = m\,(x - x_1)$$

$$y - 0 = \frac{-12}{5}\left(x + \frac{7}{24}\right)$$

$$5y = -12x - \frac{7}{2}$$

$$5y = \frac{-24x - 7}{2}$$

$$10y = -24x - 7$$

$$24x + 10y + 7 = 0 \qquad \text{Ans}$$

(c) $\mathbf{x + 2y - 5 = 0} \qquad$ **..... (1)**

$$2x + 4y = 1$$

$$2x + 4y - 1 = 0 \qquad ..... (2)$$

Put $x = 0$ in eq. (1)

$$0 + 2y - 5 = 0$$

$$2y = 5$$

$$y = \frac{5}{2}$$

$\therefore$ Point is $\left(0 \ , \ \frac{5}{2}\right)$

Now we find the distance from the point $\left(0 \ , \ \frac{5}{2}\right)$ to the line (2)

$$d = \frac{\left| 2(0) + 4\left(\frac{5}{2}\right) - 1 \right|}{\sqrt{(2)^2 + (4)^2}}$$

$$d = \frac{|0 + 10 - 1|}{\sqrt{4 + 16}} = \frac{9}{\sqrt{20}} = \frac{9}{2\sqrt{5}} \qquad \text{Ans}$$

Put $y = 0$ in eq. (1)

$$x + 0 - 5 = 0$$

$$x = 5$$

$\therefore$ Point is (5, 0)

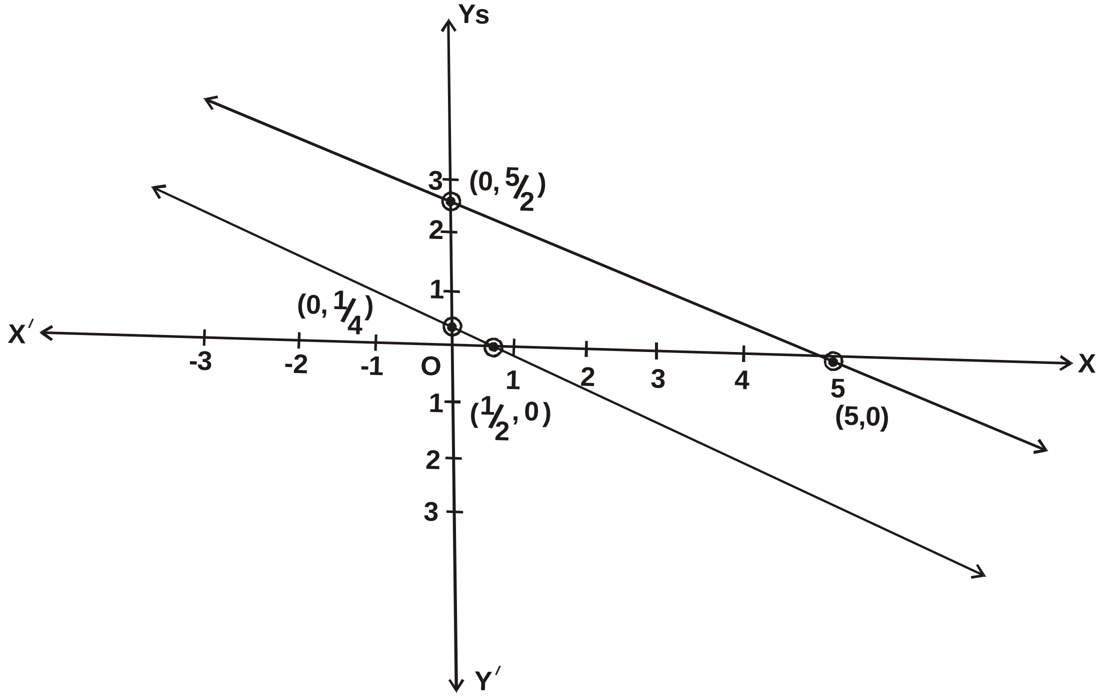
Put $y = 0$ in eq. (2)

$$2x + 0 - 1 = 0$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$\therefore$ Point is $\left(\frac{1}{2}, 0\right)$



Mid point of $\left(\frac{1}{2}, 0\right)$ and (5, 0) is

$$= \left(\frac{\frac{1}{2}+5}{2}, \frac{0+0}{2}\right) = \left(\frac{\frac{1+10}{2}}{2}, \frac{0}{2}\right)$$

$$= \left(\frac{11}{4}, 0\right)$$

Slope of parallel lines $= \frac{-1}{2}$

Equation of line passing through $\left(\frac{11}{4}, 0\right)$ and having slope $\frac{-1}{2}$ is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-1}{2}\left(x - \frac{11}{4}\right)$$

$$2y = -x + \frac{11}{4}$$

$$\boxed{x + 2y - \frac{11}{4} = 0} \quad \text{Ans}$$

Q.24 Find an equation of the line through (– 4, 7) and parallel to the line 2x – 7y + 4 = 0. (Gur. Board 2007) (Lhr. Board 2007)

Solution:

$$2x - 7y + 4 = 0$$

$$\text{Slope} = \frac{-2}{-7} = \frac{2}{7}$$

$$\text{Slope of required line} = \frac{2}{7}$$

Eq. of line passing through (– 4, 7) and having slope $\frac{2}{7}$ is

$$y - y_1 = m(x - x_1)$$

$$y - 7 = \frac{2}{7}(x + 4)$$

$$7y - 49 = 2x + 8$$

$$0 = 2x + 8 - 7y + 49$$

$$\boxed{2x - 7y + 57 = 0}$$ Ans.

Q.25: Find an equation of the line through (5, – 8) and perpendicular to the join of A(– 15, – 8), B (10, 7). (Guj. Board 2006)

Solution:

A (– 15, – 8), B (10, 7)

$$\text{Slope of AB} = \frac{7 + 8}{10 + 15} = \frac{15}{25} = \frac{3}{5}$$

$$\text{Slope of required line} = \frac{-1}{\frac{3}{5}} = \frac{-5}{3}$$

Eq. of line passing through (5, – 8) and having slope $\frac{-5}{3}$ is

$$y - y_1 = m(x - x_1)$$

$$y + 8 = \frac{-5}{3}(x - 5)$$

$$3y + 24 = -5x + 25$$

$$5x + 3y + 24 - 25 = 0$$

$$\boxed{5x + 3y - 1 = 0}$$ Ans.

Q.26 Find equations of two parallel lines perpendicular to $2x - y + 3 = 0$ such that the product of the x and y – intercepts of each is 3.

Solution:

$$2x - y + 3 = 0$$

$$\text{Slope} = \frac{-2}{-1} = 2$$

$$\text{Slope of required lines} = \frac{-1}{2}$$

Equations of required lines are

$$y = mx + c$$

$$y = \frac{-1}{2}x + c$$

$$y = \frac{-x + 2c}{2}$$

$$2y = -x + 2c$$

$$x + 2y - 2c = 0 \qquad \text{..... (1)}$$

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$$x + 0 - 2c = 0$$

$$x = 2c$$

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$$2y - 2c = 0$$

$$2y = 2c$$

$$y = \frac{2c}{2} = c$$

By given condition

$$(\text{x-intercept})\,(\text{y-intercept}) = 3$$

$$(2c)\,(c) = 3$$

$$2c^2 = 3$$

$$c^2 = \frac{3}{2}$$

$$\sqrt{c^2} = \sqrt{\frac{3}{2}}$$

$$c = \pm\sqrt{\frac{3}{2}}$$

Put in eq. (1)

$$x + 2y \pm 2\sqrt{\frac{3}{2}} = 0$$

$$x + 2y \pm \sqrt{2}\,.\sqrt{3} = 0$$

$$x + 2y \pm \sqrt{6} = 0$$

$$\boxed{x + 2y + \sqrt{6} = 0 \quad \text{and} \quad x + 2y - \sqrt{6} = 0} \qquad \text{Ans.}$$

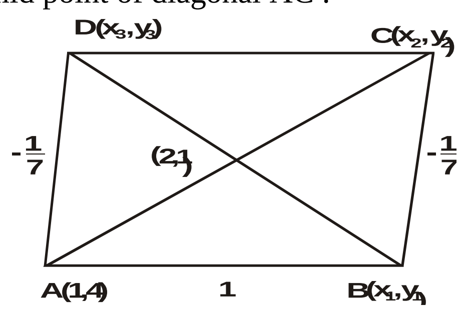
Q.27 One vertex of a parallelogram is (1, 4); the diagonals intersects at (2, 1) and the sides have slopes 1 and $\frac{-1}{7}$. Find the other three vertices.

Solution:

A (1, 4)

Let B (x_1, y_1), C (x_2, y_2) and D (x_3, y_3) be the required vertices of parallelogram.

Since (2, 1) is the mid point of diagonal $\overline{AC}$.



$$2 = \frac{1 + x_2}{2} \quad , \quad 1 = \frac{4 + y_2}{2}$$

$$1 + x_2 = 4 \quad , \quad 2 = 4 + y_2$$

$$x_2 = 4 - 1 \quad , \quad y_2 = 2 - 4$$

$$x_2 = 3 \quad , \quad y_2 = -2$$

$\therefore$ $C(x_2, y_2) = C(3, -2)$

Now slope of AB $= 1$

$$\frac{y_1 - 4}{x_1 - 1} = 1$$

$$y_1 - 4 = x_1 - 1$$

$$-4 + 1 = x_1 - y_1$$

$$x_1 - y_1 = -3 \qquad \text{.... (1)}$$

and Slope of BC $= \frac{-1}{7}$

$$\frac{-2-y_1}{3-x_1} = \frac{-1}{7}$$

$$-14-7y_1 = -3+x_1$$

$$-14+3 = x_1+7y_1$$

$$x_1+7y_1 = -11 \quad .... (2)$$

Eq. (1) – Eq. (2), we get

$$\begin{array}{rcl} x_1 - y_1 & = & -3 \\ \underset{-}{}x_1 \pm 7y_1 & = & \mp 11 \\ \hline -8y_1 & = & 8 \\ y_1 & = & \dfrac{8}{-8} = -1 \end{array}$$

Put $y_1 = -1$ in eq. (1)

$$x_1 + 1 = -3$$

$$x_1 = -3-1$$

$$= -4$$

$\therefore$ $B(x_1, y_1) = B(-4, -1)$

Now (2, 1) is the mid points of diagonal $\overline{BD}$.

$$2 = \frac{-4+x_3}{2} \quad , \quad 1 = \frac{-1+y_3}{2}$$

$$4 = -4+x_3 \quad , \quad 2 = -1+y_3$$

$$4+4 = x_3 \quad , \quad 2+1 = y_3$$

$$x_3 = 8 \quad , \quad y_3 = 3$$

$\therefore$ $D(x_3, y_3) = D(8, 3)$

Hence B (– 4, – 1), C (3, – 2) and D (8, 3) are the required vertices of parallelogram.

Q.28 Find whether the given points lies above and below the given line.

(a) (5, 8) ; $2x-3y+6 = 0$

(b) (– 7, 6) ; $4x+3y-9 = 0$

Solution:

(a) $2x-3y+6 = 0$

To make coefficient of y positive we multiply above equation with -1

$$-2x+3y-6 = 0$$

Put (5, 8) on L.H.S

$$-2(5)+3(8)-6 = -10+24-6 = 8 > 0$$

Hence (5, 8) lies above the line.

(b) $\mathbf{4x + 3y - 9 = 0}$ **(Guj. Board 2008)**

Put $(-7, 6)$ on L.H.S

$4(-7) + 3(6) - 9 = -28 + 18 - 9 = -19$ .... (1)

Since the coefficient of y and expression (1) have opposite signs therefore $(-7, 6)$ lies below the line.

Q.29 Check whether the given points are on the same or opposite sides of the given line.

(a) $\mathbf{(0, 0)}$ **and** $\mathbf{(-4, 7)}$ **;** $\mathbf{6x - 7y + 70 = 0}$

(b) $\mathbf{(2, 3)}$ **and** $\mathbf{(-2, 3)}$ **;** $\mathbf{3x - 5y + 8 = 0}$

Solution:

(a) $\mathbf{6x - 7y + 70 = 0}$

To make coefficient of y positive we multiply above eq. with -1

$-6x + 7y - 70 = 0$ ..... (1)

Put $(0, 0)$ on L.H.S of eq. (1)

$-6(0) + 7(0) - 70 = -70 < 0$

=> $(0, 0)$ lies below the line.

Put $(-4, 7)$ on L.H.S of eq. (1)

$-6(-4) - 7(7) + 70 = 24 - 49 + 70 = 45 > 0$

$\therefore$ $(-4, 7)$ lies above the line.

Hence $(0, 0)$ $(-4, 7)$ lies on the opposite side of line.

(b) $3x - 5y + 8 = 0$

To make coefficient of y positive we multiply above equation with -1

$-3x + 5y - 8 = 0$ .... (1)

Put $(2, 3)$ on L.H.S. of eq. (1)

$-3(2) + 5(3) - 8 = -6 + 15 - 8 = 1 > 0$

$\therefore$ $(2, 3)$ lies above the line.

Put $(-2, 3)$ on L.H.S of eq. (1)

$-3(-2) + 5(3) + 8 = 6 + 15 - 8 = 13 > 0$

$\therefore$ $(-2, 3)$ lies above the line.

Hence $(2, 3)$ and $(-2, 3)$ lies on the same side of line.

Q.30 Find the distance from the point P $\mathbf{(6, -1)}$ **to the line** $\mathbf{6x - 4y + 9 = 0}$ **(Guj. Board 2007)**

Solution:

Let d be the distance from P $(6, -1)$ to the line $6x - 4y + 9 = 0$

$$d = \frac{|6(6) - 4(-1) + 9|}{\sqrt{(6)^2 + (-4)^2}}$$

$$d = \frac{|36 + 4 + 9|}{\sqrt{36 + 16}} = \frac{49}{\sqrt{52}}$$

$$\boxed{d = \frac{49}{2\sqrt{13}}} \quad \text{Ans.}$$

Q.31 Find the area of the triangular region whose vertices are A (5, 3), B (– 2, 2), C (4, 2). (Lhr. Board 2008)

Solution:

A (5, 3), B (– 2, 2), C (4, 2)

$$\text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 5 & 3 & 1 \\ -2 & 2 & 1 \\ 4 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[5\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} -2 & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} -2 & 2 \\ 4 & 2 \end{vmatrix}\right]$$

$$= \frac{1}{2}\,[\,5\,(2 - 2) - 3\,(-2 - 4) + 1\,(-4 - 8)\,]$$

$$= \frac{1}{2}\,[\,5(0) - 3\,(-6) + 1\,(-12)\,]$$

$$= \frac{1}{2}\,(0 + 18 - 12) = \frac{6}{2} = \boxed{3 \text{ Square unit}} \quad \text{Ans.}$$

Q.32 The coordinates of three points are A (2, 3), B (– 1, 1) and C (4, – 5). By computing the area bounded by ABC check whether the points are collinear.

Solution:

A (2, 3), B (– 1, 1), C (4, – 5)

$$\text{Area of } \Delta\, ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & 1 & 1 \\ 4 & -5 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[2\begin{vmatrix} 1 & 1 \\ -5 & 1 \end{vmatrix} - 3\begin{vmatrix} -1 & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} -1 & 1 \\ 4 & -5 \end{vmatrix}\right]$$

$$= \frac{1}{2}\ [\,2\,(1+5) - 3\,(-1-4) + 1\,(5-4)\,]$$

$$= \frac{1}{2}\ [\,2(6) - 3\,(-5) + 1\,(1)\,]$$

$$= \frac{1}{2}\ (12 + 15 + 1)$$

$$= \frac{28}{2} = \boxed{14 \quad \text{Square unit}} \quad \text{Ans.}$$

Since Area of $\Delta ABC \neq 0$ so the points are not collinear.

EXERCISE 4.4

Q.1: Find the point of intersection of the lines.

(i) $x - 2y + 1 = 0$ **and** $2x - y + 2 = 0$

(ii) $3x + y + 12 = 0$ **and** $x + 2y - 1 = 0$

(iii) $x + 4y - 12 = 0$ **and** $x - 3y + 3 = 0$

Solution:

(i) $x - 2y + 1 = 0$ **(Guj. Board 2005, 2007)**

$2x - y + 2 = 0$

Let P (x, y) be the point of intersection of given lines.

$$\frac{x}{(-2)(2) - 1(-1)} = \frac{-y}{(1)(2) - 1(2)} = \frac{1}{1(-1) - (2)(-2)}$$

$$\frac{x}{-4+1} = \frac{-y}{2-2} = \frac{1}{-1+4}$$

$$\frac{x}{-3} = \frac{-y}{0} = \frac{1}{3}$$

$$\Rightarrow \frac{x}{-3} = \frac{1}{3} \quad \text{and} \quad \frac{-y}{0} = \frac{1}{3}$$