$$= \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & 1 & 1 \\ 4 & -5 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[2\begin{vmatrix} 1 & 1 \\ -5 & 1 \end{vmatrix} - 3\begin{vmatrix} -1 & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} -1 & 1 \\ 4 & -5 \end{vmatrix}\right]$$

$$= \frac{1}{2}\quad [\,2\,(1+5) - 3\,(-1-4) + 1\,(5-4)\,]$$

$$= \frac{1}{2}\quad [\,2(6) - 3\,(-5) + 1\,(1)\,]$$

$$= \frac{1}{2}\quad (12 + 15 + 1)$$

$$= \frac{28}{2} \quad = \quad \boxed{14 \quad \text{Square unit}} \qquad \text{Ans.}$$

Since Area of $\Delta ABC \neq 0$ so the points are not collinear.

EXERCISE 4.4

Q.1: Find the point of intersection of the lines.

(i) $x - 2y + 1 = 0$ and $2x - y + 2 = 0$

(ii) $3x + y + 12 = 0$ and $x + 2y - 1 = 0$

(iii) $x + 4y - 12 = 0$ and $x - 3y + 3 = 0$

Solution:

(i) $x - 2y + 1 = 0$ (Guj. Board 2005, 2007)

$2x - y + 2 = 0$

Let P (x, y) be the point of intersection of given lines.

$$\frac{x}{(-2)(2) - 1(-1)} = \frac{-y}{(1)(2) - 1(2)} = \frac{1}{1(-1) - (2)(-2)}$$

$$\frac{x}{-4+1} = \frac{-y}{2-2} = \frac{1}{-1+4}$$

$$\frac{x}{-3} = \frac{-y}{0} = \frac{1}{3}$$

$$\Rightarrow \frac{x}{-3} = \frac{1}{3} \quad \text{and} \quad \frac{-y}{0} = \frac{1}{3}$$

$3x = -3$ $\qquad$ $-3y = 0$

$x = \frac{-3}{3}$ $\qquad$ $y = \frac{0}{-3}$

$x = -1$ $\qquad$ $y = 0$

$\therefore$ P (x, y) = P (– 1, 0) Ans

(ii) $\mathbf{3x + y + 12 = 0}$

$\mathbf{x + 2y - 1 = 0}$

Let P (x, y) be the point of intersection of given lines.

$$\frac{x}{1(-1)-2(12)} = \frac{-y}{3(-1)-1(12)} = \frac{1}{3(2)-1(1)}$$

$$\frac{x}{-1-24} = \frac{-y}{-3-12} = \frac{1}{6-1}$$

$$\frac{x}{-25} = \frac{-y}{-15} = \frac{1}{5}$$

$\Rightarrow \frac{x}{-25} = \frac{1}{5}$ and $\frac{-y}{-15} = \frac{1}{5}$

$x = \frac{-25}{5}$ $\qquad$ $y = \frac{15}{5}$

$x = -5$ $\qquad$ $y = 3$

$\therefore$ P (x, y) = P (– 5, 3) Ans

(iii) $\mathbf{x + 4y - 12 = 0}$

$\mathbf{x - 3y + 3 = 0}$

Let P (x, y) be the point of intersection of given lines.

$$\frac{x}{4(3)-(-3)(-12)} = \frac{-y}{1(3)-1(-12)} = \frac{1}{1(-3)-4(1)}$$

$$\frac{x}{12-36} = \frac{-y}{3+12} = \frac{1}{-3-4}$$

$$\frac{x}{-24} = \frac{-y}{15} = \frac{1}{-7}$$

$\Rightarrow \frac{x}{-24} = \frac{1}{-7}$ and $\frac{-y}{15} = \frac{1}{-7}$

$x = \frac{-24}{-7}$ $\qquad$ $y = \frac{15}{7}$

$x = \frac{24}{7}$

$\therefore$ P (x, y) = $P\left(\frac{24}{7}, \frac{15}{7}\right)$ Ans

Q.2 Find an equation of the line through

(i) the point (2, – 9) and the intersection of the lines 2x + 5y – 8 = 0 and 3x – 4y – 6 = 0

(ii) the intersection of the lines x – y – 4 = 0 and 7x + y + 20 = 0 and

(a) Parallel (b) Perpendicular

to the line 6x + y – 14 = 0 (Lhr. Board 2009 (S)) (Guj. Board 2005)

(iii) through the intersection of lines x + 2y + 3 = 0, 3x + 4y + 7 = 0 and making equal intercepts on the axes.

Solution:

(i) 2x + 5y – 8 = 0

3x – 4y – 6 = 0

Let P (x, y) be the point of intersection of given lines.

$$\frac{x}{5(-6)-(-8)(-4)} = \frac{-y}{2(-6)-3(-8)} = \frac{1}{2(-4)-3(5)}$$

$$\frac{x}{-30-32} = \frac{-y}{-12+24} = \frac{1}{-8-15}$$

$$\frac{x}{-62} = \frac{-y}{12} = \frac{1}{-23}$$

$$\Rightarrow \quad \frac{x}{-62} = \frac{1}{-23} \quad \text{and} \quad \frac{-y}{12} = \frac{1}{-23}$$

$$x = \frac{62}{23} \qquad y = \frac{12}{23}$$

$$\therefore\ P(x, y) = P\left(\frac{62}{23}, \frac{12}{23}\right)$$

Equation of the line passing through (2, – 9) and $\left(\frac{62}{23}, \frac{12}{23}\right)$ is

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y+9}{\frac{12}{23}+9} = \frac{x-2}{\frac{62}{23}-2}$$

$$\frac{y+9}{\frac{12+207}{23}} = \frac{x-2}{\frac{62-46}{23}}$$

$$\frac{23(y+9)}{219} = \frac{23(x-2)}{16}$$

$$16(y+9) = 219(x-2)$$

$$16y + 144 = 219x - 438$$
$$0 = 219x - 438 - 16y - 144$$
$$\boxed{219x - 16y - 582 = 0} \quad \text{Ans}$$

(ii) $\mathbf{x - y - 4 = 0}$

$\mathbf{7x + y + 20 = 0}$

Let P (x, y) be the point of intersection of given lines.

$$\frac{x}{(-1)(20) - 1(-4)} = \frac{-y}{1(20) - 7(-4)} = \frac{1}{1(1) - 7(-1)}$$
$$\frac{x}{-20 + 4} = \frac{-y}{20 + 28} = \frac{1}{1 + 7}$$
$$\frac{x}{-16} = \frac{-y}{48} = \frac{1}{8}$$
$$\Rightarrow \frac{x}{-16} = \frac{1}{8} \quad \text{and} \quad \frac{-y}{48} = \frac{1}{8}$$
$$x = \frac{-16}{8} \qquad y = \frac{-48}{8}$$
$$x = -2 \qquad y = -6$$

$\therefore$ P (x, y) = P (– 2, – 6)

Given line is

$6x + y - 14 = 0$

Let m_1 be the slope of given line and m_2 be the slope of required line.

$$m_1 = -\frac{6}{1} = -6$$

(a) Since the given line and the required line is parallel.

$\therefore m_1 = m_2$

$m_2 = -6$

Equation of line passing through (– 2, – 6) and having slope $m_2 = -6$ is

$$y - y_1 = m_2(x - x_1)$$
$$y + 6 = -6(x + 2)$$
$$y + 6 = -6x - 12$$
$$6x + y + 6 + 12 = 0$$
$$\boxed{6x + y = 0}$$

(b) Since the given line and the required line is perpendicular.

$\therefore m_1 \times m_2 = -1$

$-6 \times m_2 = -1$

$$m_2 = \frac{-1}{-6} = \frac{1}{6}$$

Equation of line passing through $(-2, -6)$ and having slope $m_2 = \frac{1}{6}$ is

$$y - y_1 = m_2(x - x_1)$$

$$y + 6 = \frac{1}{6}(x + 2)$$

$$6y + 36 = x + 2$$

$$0 = x + 2 - 6y - 36$$

$$\boxed{x - 6y - 34 = 0} \qquad \text{Ans}$$

(iii) $\mathbf{x + 2y + 3 = 0}$

$\mathbf{3x + 4y + 7 = 0}$

Let $P(x, y)$ be the point of intersection of given lines.

$$\frac{x}{2(7) - 3(4)} = \frac{-y}{1(7) - 3(3)} = \frac{1}{1(4) - 2(3)}$$

$$\frac{x}{14 - 12} = \frac{-y}{7 - 9} = \frac{1}{4 - 6}$$

$$\frac{x}{2} = \frac{-y}{-2} = \frac{1}{-2}$$

$$\Rightarrow \frac{x}{2} = \frac{1}{-2} \qquad \text{and} \qquad \frac{-y}{-2} = \frac{1}{-2}$$

$$x = \frac{2}{-2} \qquad\qquad y = \frac{2}{-2}$$

$$x = -1 \qquad\qquad y = -1$$

$$\therefore P(x, y) = P(-1, -1)$$

Equation of line passing through $P(-1, -1)$ and having slope m is

$$y - y_1 = m(x - x_1)$$

$$y + 1 = m(x + 1)$$

$$y + 1 = mx + m$$

$$y - mx + 1 - m = 0 \qquad \text{..... (1)}$$

<u>x–intercept</u>

Put $y = 0$ in equation (1)

$$0 - mx + 1 - m = 0$$

$$1 - m = mx$$

$$x = \frac{1 - m}{m}$$

<u>y–intercept</u>

Put $x = 0$ in equation (1)

$$y + 1 - m = 0$$

$$y = m - 1$$

Since x – intercept = y intercept

$$\frac{1-m}{m} = m - 1$$

$$1 - m = m^2 - m$$

$$1 = m2 - m + m$$

$$m^2 = 1$$

$$m = \pm 1$$

Put $m = 1$ in eq. (1)

$$y - x + 1 - 1 = 0$$

$$y - x = 0$$

$$y = x$$

This equation passing through origin and having no intercepts on the axes. So we neglect it.

Put $m = -1$ in eq. (1)

$$y + x + 1 + 1 = 0$$

$$\boxed{x + y + 2 = 0} \quad \text{Ans}$$

Q.3 Find an equation of the line through the intersection of 16x – 10y – 33 = 0; 12x + 14y + 29 = 0 and the intersection of x – y + 4 = 0 ; x – 7y + 2 = 0

Solution:

$$16x - 10\,y - 33 = 0$$

$$12x + 14y + 29 = 0$$

Let P(x, y) be the point of intersection of above lines.

$$\frac{x}{(-10)(29) - 14(-33)} = \frac{-y}{16(29) - 12(-33)} = \frac{1}{16(14) - 12(-10)}$$

$$\frac{x}{-290 + 462} = \frac{-y}{464 + 396} = \frac{1}{224 + 120}$$

$$\frac{x}{172} = \frac{-y}{860} = \frac{1}{344}$$

$$\Rightarrow \frac{x}{172} = \frac{1}{344}, \quad \frac{-y}{860} = \frac{1}{344}$$

$$x = \frac{172}{344} \qquad y = \frac{-860}{344}$$

$$x = \frac{1}{2} \qquad y = \frac{-5}{2}$$

$\therefore$ P (x, y) $= P\left(\frac{1}{2}, \frac{-5}{2}\right)$

$x - y + 4 = 0 \qquad x - 7y + 2 = 0$

Let P_1 (x, y) be the point of intersection of above lines.

$$\frac{x}{(-1)(2) - 4(-7)} = \frac{-y}{1(2) - 1(4)} = \frac{1}{1(-7) - 1(-1)}$$

$$\frac{x}{-2 + 28} = \frac{-y}{2 - 4} = \frac{1}{-7 + 1}$$

$$\frac{x}{26} = \frac{-y}{-2} = \frac{1}{-6}$$

$$\Rightarrow \frac{x}{26} = \frac{1}{-6} \quad \text{and} \quad \frac{-y}{-2} = \frac{1}{-6}$$

$$x = \frac{-26}{6} \qquad y = \frac{2}{-6}$$

$$x = \frac{-13}{3} \qquad y = \frac{-1}{3}$$

$$\therefore P_1(x, y) = P_1\left(\frac{-13}{3}, \frac{-1}{3}\right)$$

Equation of line passing through $\left(\frac{1}{2}, \frac{-5}{2}\right)$ and $\left(\frac{-13}{3}, \frac{-1}{3}\right)$ is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y + \frac{5}{2}}{\frac{-1}{3} + \frac{5}{2}} = \frac{x - \frac{1}{2}}{\frac{-13}{3} - \frac{1}{2}}$$

$$\frac{\frac{2y + 5}{2}}{\frac{-2 + 15}{6}} = \frac{\frac{2x - 1}{2}}{\frac{-26 - 3}{6}}$$

$$\frac{6(2y + 5)}{2(13)} = \frac{6(2x - 1)}{2(-29)}$$

$$-29(2y + 5) = 13(2x - 1)$$

$$-58y - 145 = 26x - 13$$

$$0 = 26x - 13 + 58y + 145$$

$$26x + 58y + 132 = 0$$

$$2(13x + 29y + 66) = 0 \qquad \boxed{13x + 29y + 66 = 0} \qquad \text{Ans}$$

Q.4: Find the condition that the lines $y = m_1x + c_1$; $y = m_2x + c_2$ and $y = m_3x + c_3$ are concurrent. (Lhr. Board 2007)

Solution:

$$y = m_1x + c_1$$
$$y = m_2x + c_2$$
$$y = m_3x + c_3$$
$$m_1x - y + c_1 = 0$$
$$m_2x - y + c_2 = 0$$
$$m_3x - y + c_3 = 0$$

Since the lines are concurrent

$$\therefore \begin{vmatrix} x_1 & y_1 & c_1 \\ x_2 & y_2 & c_2 \\ x_3 & y_3 & c_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} m_1 & -1 & c_1 \\ m_2 & -1 & c_2 \\ m_3 & -1 & c_3 \end{vmatrix} = 0$$

$$- \begin{vmatrix} m_1 & 1 & c_1 \\ m_2 & 1 & c_2 \\ m_3 & 1 & c_3 \end{vmatrix} = 0$$

$R_2 - R_1$, $R_3 - R_1$

$$\begin{vmatrix} m_1 & 1 & c_1 \\ m_2 - m_1 & 0 & c_2 - c_1 \\ m_3 - m_1 & 0 & c_3 - c_1 \end{vmatrix} = 0$$

Expanding from C_2

$$-(-1) \begin{vmatrix} m_2 - m_1 & c_2 - c_1 \\ m_3 - m_1 & c_3 - c_1 \end{vmatrix} = 0$$

$$(m_2 - m_1)(c_3 - c_1) - (m_3 - m_1)(c_2 - c_1) = 0$$

$(m_2 - m_1)(c_3 - c_1) = (m_3 - m_1)(c_2 - c_1)$ is the required condition.

Q.5 Determine the value of P such that the lines 2x – 3y – 1 = 0, 3x – y – 5 = 0 and 3x + py + 8 = 0 meet at a point.

(Lhr. Board 2007, 2009) (Guj. Board 2008)

Solution:

$2x - 3y - 1 = 0$

$3x - y - 5 = 0$

$3x + Py + 8 = 0$

Since the lines are concurrent

$$\therefore \begin{vmatrix} 2 & -3 & -1 \\ 3 & -1 & -5 \\ 3 & P & 8 \end{vmatrix} = 0$$

$$2\begin{vmatrix} -1 & -5 \\ P & 8 \end{vmatrix} - (-3)\begin{vmatrix} 3 & -5 \\ 3 & 8 \end{vmatrix} + (-1)\begin{vmatrix} 3 & -1 \\ 3 & P \end{vmatrix} = 0$$

$2(-8 + 5P) + 3(24 + 15) - 1(3P + 3) = 0$

$-16 + 10P + 3(39) - 3P - 3 = 0$

$-16 + 10P + 117 - 3P - 3 = 0$

$7P + 98 = 0$

$7P = -98$

$P = \dfrac{-98}{7}$

$\boxed{P = -14}$ Ans

Q.6 Show that the lines 4x – 3y – 8 = 0, 3x – 4y – 6 = 0 and x – y – 2 = 0 are concurrent and third line bisects the angle formed by the first two lines.

(Lhr. Board 2011) (Guj. Board 2008)

Solution:

$4x - 3y - 8 = 0$ ... (1)

$3x - 4y - 6 = 0$ ... (2)

$x - y - 2 = 0$ ... (3)

Taking

$$\begin{vmatrix} 4 & -3 & -8 \\ 3 & -4 & -6 \\ 1 & -1 & -2 \end{vmatrix}$$

$$= 4\begin{vmatrix} -4 & -6 \\ -1 & -2 \end{vmatrix} - (-3)\begin{vmatrix} 3 & -6 \\ 1 & -2 \end{vmatrix} + (-8)\begin{vmatrix} 3 & -4 \\ 1 & -1 \end{vmatrix}$$

$$= 4(8-6) + 3(-6+6) - 8(-3+4)$$

$$= 4(2) + 3(0) - 8(1) = 8 + 0 - 8 = 0$$

Shows the lines are concurrent.

Let m_1, m_2 and m_3 be the slopes of lines (1), (2) and (3).

$$m_1 = \frac{-4}{-3} = \frac{4}{3}$$

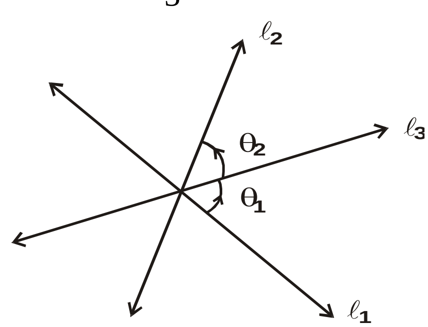
$$m_2 = \frac{-3}{-4} = \frac{3}{4}$$

$$m_3 = \frac{-1}{-1} = 1$$

Let θ_1 be an angle from ℓ_1 to ℓ_3 and θ_2 be an angle from ℓ_3 to ℓ_2.

$$\tan\theta_1 = \frac{m_3 - m_1}{1 + m_3 m_1}$$

$$= \frac{1 - \frac{4}{3}}{1 + (1)\left(\frac{4}{3}\right)} = \frac{\frac{3-4}{3}}{\frac{3+4}{3}}$$



$$\tan\theta_1 = \frac{-1}{7}$$

$$\tan\theta_2 = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{\frac{3}{4} - 1}{1 + \left(\frac{3}{4}\right)(1)}$$

$$= \frac{\frac{3-4}{4}}{\frac{4+3}{4}}$$

$\tan\theta_2 = \dfrac{-1}{7}$

$\therefore\ \tan\theta_1 = \tan\theta_2$

$\Rightarrow \theta_1 = \theta_2$

$\Rightarrow \ell_3$ bisect the angle formed by the first two lines.

Q.7 The vertices of a triangle are A (– 2, 3), B (– 4, 1) and C (3, 5). Find coordinates of the

(i) centroid (L.B 2006) (ii) Orthocentre (L.B 2009 (S))

(iii) Circumcentre of the triangle. Are these three points are collinear?

Solution:

A (– 2, 3), B (– 4, 1) C (3, 5)

(i) Centroid of a triangle is the intersection of medians is

$$= \left(\frac{x_1 + x_2 + x_3}{2}, \frac{y_1 + y_2 + y_3}{2}\right)$$

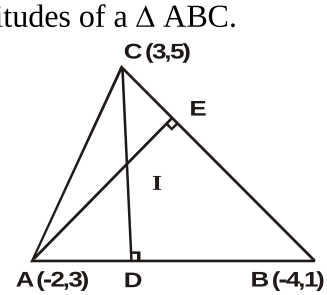
$$= \left(\frac{-2-4+3}{3}, \frac{3+1+5}{3}\right) = \left(\frac{-3}{3}, \frac{9}{3}\right)$$

$= (-1, 3)$ is the centroid of Δ ABC.

(ii) Orthocentre is the point of intersection of altitudes.

Let I be an orthocentre of a Δ ABC.

Let $\overline{CD}$ and $\overline{AE}$ be the altitudes of a Δ ABC.



Slope of side BC $= \dfrac{5-1}{3+4} = \dfrac{4}{7}$

Slope of side AB $= \dfrac{1-3}{-4+2} = \dfrac{-2}{-2} = 1$

Since altitudes are $\perp$ to sides.

$\therefore$ Slope of altitude $\overline{AE} = \dfrac{-1}{\frac{4}{7}} = \dfrac{-7}{4}$

Slope of altitude $\overline{CD} = \frac{-1}{1} = -1$

Equation of altitude $\overline{AE}$ is

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{-7}{4}(x + 2)$$

$$4(y - 3) = -7x - 14$$

$$4y - 12 + 7x + 14 = 0$$

$$7x + 4y + 2 = 0 \quad \text{.......} (1)$$

Equation of altitude $\overline{CD}$ is

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -1(x - 3)$$

$$y - 5 = -x + 3$$

$$x + y - 5 - 3 = 0$$

$$x + y - 8 = 0 \quad \text{.......} (2)$$

To find the point of intersection solving equation (1) and equation (2)

$$\frac{x}{4(-8) - 1(2)} = \frac{-y}{7(-8) - 1(2)} = \frac{1}{7(1) - 1(4)}$$

$$\frac{x}{-32 - 2} = \frac{-y}{-56 - 2} = \frac{1}{7 - 4}$$

$$\frac{x}{-34} = \frac{-y}{-58} = \frac{1}{3}$$

$$\Rightarrow \frac{x}{-34} = \frac{1}{3} \quad \text{and} \quad \frac{-y}{-58} = \frac{1}{3}$$

$$x = \frac{-34}{3} \qquad y = \frac{58}{3}$$

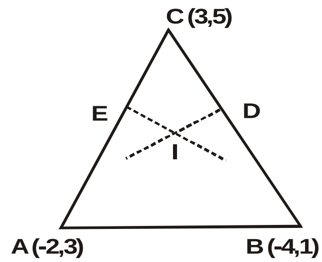
$\therefore \left(\frac{-34}{3}, \frac{58}{3}\right)$ is orthocenter of ΔABC.

(iii) Circumcentre of the triangle is the point of intersection of perpendicular bisector.

Let I be the centre of the circumcircle.

Since D and E are the mid points of sides BC and AC respectively.

Let $\overline{CE}$ and $\overline{CD}$ are the perpendicular bisectors of sides $\overline{AC}$ and $\overline{BC}$ respectively.



Coordinates of D $= \left(\dfrac{3-4}{2}\ ,\ \dfrac{5+1}{2}\right)$

$= \left(\dfrac{-1}{2}\ ,\ \dfrac{6}{2}\right) = \left(\dfrac{-1}{2}\ ,\ 3\right)$

Coordinates of E $= \left(\dfrac{3-2}{2}\ ,\ \dfrac{5+3}{2}\right)$

$= \left(\dfrac{1}{2}\ ,\ \dfrac{8}{2}\right) = \left(\dfrac{1}{2}\ ,\ 4\right)$

Slope of side $\overline{AC} = \dfrac{5-3}{3+2} = \dfrac{2}{5}$

Slope of side $\overline{BC} = \dfrac{5-1}{3+4} = \dfrac{4}{7}$

Since perpendicular bisectors are perpendicular to the sides.

$\therefore$ Slope of $\overline{CE} = \dfrac{-1}{\frac{2}{5}} = \dfrac{-5}{2}$

Slope of $\overline{CD} = \dfrac{-1}{\frac{4}{7}} = \dfrac{-7}{4}$

Equation of perpendicular bisector $\overline{CE}$ is

$y - y_1 = m(x - x_1)$

$y - 4 = \dfrac{-5}{2}\left(x - \dfrac{1}{2}\right)$

$2(y-4) = -5\left(\dfrac{2x-1}{2}\right)$

$10x + 4y - 21 = 0 \quad \text{.... (1)}$

Equation of perpendicular bisector $\overline{CD}$ is

$y - y_1 = m(x - x_1)$

$$y - 3 = \frac{-7}{4}\left(x + \frac{1}{2}\right)$$

$$y - 3 = \frac{-7}{4}\left(\frac{2x+1}{2}\right)$$

$$8(y-3) = -14x - 7$$

$$8y - 24 + 14x + 7 = 0$$

$$14x + 8y - 17 = 0 \qquad .... (2)$$

For point of intersection solving equation (1) and equation (2)

$$\frac{x}{4(-17) - 8(-21)} = \frac{-y}{(-17)(10) - 14(-21)} = \frac{1}{10(8) - 4(14)}$$

$$\frac{x}{-68 + 168} = \frac{-y}{-170 + 294} = \frac{1}{80 - 56}$$

$$\frac{x}{100} = \frac{-y}{124} = \frac{1}{24}$$

$$\Rightarrow \frac{x}{100} = \frac{1}{24} \quad \text{and} \quad \frac{-y}{124} = \frac{1}{24}$$

$$x = \frac{100}{24} \qquad y = \frac{-124}{24}$$

$$x = \frac{25}{6} \qquad y = \frac{-31}{6}$$

$\therefore \left(\frac{25}{6}, \frac{-31}{6}\right)$ is circumcentre of Δ ABC.

$$\text{Taking} \begin{vmatrix} -1 & 3 & 1 \\ \frac{-34}{3} & \frac{58}{3} & 1 \\ \frac{25}{6} & \frac{-31}{6} & 1 \end{vmatrix}$$

$$= -1\begin{vmatrix} \frac{58}{3} & 1 \\ \frac{-31}{6} & 1 \end{vmatrix} - 3\begin{vmatrix} \frac{-34}{3} & 1 \\ \frac{25}{6} & 1 \end{vmatrix} + 1\begin{vmatrix} \frac{-34}{3} & \frac{58}{3} \\ \frac{25}{6} & \frac{-31}{6} \end{vmatrix}$$

$$= -1\left(\frac{58}{3} + \frac{31}{6}\right) - 3\left(\frac{-34}{3} - \frac{25}{6}\right) + 1\left(\frac{1054}{18} - \frac{1450}{18}\right)$$

$$= -1\left(\frac{116+31}{6}\right) - 3\left(\frac{-68-25}{6}\right) + 1\left(\frac{-396}{18}\right)$$

$$= -\frac{147}{6} - 3\left(\frac{-93}{6}\right) - 22 \qquad = -\frac{49}{2} + \frac{93}{2} - 22$$

$$= -\frac{49}{2} - 3\left(\frac{93}{2}\right) - 22 \qquad = \frac{-49+93-44}{2} \quad = \frac{0}{2} \quad = 0$$

Hence centroid, orthocentre and circumcentre of ΔABC are concurrent.

Q.8 Check whether the lines $4x - 3y - 8 = 0$; $3x - 4y - 6 = 0$; $x - y - 2 = 0$ are concurrent. If so, find the point where they meet.

(Lhr. Board 2005) (Guj. Board 2006)

Solution:

$$4x - 3y - 8 = 0 \qquad \text{.... (1)}$$
$$3x - 4y - 6 = 0 \qquad \text{.... (2)}$$
$$x - y - 2 = 0 \qquad \text{.... (3)}$$

Taking $\begin{vmatrix} 4 & -3 & -8 \\ 3 & -4 & -6 \\ 1 & -1 & -2 \end{vmatrix}$

$$= 4\begin{vmatrix} -4 & -6 \\ -1 & -2 \end{vmatrix} - (-3)\begin{vmatrix} 3 & -6 \\ 1 & -2 \end{vmatrix} + (-8)\begin{vmatrix} 3 & -4 \\ 1 & -1 \end{vmatrix}$$

$$= 4(8-6) + 3(-6+6) - 8(-3+4)$$

$$= 4(2) + 3(0) - 8(1) \quad = 8 + 0 - 8 \quad = 0$$

Hence the lines are concurrent. Now we find the point of intersection solving eq. (1) & eq. (2)

$$\frac{x}{-3(-6)-(-4)(-8)} = \frac{-y}{4(-6)-3(-8)} = \frac{1}{4(-4)-3(-3)}$$

$$\frac{x}{18-32} = \frac{-y}{-24+24} = \frac{1}{-16+9}$$

$$\frac{x}{-14} = \frac{-y}{0} = \frac{1}{-7}$$

$$\Rightarrow \frac{x}{-14} = \frac{1}{-7} \quad \text{and} \quad \frac{-y}{0} = \frac{1}{-7}$$

$$x = \frac{-14}{-7} \quad , \quad 7y = 0$$

$$x = 2 \quad , \quad y = 0$$

$\therefore$ (2, 0) is the point of concurrency of three lines.

Q.9 Find the coordinates of the vertices of the triangle formed by the lines x – 2y – 6 = 0 ; 3x – y + 3 = 0; 2x + y – 4 = 0. Also find measures of the angles of the triangle. (Lhr. Board 2005) (Guj. Board 2005)

Solution:

$x - 2y - 6 = 0$ .... (1)

$3x - y + 3 = 0$ .... (2)

$2x + y - 4 = 0$ .... (3)

For point of intersection solving (1) and (2)

$$\frac{x}{-2(3)-(-1)(-6)} = \frac{-y}{1(3)-(3)(-6)} = \frac{1}{1(-1)-3(-2)}$$

$$\frac{x}{-6-6} = \frac{-y}{3+18} = \frac{1}{-1+6}$$

$$\frac{x}{-12} = \frac{-y}{21} = \frac{1}{5}$$

$$\Rightarrow \frac{x}{-12} = \frac{1}{5} \quad \text{and} \quad \frac{-y}{21} = \frac{1}{5}$$

$$x = \frac{-12}{5} \qquad y = \frac{-21}{5}$$

$\therefore \left(\frac{-12}{5}, \frac{-21}{5}\right)$ is the point of intersection of (1) and (2).

For point of intersection solving equation (2) and equation (3)

$$\frac{x}{(-1)(-4)-1(3)} = \frac{-y}{3(-4)-2(3)} = \frac{1}{3(1)-2(-1)}$$

$$\frac{x}{4-3} = \frac{-y}{-12-6} = \frac{1}{3+2}$$

$$\frac{x}{1} = \frac{-y}{-18} = \frac{1}{5}$$

$$\Rightarrow \frac{x}{1} = \frac{1}{5} \quad \text{and} \quad \frac{-y}{-18} = \frac{1}{5}$$

$$x = \frac{1}{5} \qquad y = \frac{18}{5}$$

$\therefore \left(\frac{1}{5}, \frac{18}{5}\right)$ is the point of intersection of (2) & (3)

For point of intersection solving equation (1) & equation (3)

$$\frac{x}{(-2)(-4)-1(-6)} = \frac{-y}{1(-4)-2(-6)} = \frac{1}{1(1)-2(-2)}$$

$$\frac{x}{8+6} = \frac{-y}{-4+12} = \frac{1}{1+4}$$

$$\frac{x}{14} = \frac{-y}{8} = \frac{1}{5}$$

$$\frac{x}{14} = \frac{1}{5} \quad \text{and} \quad \frac{-y}{8} = \frac{1}{5}$$

$$x = \frac{14}{5} \qquad y = \frac{-8}{5}$$

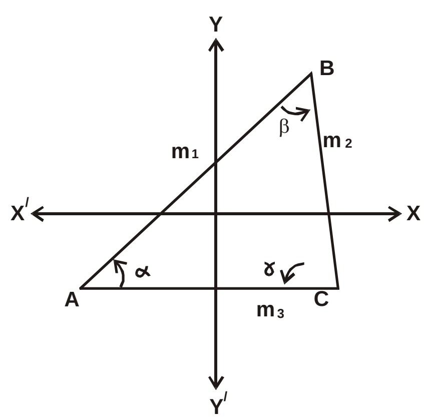
$\therefore \left(\frac{14}{5}, \frac{-8}{5}\right)$ is the point of intersection of equation (1) and equation (3).

Let m_1, m_2 and m_3 be the slopes of sides AB, BC and CA respectively.

$$m_1 = \frac{\frac{18}{5}+\frac{21}{5}}{\frac{1}{5}+\frac{12}{5}} = \frac{\frac{18+21}{5}}{\frac{1+12}{5}} = \frac{39}{13} = 3$$

$$m_2 = \frac{\frac{-8}{5}+\frac{18}{5}}{\frac{14}{5}-\frac{1}{5}} = \frac{\frac{-8-18}{5}}{\frac{14-1}{5}} = \frac{-26}{13} = -2$$

$$m_3 = \frac{\frac{-21}{5}+\frac{8}{5}}{\frac{-12}{5}-\frac{14}{5}} = \frac{\frac{-21+8}{5}}{\frac{-12-14}{5}} = \frac{13}{26} = \frac{1}{2}$$



Let α, β and γ be the angles of a triangle ABC.

$$\tan\alpha = \frac{m_1 - m_3}{1 + m_1 m_3} = \frac{3 - \frac{1}{2}}{1 + (3)\left(\frac{1}{2}\right)}$$

$$= \frac{\frac{6-1}{2}}{\frac{2+3}{2}} = \frac{5}{5}$$

$$\tan\alpha = 1$$

$$\alpha = \tan^{-1}(1) = 45^o$$

$$\tan\beta = \frac{m_2 - m_1}{1 + m_2 m_1}$$

$$= \frac{-2-3}{1+(-2)(3)}$$

$$= \frac{-5}{1-6}$$

$$= \frac{-5}{-5}$$

$$\tan\beta = 1$$

$$\beta = \tan^{-1}(1) = 45^o$$

$$\tan\gamma = \frac{m_3 - m_2}{1 + m_3 m_2}$$

$$= \frac{\frac{1}{2}+2}{1+\left(\frac{1}{2}\right)(-2)} = \frac{\frac{1+4}{2}}{1-1} = \frac{\frac{5}{2}}{0}$$

$$\tan\gamma = \infty$$

$$\gamma = \tan^{-1}(\infty) = 90^o$$

Q.10 Find the angle measured from the line ℓ_1 to the line ℓ_2 where

(a) ℓ_1 : joining (2, 7) and (7, 10)

ℓ_2 : joining (1, 1) and (– 5, 3)

(b) ℓ_1 : joining (3, – 1) and (5, 7)

ℓ_2 : joining (2, 4) and (– 8, 2)

(c) ℓ_1 : joining (1, – 7) and (6, – 4)

ℓ_2 : joining (– 1, 2) and (– 6, – 1)

(d) ℓ_1 : joining (– 9, – 1) and (3, – 5)

ℓ_2 : joining (2, 7) and (– 6, – 7)

Also find the acute angle in each case.

Solution:

(a) Let m_1 be the slope of line ℓ_1 and m_2 be the slope of line ℓ_2.

$$m_1 = \frac{10-7}{7-2} = \frac{3}{5}$$

$$m_2 = \frac{3-1}{-5-1} = \frac{2}{-6} = \frac{-1}{3}$$

Let θ be an angle from line ℓ_1 to ℓ_2.

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$= \frac{\frac{-1}{3} - \frac{3}{5}}{1 + \left(\frac{-1}{3}\right)\left(\frac{3}{5}\right)} = \frac{\frac{-5-9}{15}}{\frac{15-3}{15}} = \frac{-14}{12}$$

$$\tan\theta = \frac{-7}{6}$$

$$\theta = \tan^{-1}\left(\frac{-7}{6}\right) = -49.4^\circ$$

$$= 180^\circ - 49.4^\circ$$

$$= 130.6^\circ \quad \text{Ans}$$

$$\text{Acute angle} = \theta = \tan^{-1}\left|\frac{-7}{6}\right| = 49.4^\circ \text{ Ans.}$$

(b) Let m_1 be the slope of line ℓ_1 and m_2 be the slope of line ℓ_2.

$$m_1 = \frac{7+1}{5-3} = \frac{8}{2} = 4$$

$$m_2 = \frac{2-4}{-8-2} = \frac{-2}{-10} = \frac{1}{5}$$

Let θ be an angle from line ℓ_1 to ℓ_2.

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$= \frac{\frac{1}{5} - 4}{1 + 4\left(\frac{1}{5}\right)} = \frac{\frac{1-20}{5}}{\frac{5+4}{5}} = \frac{-19}{9}$$

$$\theta = \tan^{-1}\left(\frac{-19}{9}\right) = -64.65^\circ$$

$= 180^\circ - 64.5^\circ$

$= 115.35^\circ$ Ans

Acute angle $= \theta = \tan^{-1}\left|\frac{-19}{9}\right| = 64.65^{o}$ Ans.

(c) Let m_1 and m_2 be the slopes of lines ℓ_1 and ℓ_2.

$$m_1 = \frac{-4+7}{6-1} = \frac{3}{5}$$

$$m_2 = \frac{-1-2}{-6+1} = \frac{-3}{-5} = \frac{3}{5}$$

Let θ be an angle from ℓ_1 to ℓ_2.

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$= \frac{\frac{3}{5} - \frac{3}{5}}{1 + \left(\frac{3}{5}\right)\left(\frac{3}{5}\right)} = \frac{0}{1 + \frac{9}{25}}$$

$$\tan\theta = 0$$

$$\theta = \tan^{-1}(0) = 0^{o} \quad \text{Ans}$$

(d) Let m_1 and m_2 be the slopes of lines ℓ_1 and ℓ_2.

$$m_1 = \frac{-5+1}{3+9} = \frac{-4}{12} = \frac{-1}{3}$$

$$m_2 = \frac{-7-7}{-6-2} = \frac{-14}{-8} = \frac{7}{4}$$

Let θ be an angle between from ℓ_1 to ℓ_2.

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$= \frac{\frac{7}{4} + \frac{1}{3}}{1 + \left(\frac{-1}{3}\right)\left(\frac{7}{4}\right)} = \frac{\frac{21+4}{12}}{\frac{12-7}{12}} = \frac{25}{5}$$

$$\tan\theta = 5$$

$$\theta = \tan^{-1}(5)$$

$$\theta = 78.69^{o} \quad \text{Ans}$$

Q.11: Find the interior angles of the triangle whose vertices are

(a) A (– 2, 11), B (– 6, – 3), C (4, – 9) (b) A (6, 1), B (2, 7), C (– 6, – 7)

(c) A (2, – 5), B (– 4, – 3), C (– 1, 5) (d) A (2, 8), B (– 5, 4), C (4, – 9)

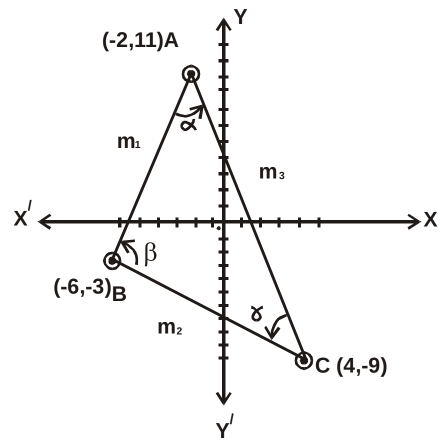
Solution:

(a)A (– 2, 11), B (– 6, – 3), C (4, – 9) (Lhr. Board 2008)

Let m_1, m_2 and m_3 be the slopes of sides AB, BC and AC respectively.

$$m_1 = \frac{-3-11}{-6+2} = \frac{-14}{-4} = \frac{7}{2}$$

$$m_2 = \frac{-9+3}{4+6} = \frac{-6}{10} = \frac{-3}{5}$$



$$m_3 = \frac{-9-11}{4+2} = \frac{-20}{6} = \frac{-10}{3}$$

Let α, β and γ be the angles of a Δ ABC.

Let θ be an angle from line ℓ_1 to ℓ_2.

$$\tan\alpha = \frac{m_3 - m_1}{1 + m_3m_1}$$

$$= \frac{\frac{-10}{3} - \frac{7}{2}}{1 + \left(\frac{-10}{3}\right)\left(\frac{7}{2}\right)} = \frac{\frac{-20-21}{6}}{\frac{6-70}{6}} = \frac{-41}{-64} = \frac{41}{64}$$

$$\alpha = \tan^{-1}\left(\frac{41}{64}\right) = 32.64^\circ \quad \text{Ans.}$$

$$\tan\beta = \frac{m_1 - m_2}{1 + m_1m_2}$$

$$= \frac{\frac{7}{2} + \frac{3}{5}}{1 + \left(\frac{7}{2}\right)\left(\frac{-3}{5}\right)} = \frac{\frac{35+6}{10}}{\frac{10-21}{10}} = \frac{41}{-11}$$

$$\beta = \tan^{-1}\left(\frac{-41}{11}\right)$$

$$= -74.98^\circ$$

$$= 180^\circ - 74.98^\circ$$

$$= 105.02^\circ \qquad \text{Ans.}$$

$$\tan\gamma = \frac{m_2 - m_3}{1 + m_2 m_3}$$

$$= \frac{\frac{-3}{5} + \frac{10}{3}}{1 + \left(\frac{-3}{5}\right)\left(\frac{-10}{3}\right)} = \frac{\frac{-9+50}{15}}{\frac{15+30}{15}}$$

$$\tan\gamma = \frac{41}{45}$$

$$\gamma = \tan^{-1}\left(\frac{41}{45}\right)$$

$$\gamma = 42.34^\circ \qquad \text{Ans}$$

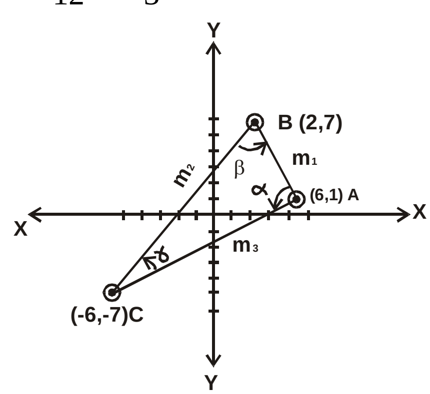
(b) A (6, 1), B (2, 7), C(– 6, – 7)

Let m_1, m_2 and m_3 be the slopes of sides AB, BC and AC respectively.

$$m_1 = \frac{7-1}{2-6} = \frac{6}{-4} = \frac{-3}{2}$$

$$m_2 = \frac{-7-7}{-6-2} = \frac{-14}{-8} = \frac{7}{4}$$

$$m_3 = \frac{-7-1}{-6-6} = \frac{-8}{-12} = \frac{2}{3}$$



Let α, β and γ be the angles of a Δ ABC.

$$\tan\alpha = \frac{m_3 - m_1}{1 + m_3 m_1}$$

$$= \frac{\frac{2}{3}+\frac{3}{2}}{1+\left(\frac{2}{3}\right)\left(\frac{-3}{2}\right)} = \frac{\frac{4+9}{6}}{1-1} = \frac{\frac{13}{6}}{0}$$

$$\tan\alpha = \infty$$

$$\alpha = \tan^{-1}(\infty) = 90^{o} \quad \text{Ans}$$

$$\tan\beta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \frac{\frac{-3}{2}-\frac{7}{4}}{1+\left(\frac{-3}{2}\right)\left(\frac{7}{4}\right)} = \frac{\frac{-12-14}{8}}{\frac{8-21}{8}} = \frac{-26}{-13} = 2$$

$$\beta = \tan^{-1}(2) = 63.43^{o} \quad \text{Ans}$$

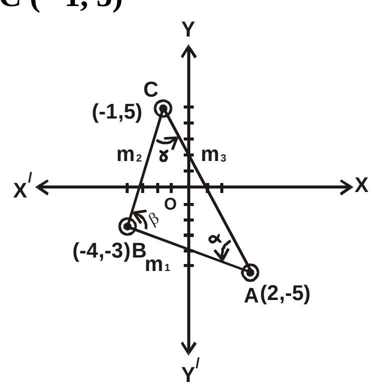
$$\tan\gamma = \frac{m_2 - m_3}{1 + m_2 m_3}$$

$$= \frac{\frac{7}{4}-\frac{2}{3}}{1+\left(\frac{7}{4}\right)\left(\frac{2}{3}\right)} = \frac{\frac{21-8}{12}}{\frac{12+14}{12}} = \frac{13}{26} = \frac{1}{2}$$

$$\gamma = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\boxed{\gamma = 26.57^{o}} \quad \text{Ans}$$

(c) A (2, – 5), B (– 4, – 3), C (– 1, 5)



Let m_1, m_2 and m_3 be the slopes of sides AB, BC and AC respectively.

$$m_1 = \frac{-3+5}{-4+2} = \frac{2}{-6} = \frac{-1}{3}$$

$$m_2 = \frac{5+3}{-1+4} = \frac{8}{3}$$

$$m_3 = \frac{5+5}{-1-2} = \frac{10}{-3} = \frac{-10}{3}$$

Let α, β and γ be the angles of a triangle ABC.

$$\tan\alpha = \frac{m_1 - m_3}{1 + m_1m_3}$$

$$= \frac{\frac{-1}{3} + \frac{10}{3}}{1 + \left(\frac{-1}{3}\right)\left(\frac{-10}{3}\right)} = \frac{\frac{-1+10}{3}}{\frac{9+10}{9}} = \frac{9\,(9)}{3(19)} = \frac{27}{19}$$

$$\alpha = \tan^{-1}\left(\frac{27}{19}\right) = 54.87^{o} \qquad \text{Ans.}$$

$$\tan\beta = \frac{m_2 - m_1}{1 + m_2m_1}$$

$$= \frac{\frac{8}{3} + \frac{1}{3}}{1 + \left(\frac{8}{3}\right)\left(\frac{-1}{3}\right)} = \frac{\frac{9}{3}}{\frac{9-8}{9}} = \frac{9\times 9}{3} = 27$$

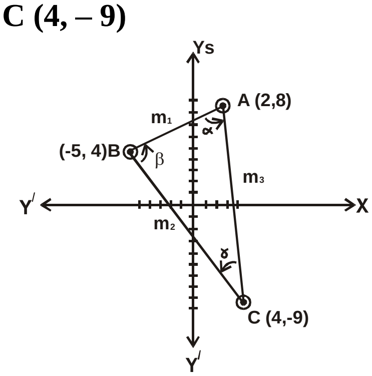
$$\beta = \tan^{-1} 27 = 87.88^{o} \qquad \text{Ans.}$$

$$\tan\gamma = \frac{m_3 - m_2}{1 + m_3m_2}$$

$$= \frac{\frac{-10}{3} - \frac{8}{3}}{1 + \left(\frac{-10}{3}\right)\left(\frac{8}{3}\right)} = \frac{\frac{-18}{3}}{\frac{9-80}{9}} = \frac{-18\times 9}{3\times -71} = \frac{54}{71}$$

$$\gamma = \tan^{-1}\left(\frac{54}{71}\right) = 37.26^{o} \qquad \text{Ans.}$$

(d) A (2, 8), B (– 5, 4), C (4, – 9)



Let m_1, m_2 and m_3 be the slope of sides AB, BC and AC respectively.

$$m_1 = \frac{4-8}{-5-2} = \frac{-4}{-7} = \frac{4}{7}$$

$$m_2 = \frac{-9-4}{4+5} = \frac{-13}{9}$$

$$m_3 = \frac{-9-8}{4-2} = \frac{-17}{2}$$

Let α, β and γ be the angles of ΔABC.

$$\tan\alpha = \frac{m_3 - m_1}{1 + m_3 m_1}$$

$$= \frac{\frac{-17}{2} - \frac{4}{7}}{1 + \left(\frac{-17}{2}\right)\left(\frac{4}{7}\right)} = \frac{\frac{-119-8}{14}}{\frac{14-68}{14}} = \frac{-127}{-54}$$

$$\alpha = \tan^{-1}\left(\frac{127}{54}\right) = 66.96^\circ \quad \text{Ans.}$$

$$\tan\beta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \frac{\frac{4}{7} + \frac{13}{9}}{1 + \left(\frac{4}{7}\right)\left(\frac{-13}{9}\right)} = \frac{\frac{36+91}{63}}{\frac{63-52}{63}} = \frac{127}{11}$$

$$\beta = \tan^{-1}\left(\frac{127}{11}\right) = 85.05^\circ \quad \text{Ans.}$$

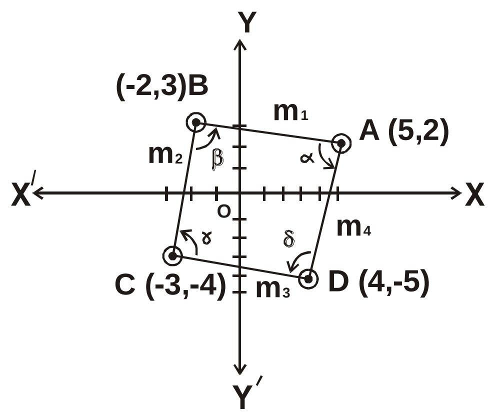
$$\tan\gamma = \frac{m_2 - m_3}{1 + m_2 m_3}$$

$$= \frac{-\frac{13}{9} + \frac{17}{2}}{1 + \left(\frac{-13}{9}\right)\left(\frac{-17}{2}\right)} = \frac{\frac{-26+153}{18}}{\frac{18+221}{18}} = \frac{127}{239}$$

$$\gamma = \tan^{-1}\left(\frac{127}{239}\right) = 27.99^\circ \quad \text{Ans}$$

Q.12 Find the interior angles of the quadrilateral whose vertices are A (5, 2), B (–2, 3), C (– 3, – 4) and D (4, – 5).

Solution:



Let m_1, m_2, m_3 and m_4 be the slopes of sides AB, BC, CD and AD respectively.

$$m_1 = \frac{3-2}{-2-5} = \frac{1}{-7} = \frac{-1}{7}$$

$$m_2 = \frac{-4-3}{-3+2} = \frac{-7}{-1} = 7$$

$$m_3 = \frac{-5+4}{4+3} = \frac{-1}{7}$$

$$m_4 = \frac{-5-2}{4-5} = \frac{-7}{-1} = 7$$

Let α, β, γ and δ be the angles of quadrilateral ABCD.

$$\tan\alpha = \frac{m_4 - m_1}{1 + m_4 m_1}$$

$$= \frac{7 + \frac{1}{7}}{1 + 7\left(\frac{-1}{7}\right)} = \frac{\frac{49+1}{7}}{1-1} = \frac{50}{7(0)} = \frac{50}{0} = \infty$$

$$\alpha = \tan^{-1}(\infty)$$

$$\alpha = 90^\circ$$

$$\tan\beta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\frac{-1}{7} - 7}{1 + \left(\frac{-1}{7}\right)(7)} = \frac{\frac{-1-49}{7}}{1-1} = \frac{-50}{7(0)}$$

$$= \infty$$

$$\beta = \tan^{-1}(\infty) = 90^\circ$$

$$\tan\gamma = \frac{m_2 - m_3}{1 + m_2m_3}$$

$$= \frac{7 + \frac{1}{7}}{1 + (7)\left(\frac{-1}{7}\right)} = \frac{\frac{49 + 1}{7}}{1 - 1} = \frac{50}{7(0)}$$

$$\tan\gamma = \infty$$

$$\gamma = \tan^{-1}(\infty) = 90^{\circ}$$

$$\tan\delta = \frac{m_3 - m_4}{1 + m_3m_4}$$

$$= \frac{\frac{-1}{7} - 7}{1 + \left(\frac{-1}{7}\right)(7)} = \frac{\frac{-1-49}{7}}{1 - 1} = \frac{-50}{7(0)}$$

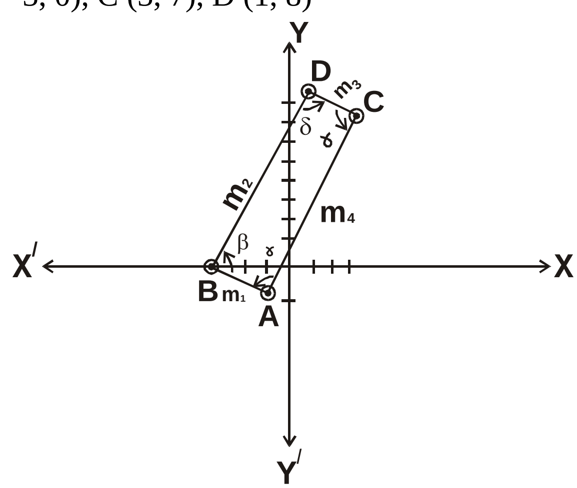
$$\tan\delta = \infty$$

$$\delta = \tan^{-1}(\infty) = 90^{\circ}$$

Q.13: Show that the points A (– 1, – 1), B (– 3, 0), C (3, 7), D (1, 8) are the vertices of a rectangle. Find its interior angles.

Solution:

A (– 1, – 1), B (– 3, 0), C (3, 7), D (1, 8)



Let m_1, m_2, m_3 and m_4 be the slopes of sides AB, BD, DC and AC respectively

$$m_1 = \frac{0 + 1}{-3 + 1} = \frac{1}{-2} = \frac{-1}{2}$$

$$m_2 = \frac{8-0}{1+3} = \frac{8}{4} = 2$$

$$m_3 = \frac{7-8}{3-1} = \frac{-1}{2}$$

$$m_4 = \frac{7+1}{3+1} = \frac{8}{4} = 2$$

$$\tan\alpha = \frac{m_1 - m_4}{1 + m_1 m_4}$$

$$= \frac{\frac{-1}{2} - 2}{1 + \left(\frac{-1}{2}\right)(2)} = \frac{\frac{-1-4}{2}}{1-1} = \frac{\frac{-5}{2}}{0}$$

$$\tan\alpha = \infty$$

$$\alpha = \tan^{-1}(\infty) = 90^{o}$$

$$\tan\beta = \frac{m_2 - m_1}{1 + m_2 m_1}$$

$$= \frac{2 + \frac{1}{2}}{1 + (2)\left(\frac{-1}{2}\right)} = \frac{\frac{4+1}{2}}{1-1} = \frac{\frac{5}{2}}{0}$$

$$\tan\beta = \infty$$

$$\beta = \tan^{-1}(\infty) = 90^{o}$$

$$\tan\delta = \frac{m_3 - m_2}{1 + m_3 m_2}$$

$$= \frac{\frac{-1}{2} - 2}{1 + \left(\frac{-1}{2}\right)(2)} = \frac{\frac{-1-4}{2}}{1-1} = \frac{\frac{5}{2}}{0}$$

$$\tan\delta = \infty$$

$$\delta = \tan^{-1}(\infty) = 90^{o}$$

$$\tan\gamma = \frac{m_4 - m_3}{1 + m_4 m_3}$$

$$= \frac{2 + \frac{1}{2}}{1 + (2)\left(\frac{-1}{2}\right)} = \frac{\frac{4+1}{2}}{1-1} = \frac{\frac{5}{2}}{0}$$

$$\tan\gamma = \infty$$
$$\gamma = \tan^{-1}(\infty) = 90^o$$

Q.14 Find the area of a region bounded by the triangle whose sides are $7x - y - 10 = 0$; $10x + y - 41 = 0$; $3x + 2y + 3 = 0$.

Solution:

$$7x - y - 10 = 0 \quad .... (1)$$
$$10x + y - 41 = 0 \quad .... (2)$$
$$3x + 2y + 3 = 0 \quad .... (3)$$

For point of intersection solving equation (1) and equation (2)

$$\frac{x}{(-1)(-41) - 1(-10)} = \frac{-y}{7(-41) - 10(-10)} = \frac{1}{7(1) - 10(-1)}$$

$$\frac{x}{41 + 10} = \frac{-y}{-287 + 100} = \frac{1}{7 + 10}$$

$$\frac{x}{51} = \frac{-y}{-187} = \frac{1}{17}$$

$$\Rightarrow \quad \frac{x}{51} = \frac{1}{17} \quad \text{and} \quad \frac{-y}{-187} = \frac{1}{17}$$

$$x = \frac{51}{17} \qquad y = \frac{187}{17}$$

$$x = 3 \qquad y = 11$$

$\therefore$ (3, 11) is the point of intersection of equation (1) and equation (2).

For point of intersection solving equation (2) and equation (3)

$$\frac{x}{1(3) - 2(-41)} = \frac{-y}{3(10) - 3(-41)} = \frac{1}{10(2) - 3(1)}$$

$$\frac{x}{3 + 82} = \frac{-y}{30 + 123} = \frac{1}{20 - 3}$$

$$\frac{x}{85} = \frac{-y}{153} = \frac{1}{17}$$

$$\Rightarrow \quad \frac{x}{85} = \frac{1}{17} \quad \text{and} \quad \frac{-y}{153} = \frac{1}{17}$$

$$x = \frac{85}{17} \qquad y = \frac{-153}{17}$$

$$x = 5 \qquad y = -9$$

$\therefore$ (5, – 9) is the point of intersection of equation (2) and equation (3).

For point of intersection solving equation (1) and equation (3).

$$\frac{x}{3(-1)-2(-10)} = \frac{-y}{3(7)-3(-10)} = \frac{1}{7(2)-3(-1)}$$

$$\frac{x}{-3+20} = \frac{-y}{21+30} = \frac{1}{14+3}$$

$$\frac{x}{17} = \frac{-y}{51} = \frac{1}{17}$$

$$\Rightarrow \quad \frac{x}{17} = \frac{1}{17} \quad \text{and} \quad \frac{-y}{51} = \frac{1}{17}$$

$$x = \frac{17}{17} \qquad y = \frac{-51}{17}$$

$$x = 1 \qquad y = -3$$

$\therefore$ (1, – 3) is the point of intersection of equation (1) and equation (2).

$$\text{Area of } \Delta\, ABC = \frac{1}{2}\begin{vmatrix} 3 & 11 & 1 \\ 5 & -9 & 1 \\ 1 & -3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[3\begin{vmatrix} -9 & 1 \\ -3 & 1 \end{vmatrix} - 11\begin{vmatrix} 5 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 5 & -9 \\ 1 & -3 \end{vmatrix}\right]$$

$$= \frac{1}{2}\,[\,3(-9+3) - 11\,(5-1) + 1\,(-15+9)\,]$$

$$= \frac{1}{2}\,[\,3(-6) - 11\,(4) + 1\,(-6)\,]$$

$$= \frac{1}{2}\,[-18-44-6\,] = \frac{-68}{2} = -34$$

= 34 sq. unit Neglecting negative sign. Ans.

Q.15 The vertices of a triangle are A (– 2, 3), B (– 4, 1) and C (3, 5). Find the centre of the circum circle of the triangle.

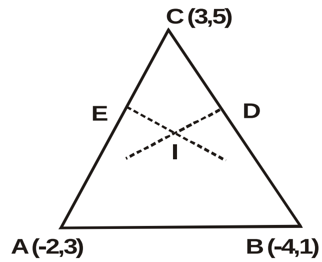
Solution:

A (– 2, 3), B (– 4, 1), C (3, 5)

Let I be the center of the circum circle.

Since D and E are the mid points of sides $\overline{BC}$ and $\overline{AC}$ respectively.

Let $\overline{CE}$ and $\overline{CD}$ be the perpendicular bisectors of sides $\overline{AC}$ and $\overline{BC}$ respectively.



Coordinates of D $= \left(\frac{3-4}{2}\ ,\ \frac{5+1}{2}\right)$

Coordinates of D $= \left(\frac{-1}{2}\ ,\ \frac{6}{2}\right)$

Coordinates of D $= \left(\frac{-1}{2}\ ,\ 3\right)$

Coordinates of E $= \left(\frac{3-2}{2}\ ,\ \frac{5+3}{2}\right)$

Coordinates of E $= \left(\frac{1}{2}\ ,\ \frac{8}{2}\right)$

Coordinates of E $= \left(\frac{1}{2}\ ,\ 4\right)$

Slope of $\overline{AC}$ $= \frac{5-3}{3+2}$

$= \frac{2}{5}$

Slope of $\overline{BC}$ $= \frac{5-1}{3+4} = \frac{4}{7}$

Since perpendicular bisectors are $\perp$ to the sides

$\therefore$ Slope of $\overline{CE}$ $= \frac{-1}{\frac{2}{5}} = \frac{-5}{2}$

Slope of $\overline{CD}$ $= \frac{-1}{\frac{4}{7}} = \frac{-7}{4}$

Equation of perpendicular bisector $\overline{CE}$ is

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{-5}{2}\left(x - \frac{1}{2}\right)$$

$$2(y-4) = -5\left(\frac{2x-1}{2}\right)$$
$$4(y-4) = -10x+5$$
$$4y-16+10x-5 = 0$$
$$10x+4y-21 = 0 \quad \text{.... (1)}$$

Equation of perpendicular bisector $\overline{CD}$ is

$$y-y_1 = m(x-x_1)$$
$$y-3 = \frac{-7}{4}\left(x+\frac{1}{2}\right)$$
$$y-3 = \frac{-7}{4}\left(\frac{2x+1}{2}\right)$$
$$8(y-3) = -14x-7$$
$$8y-24+14x+7 = 0$$
$$14x+8y-17 = 0 \quad \text{..... (2)}$$

Since I be the point of intersection of equation (1) and equation (2)

Solving eq. (1) and eq. (2) for point for intersection.

$$\frac{x}{4(-17)-8(-21)} = \frac{-y}{(-17)(10)-14(-21)} = \frac{1}{10(8)-4(14)}$$
$$\frac{x}{-68+168} = \frac{-y}{-170+294} = \frac{1}{80-56}$$
$$\frac{x}{100} = \frac{-y}{124} = \frac{1}{24}$$

=> $\frac{x}{100} = \frac{1}{24}$ and $\frac{-y}{124} = \frac{1}{24}$

$x = \frac{100}{24}$ $\quad$ $y = \frac{-124}{24}$

$x = \frac{25}{6}$ $\quad$ $y = \frac{-31}{6}$

$\therefore$ $I\left(\frac{25}{6}, \frac{-31}{6}\right)$ is the centre of circum circle.

Q.16 Express the given system of equations in matrix form. Find in each case whether the lines are concurrent.

(a) $x+3y-2 = 0$; $2x-y+4 = 0$; $x-11y+14 = 0$

(b) $2x+3y+4 = 0$; $x-2y-3 = 0$; $3x+y-8 = 0$

(c) $3x-4y-2 = 0$; $x+2y-4 = 0$; $3x-2y+5 = 0$

Solution:

(a)

$$x + 3y - 2 = 0$$
$$2x - y + 4 = 0$$
$$x - 11y + 14 = 0$$

This system of equations can be written in matrix form as

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Taking $\begin{vmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{vmatrix}$

$$= 1\begin{vmatrix} -1 & 4 \\ -11 & 14 \end{vmatrix} - 3\begin{vmatrix} 2 & 4 \\ 1 & 14 \end{vmatrix} + (-2)\begin{vmatrix} 2 & -1 \\ 1 & -11 \end{vmatrix}$$

$$= 1(-14 + 44) - 3(28 - 4) - 2(-22 + 1)$$

$$= 30 - 3(24) - 2(-21) = 30 - 72 + 42 = 0$$

$\therefore$ The given lines are concurrent.

(b)

$$2x + 3y + 4 = 0$$
$$x - 2y - 3 = 0$$
$$3x + y - 8 = 0$$

This system of equations can be written in matrix form as

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & -2 & -3 \\ 3 & 1 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Taking $\begin{vmatrix} 2 & 3 & 4 \\ 1 & -2 & -3 \\ 3 & 1 & -8 \end{vmatrix}$

$$= 2\begin{vmatrix} -2 & -3 \\ 1 & -8 \end{vmatrix} - 3\begin{vmatrix} 1 & -3 \\ 3 & -8 \end{vmatrix} + 4\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix}$$

$$= 2(16 + 3) - 3(-8 + 9) + 4(1 + 6)$$

$= 2(19) - 3(1) + 4(7)$

$= 38 - 3 + 28$

$= 63 \neq 0$

$\therefore$ The given lines are not concurrent.

(c)

$3x - 4y - 2 = 0$

$x + 2y - 4 = 0$

$3x - 2y + 5 = 0$

This system of equations can be written in matrix form as

$$\begin{bmatrix} 3 & -4 & -2 \\ 1 & 2 & -4 \\ 3 & -2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Taking $\begin{vmatrix} 3 & -4 & -2 \\ 1 & 2 & -4 \\ 3 & -2 & 5 \end{vmatrix}$

$$= 3\begin{vmatrix} 2 & -4 \\ -2 & 5 \end{vmatrix} - (-4)\begin{vmatrix} 1 & -4 \\ 3 & 5 \end{vmatrix} + (-2)\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix}$$

$= 3(10 - 8) + 4(5 + 12) - 2(-2 - 6)$

$= 3(2) + 4(17) - 2(-8)$

$= 6 + 68 + 16$

$= 90 \neq 0$

$\therefore$ The given lines are not concurrent.

Q.17: Find a system of linear equations corresponding to the given matrix form. Check whether the lines represented by the system are concurrent.

(a) $\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Solution:

(a) $\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$x - 1 = 0$$
$$2x + 1 = 0$$
$$-y + 2 = 0$$

Taking $\begin{vmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{vmatrix}$

$$= 1\begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} - 0\begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} + (-1)\begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix}$$
$$= 1(0+1) - 0 - 1(-2-0)$$
$$= 1 + 2$$
$$= 3 \neq 0$$

$\therefore$ The lines are not concurrent.

(b) $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$x + y + 2 = 0$$
$$2x + 4y - 3 = 0$$
$$3x + 6y - 5 = 0$$

Taking $\begin{vmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{vmatrix}$

$$= 1\begin{vmatrix} 4 & -3 \\ 6 & -5 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 3 & -5 \end{vmatrix} + 2\begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix}$$
$$= 1(-20+18) - 1(-10+9) + 2(12-12)$$
$$= 1(-2) - 1(-1) + 2(0)$$
$$= -2 + 1$$
$$= -1 \neq 0$$

$\therefore$ The lines are not concurrent.