$0 < 12$

Which is true.

$\therefore$ Graph of an inequality $x + 4y \leq 12$ will be towards the origin side.

Put (0, 0) in

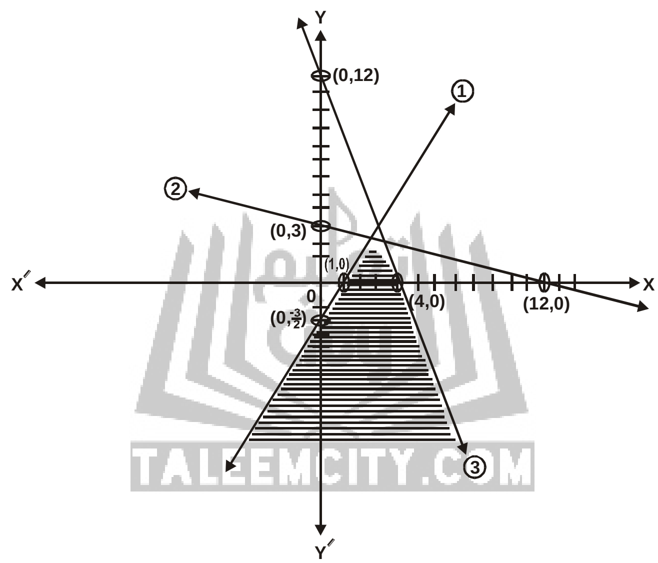
$3x + y < 12$

$3(0) + 0 < 12$

$0 < 12$

Which is true.

$\therefore$ Graph of an inequality $3x + y \leq 12$ will be towards the origin side.



EXERCISE 5.2

Q.4: Graph the feasible region of the following system of linear inequalities and find the corner points in each case.

(i)	**$2x - 3y \leq 6$**	**(ii)**	**$x + y \leq 5$**	**(iii)**	**$x + y \leq 5$**
	$2x + 3y \leq 12$		**$-2x + y \leq 2$**		**$-2x + y \geq 2$**
	$x \geq 0$, $y \geq 0$		**$x \geq 0$, $y \geq o$**		**$x \geq 0$, $y \geq 0$**
(iv)	**$3x + 7y \leq 21$**	**(v)**	**$3x + 2y \geq 6$**	**(vi)**	**$5x + 7y \leq 35$**
	$x - y \leq 3$		**$x + y \leq 4$**		**$x - 2y \leq 4$**
	$x \geq 0$, $y \geq 0$		**$x \geq 0$, $y \geq 0$**		**$x \geq 0$, $y \geq 0$**

Solution:

(i) $\mathbf{2x - 3y \leq 6}$ **(Lhr. Board 2005)**

$\mathbf{2x + 3y \leq 12}$

$\mathbf{x \geq 0 \quad , \quad y \geq 0}$

The associated equations are

$2x - 3y = 6$ ...... (1)

$2x + 3y = 12$ ...... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$$2x - 3(0) = 6$$

$$2x = 6$$

$$x = \frac{6}{2} = 3$$

$\therefore$ Point is (3, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$$2(0) - 3y = 6$$

$$-3y = 6$$

$$y = \frac{6}{-3} = -2$$

$\therefore$ Point is (0, – 2)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$$2x + 3(0) = 12$$

$$x = 12$$

$$x = \frac{12}{2} = 6$$

$\therefore$ Point is (6, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$$2(0) + 3y = 12$$

$$3y = 12$$

$$y = \frac{12}{3} = 4$$

$\therefore$ Point is (0, 4)

<u>Test Point</u>

Put (0, 0) in

$2x - 3y < 6$

$2(0) - 3(0) < 6$

$0 < 6$

Which is true.

$\therefore$ Graph of an inequality $2x - 3y \leq 6$ will be towards the origin side.

Put (0, 0) in

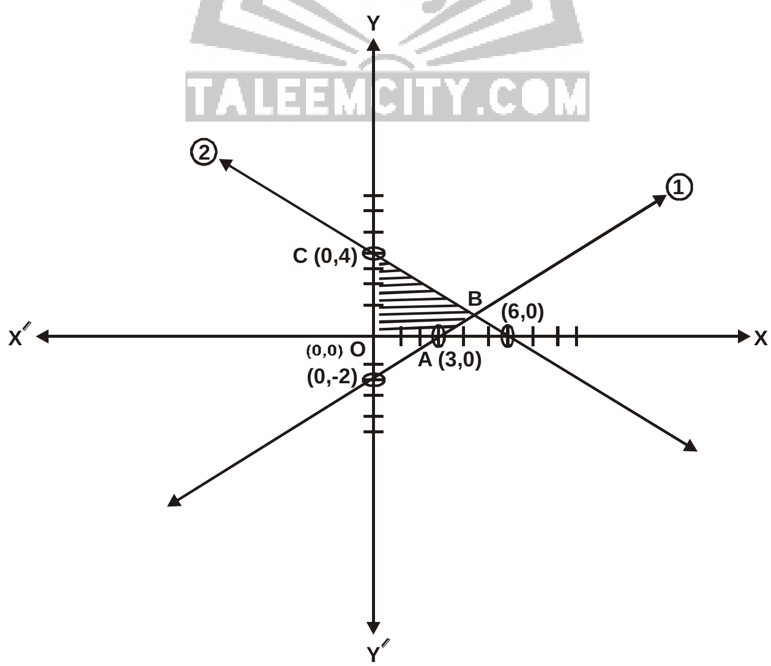
$2x + 3y < 12$

$2(0) + 3(0) < 12$

$0 < 12$

Which is true.

$\therefore$ Graph of an inequality $2x + 3y \leq 12$ will be towards the origin side.



$\therefore$ OABC is the feasible solution region so corner points are

O(0, 0), A(3, 0), C(0, 4)

To find B solving eq. (1) & eq. (2)

Adding eq. (1) & eq. (2)

$$2x - 3y = 6$$
$$\underline{2x + 3y = 12}$$
$$4x = 18$$
$$x = \frac{18}{4} = \frac{9}{2}$$

Put

$$x = \frac{9}{2} \text{ in eq. (1)}$$

$$2\left(\frac{9}{2}\right) - 3y = 6$$
$$9 - 6 = 3y$$
$$y = \frac{3}{3} = 1$$
$$\therefore \quad B\left(\frac{9}{2}, 1\right)$$

(ii) $\mathbf{x + y \le 5}$

$\mathbf{-2x + y \le 2}$

$\mathbf{x \ge 0 \quad , \quad y \ge 0}$

The associated equations are

$$x + y = 5 \quad \text{...... (1)}$$
$$y - 2x = 2 \quad \text{...... (2)}$$

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$$x + 0 = 5$$
$$x = 5$$

$\therefore$ Point is (5, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$$0 + y = 5$$
$$y = 5$$

$\therefore$ Point is (0, 5)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$0 - 2x = 2$

$$x = \frac{2}{-2} = -1$$

$\therefore$ Point is (– 1, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$y - 2(0) = 2$

$y = 2$

$\therefore$ Point is (0, 2)

<u>Test Point</u>

Put (0, 0) in

$x + y < 5$

$0 + 0 < 5$

$0 < 5$

Which is true.

$\therefore$ Graph of an inequality $x + y \leq 5$ will towards the origin side.

Put (0, 0) in

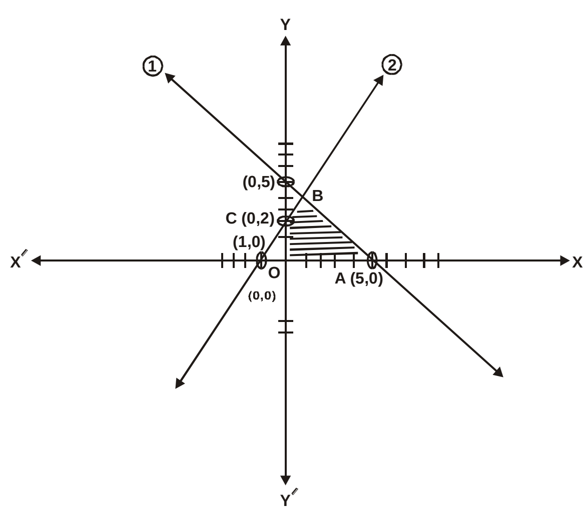
$y - 2x < 2$

$0 - 2(0) < 2$

$0 < 2$

Which is true.

$\therefore$ Graph of an inequality $y - 2x \leq 2$ will towards the origin side.



$\therefore$ OABC is the feasible solution region so corner points are

O(0, 0), A(5, 0), C(0, 2)

To find B solving eq. (1) & eq. (2)

Eq. (1) – Eq. (2) we get

$$\begin{array}{rcl} x + y &=& 5 \\ \underline{\mp 2x \pm y} &\underline{=}& \underline{\mp 2} \\ 3x &=& 3 \end{array}$$

$$x = \frac{3}{3} = 1$$

Put

$x = 1$ in eq. (1)

$1 + y = 5$

$y = 5 - 1 = 4$

$\therefore$ B (1, 4)

(iii) $\mathbf{x + y \leq 5}$

$\mathbf{-2x + y \geq 2}$

$\mathbf{x \geq 0}$ **,** $\mathbf{y \geq 0}$

The associated equations are

$x + y = 5$ ..... (1)

$-2x + y = 2$ ..... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$x + 0 = 5$

$x = 5$

$\therefore$ Point is (5, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$0 + y = 5$

$y = 5$

$\therefore$ Point is (0, 5)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$-2x + 0 = 2$

$$x = \frac{2}{-2} = -1$$

$\therefore$ Point is (– 1, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$-2(0) + y = 2$

$y = 2$

$\therefore$ Point is (0, 2)

<u>Test Point</u>

Put (0, 0) in

$x + y < 5$

$0 + 0 < 5$

$0 < 5$

Which is true.

$\therefore$ Graph of an inequality $x + y \leq 5$ will be towards the origin side.

Put (0, 0) in

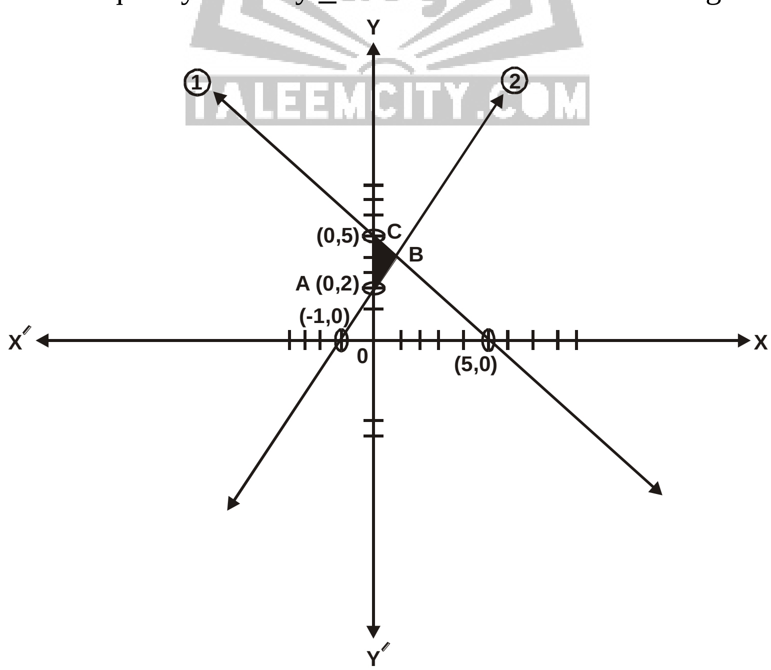
$-2x + y > 2$

$-2(0) + 0 > 2$

$0 > 2$

Which is false.

$\therefore$ Graph of an inequality $-2x + y \geq 2$ will not be towards the origin side.



$\therefore$ ABC is the feasible solution region. So corner points are A (0, 2), C (0, 5). To find B solving eq. (1) & eq. (2)

Eq. (1) – Eq. (2), we get

$$\begin{aligned} x + y &= 5 \\ \underline{\mp 2x \pm y} &= \underline{2} \\ 3x &= 3 \\ x = \frac{3}{3} &= 1 \end{aligned}$$

Put $x = 1$ in eq. (1)

$$1 + y = 5$$

$$y = 5 - 1 = 4$$

$\therefore$ B (1, 4)

(iv) $\mathbf{3x + 7y \leq 21}$

$\mathbf{x - y \leq 3}$

$\mathbf{x \geq 0 \;,\; y \geq 0}$

The associated equations are

$$3x + 7y = 21 \quad \ldots\ldots (1)$$

$$x - y = 3 \quad \ldots\ldots (2)$$

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$$3x + 7(0) = 21$$

$$3x = 21$$

$$x = \frac{21}{3} = 7$$

$\therefore$ Point is (7, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$$3(0) + 7y = 21$$

$$y = \frac{21}{7} = 3$$

$\therefore$ Point is (0, 3)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$$x - 0 = 3$$
$$x = 3$$

$\therefore$ Point is (3, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$0 - y = 3$

$y = -3$

$\therefore$ Point is (0, – 3)

<u>Test Point</u>

Put (0, 0) in

$3x + 7y < 21$

$3(0) + 7(0) < 21$

$0 < 21$

Which is true.

$\therefore$ Graph of an inequality $3x + 7y \leq 21$ will be towards the origin side.

Put (0, 0) in

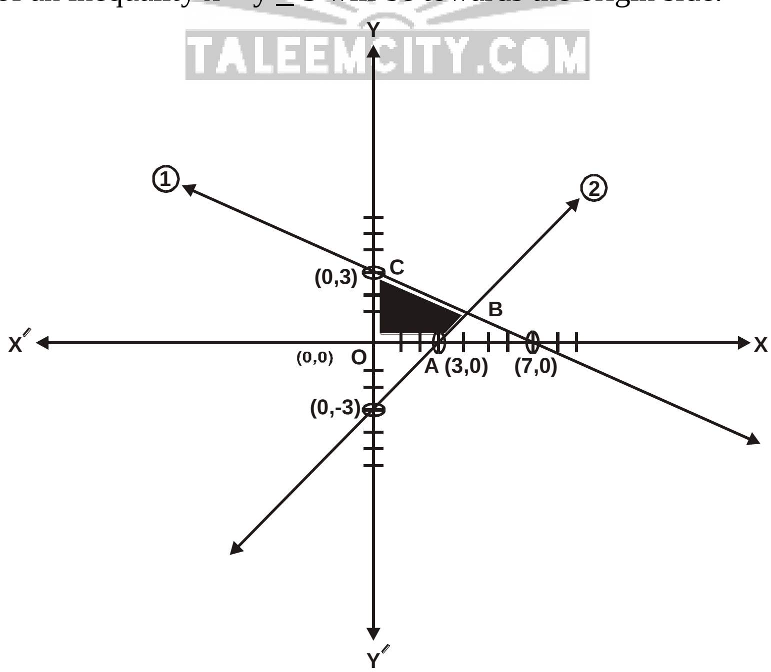
$x - y < 3$

$0 - 0 < 3$

$0 < 3$

Which is true.

$\therefore$ Graph of an inequality $x - y \leq 3$ will be towards the origin side.



$\therefore$ OABC is the feasible solution region so corner points are

O(0, 0), A(3, 0), C(0, 3)

To find B solving eq. (1) & eq. (2)

Eq. (1) + Eq. (2) × 7, we get

$$\begin{aligned} 3x + 7y &= 21 \\ \underline{7x - 7y} &\underline{= 21} \\ 10x &= 42 \\ x &= \frac{42}{10} = \frac{21}{5} \end{aligned}$$

Put $x = \frac{21}{5}$ in eq. (2)

$$\frac{21}{5} - y = 3$$

$$\frac{21}{5} - 3 = y$$

$$y = \frac{21-15}{5}$$

$$y = \frac{6}{5}$$

$\therefore$ $B\left(\frac{21}{5}, \frac{6}{5}\right)$

(v) $\mathbf{3x + 2y \geq 6}$

$\mathbf{x + y \leq 4}$

$\mathbf{x \geq 0 \;,\; y \geq 0}$

The associated equations are

$3x + 2y = 6$ .... (1)

$x + y = 4$ .... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$$3x + 2(0) = 6$$

$$x = \frac{6}{3} = 2$$

$\therefore$ Point is (2, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$3(0) + 2y = 6$

$y = \frac{6}{2} = 3$

$\therefore$ Point is (0, 3)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$x + 0 = 4$

$x = 4$

$\therefore$ Point is (4, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$0 + y = 4$

$y = 4$

$\therefore$ Point is (0, 4)

<u>Test Point</u>

Put (0, 0) in

$3x + 2y > 6$

$3(0) + 2(0) > 6$

$0 > 6$

Which is false.

$\therefore$ Graph of an inequality $3x + 2y \geq 6$ will not be towards the origin side.

Put (0, 0) in

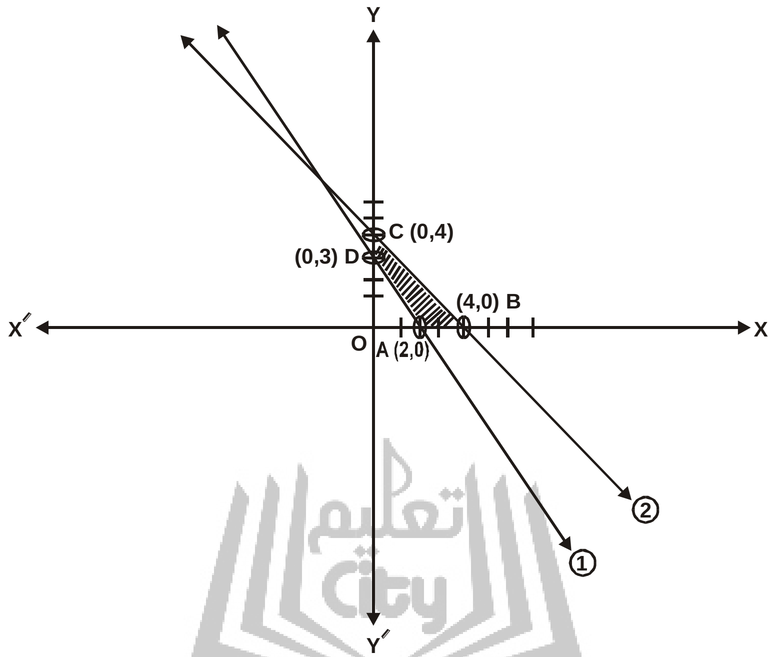
$x + y < 4$

$0 + 0 < 4$

$0 < 4$

Which is true.

$\therefore$ Graph of an inequality $x + y \leq 4$ will be towards the origin side.



$\therefore$ ABCD is the feasible solution region so corner points are

A (2, 0), B (4, 0), C(0, 4), D (0, 3)

(vi) $\mathbf{5x + 7y \leq 35}$

$\mathbf{x - 2y \leq 4}$

$\mathbf{x \geq 0 \;,\; y \geq 0}$

The associated equations are

$5x + 7y = 35$ .... (1)

$x - 2y = 4$ .... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$5x + 7(0) = 35$

$$x = \frac{35}{5} = 7$$

$\therefore$ Point is (7, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$5(0) + 7y = 35$

$y = \frac{35}{7} = 5$

$\therefore$ Point is (0, 5)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$x - 2(0) = 4$

$x = 4$

$\therefore$ Point is (4, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$0 - 2y = 4$

$y = \frac{4}{-2} = -2$

$\therefore$ Point is (0, – 2)

<u>Test Point</u>

Put (0, 0) in

$5x + 7y < 35$

$5(0) + 7(0) < 35$

$0 < 35$

Which is true.

$\therefore$ Graph of an inequality $5x + 7y \leq 35$ will be towards the origin side.

Put (0, 0) in

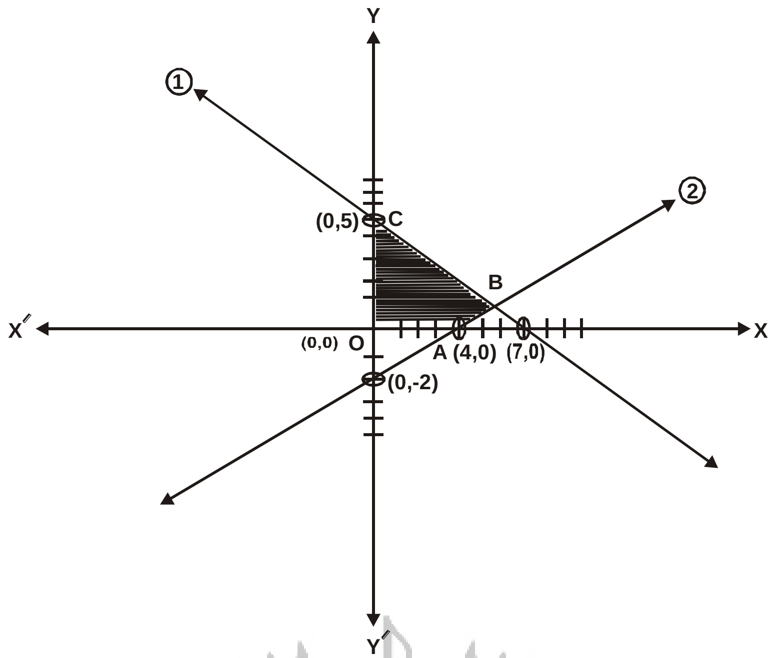
$x - 2y < 4$

$0 - 2(0) < 4$

$0 < 4$

Which is true.

$\therefore$ Graph of an inequality $x - 2y \leq 4$ will be towards the origin.



$\therefore$ OABC is the feasible solution region so corner points are

O (0, 0), A (4, 0), C (0, 5)

To find B solving eq. (1) & eq. (2)

Eq. (1) – Eq. (2) × 5, we get

$$\begin{aligned} 5x + 7y &= 35 \\ -\;5x \mp 10y &= -20 \\ \hline 17y &= 15 \\ y &= \frac{15}{17} \end{aligned}$$

Put $y = \frac{15}{17}$ in eq. (2)

$$x - 2\left(\frac{15}{17}\right) = 4$$

$$x - \frac{30}{17} = 4$$

$$x = 4 + \frac{30}{17}$$

$$x = \frac{68 + 30}{17}$$

$$x = \frac{98}{17}$$

$$\therefore \quad B = \left(\frac{98}{17}, \frac{15}{17}\right)$$

Q.2: Graph the feasible region of the following system of linear inequalities and find the corner points in each case.

(i) $2x + y \le 10$, $x + 4y \le 12$, $x + 2y \le 10$, $x \ge 0, y \ge 0$

(ii) $2x + 3y \le 18$, $2x + y \le 10$, $x + 4y \le 12$, $x \ge 0, y \ge 0$

(iii) $2x + 3y \le 18$, $x + 4y \le 12$, $3x + y \le 12$, $x \ge 0, y \ge 0$

(iv) $x + 2y \le 14$, $3x + 4y \le 36$, $2x + y \le 10$, $x \ge 0, y \ge 0$

(v) $x + 3y \le 15$, $2x + y \le 12$, $4x + 3y \le 24$, $x \ge 0, y \ge 0$

(vi) $2x + y \le 20$, $8x + 15y \le 120$, $x + y \le 11$, $x \ge 0, y \ge 0$

Solution:

(i)
$$2x + y \le 10$$
$$x + 4y \le 12$$
$$x + 2y \le 10$$
$$x \ge 0, y \ge 0$$

The associated eqs. are

$$2x + y = 10 \quad \ldots\ldots (1)$$
$$x + 4y = 12 \quad \ldots\ldots (2)$$
$$x + 2y = 10 \quad \ldots\ldots (3)$$

<u>x-intercept</u>

Put y = 0 in eqs. (1), (2) and (3)

$2x + 0 = 10$	$x + 4(0) = 12$	$x + 2(0) = 10$
$2x = 10$	$x = 12$	$x = 10$
$x = \frac{10}{2} = 5$	$\therefore$ Point is (12, 0)	$\therefore$ Point is (10, 0)
$\therefore$ Point is (5, 0)		

<u>y-intercept</u>

Put x = 0 in eqs. (1), (2) and (3)

$2(0) + y = 10$	$0 + 4y = 12$	$0 + 2y = 10$
$y = 10$	$4y = 12$	$2y = 10$

$\therefore$ Point is (0, 10)

$y = \frac{12}{4} = 3$

$\therefore$ Point is (0, 3)

$y = \frac{10}{2} = 5$

$\therefore$ Point is (0, 5)

Test Point

Put (0, 0)

$2x + y < 10$

$2(0) + 0 < 10$

$0 < 10$

Which is true.

$\therefore$ Graph of an inequality $2x + y \leq 10$ will be towards the origin side.

Put (0, 0) in

$x + 4y < 12$

$0 + 4(0) < 12$

$0 < 12$

Which is true.

$\therefore$ Graph of an inequality $x + 4y \leq 12$ will be towards the origin side.

Put (0, 0) in

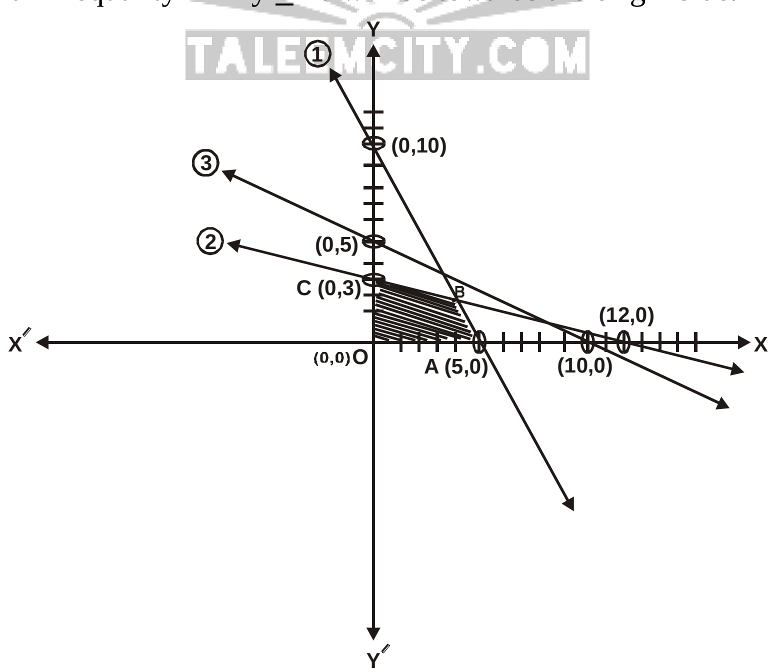
$x + 2y < 10$

$0 + 2(0) < 10$

$0 < 10$

Which is true.

$\therefore$ Graph of an inequality $x + 2y \leq 10$ will be towards the origin side.



$\therefore$ OABC is the feasible solution region so the corner points are

O (0, 0), A (5, 0), C (0, 3)

To find B solving eq. (1) & eq. (2)

Eq. (1) – Eq. (2) × 2, we get

$$\begin{array}{rcl} 2x + y &=& 10 \\ \underline{-\ 2x \pm 8y} &=& \underline{-24} \\ -7y &=& -14 \\ y &=& \frac{14}{7} = 2 \end{array}$$

Put $y = 2$ in eq. (2)

$x + 4(2) = 12$

$x + 8 = 12$

$x = 12 - 8 = 4$

$\therefore$ B = (4, 2)

(ii) $\mathbf{2x + 3y \le 18}$ **(Guj. Board 2005) (Lhr. Board 2008)**

$\mathbf{2x + y \le 10}$

$\mathbf{x + 4y \le 12}$

$\mathbf{x \ge 0 \quad , \quad y \ge 0}$

The associated equations are

$2x + 3y = 18$ .... (1)

$2x + y = 10$ .... (2)

$x + 4y = 12$ .... (3)

x-intercept

Put $y = 0$ in eqs. (1), (2) and (3)

$2x + 3(0) = 18$	$2x + 0 = 10$	$x + 4(0) = 12$
$2x = 18$	$2x = 10$	$x = 12$
$x = \frac{18}{2} = 9$	$x = \frac{10}{2} = 5$	$\therefore$ Point is (12, 0)
$\therefore$ Point is (9, 0)	$\therefore$ Point is (5, 0)	

<u>y-intercept</u>

Put $x = 0$ in eqs. (1), (2) and (3)

$$2(0) + 3y = 18$$
$$3y = 18$$
$$y = \frac{18}{3} = 6$$

$\therefore$ Point is (0, 6)

$$2(0) + y = 10$$
$$y = 10$$

$\therefore$ Point is (0, 10)

$$0 + 4y = 12$$
$$4y = 12$$
$$y = \frac{12}{4} = 3$$

$\therefore$ Point is (0, 3)

<u>Test Point</u>

Put (0, 0)

$2x + 3y < 18$

$2(0) + 3(0) < 18$

$0 < 18$

Which is true.

$\therefore$ Graph of an inequality $2x + 3y \leq 18$ will be towards the origin side.

Put (0, 0) in

$2x + y < 10$

$2(0) + 0 < 10$

$0 < 10$

Which is true.

$\therefore$ Graph of an inequality $2x + y \leq 10$ will be towards the origin side.

Put (0, 0) in

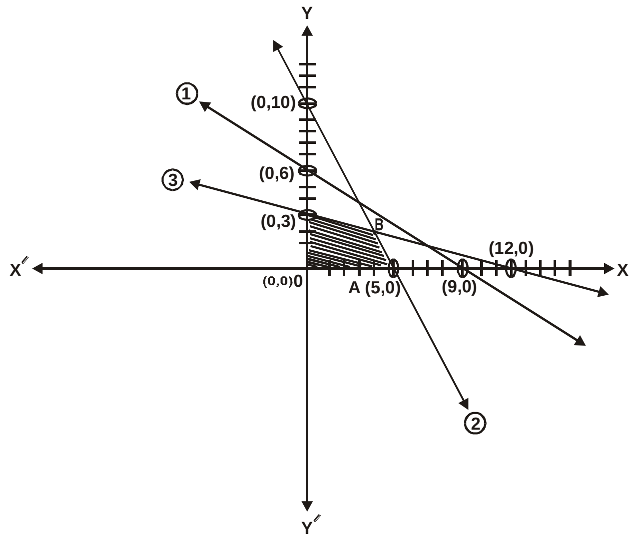
$x + 4y < 12$

$0 + 4(0) < 12$

$0 < 12$

Which is true.

$\therefore$ Graph of an inequality $x + 4y \leq 12$ will be towards the origin side.



$\therefore$ OABC is the feasible solution region so the corner points are

O (0, 0), A (5, 0), C (0, 3)

To find B solving eq. (2) & eq. (3)

Eq. (2) – Eq. (3) × 2, we get

$$\begin{array}{rrcl} & 2x + y & = & 10 \\ - & 2x \pm 8y & = & -24 \\ \hline & -7y & = & -14 \\ & y & = & \dfrac{-14}{-7} = 2 \end{array}$$

Put $y = 2$ in eq. (3)

$x + 4(2) = 12$

$x + 8 = 12$

$x = 12 - 8 = 4$

$\therefore$ B = (4, 2)

(iii) $\mathbf{2x + 3y \le 18}$

$\mathbf{x + 4y \le 12}$

$\mathbf{3x + y \le 12}$

$\mathbf{x \ge 0 \quad , \quad y \ge 0}$

The associated equations are

$2x + 3y = 18$ .... (1)

$x + 4y = 12$ .... (2)

$3x + y = 12$ .... (3)

x-intercept

Put $y = 0$ in eqs. (1), (2) and (3)

$$2x + 3(0) = 18$$
$$2x = 18$$
$$x = \frac{18}{2} = 9$$

$\therefore$ Point is (9, 0)

$$x + 4(0) = 12$$
$$x = 12$$

$\therefore$ Point is (12, 0)

$$3x + 0 = 12$$
$$3x = 12$$
$$x = \frac{12}{3} = 4$$

$\therefore$ Point is (4, 0)

y-intercept

Put $x = 0$ in eqs. (1), (2) and (3)

$$2(0) + 3y = 18$$
$$3y = 18$$
$$x = \frac{18}{3} = 6$$

$\therefore$ Point is (0, 6)

$$0 + 4y = 12$$
$$y = \frac{12}{4} = 3$$

$\therefore$ Point is (0, 3)

$$3(0) + y = 12$$
$$y = 12$$

$\therefore$ Point is (0, 12)

Test Point

Put (0, 0) in

$2x + 3y < 18$

$2(0) + 3(0) < 18$

$0 < 18$

Which is true.

$\therefore$ Graph of an inequality $2x + 3y \leq 18$ will be towards the origin side.

Put (0, 0) in

$x + 4y < 12$

$0 + 4(0) < 12$

$0 < 12$

Which is true.

$\therefore$ Graph of an inequality $x + 4y \leq 12$ will be towards the origin side.

Put (0, 0) in

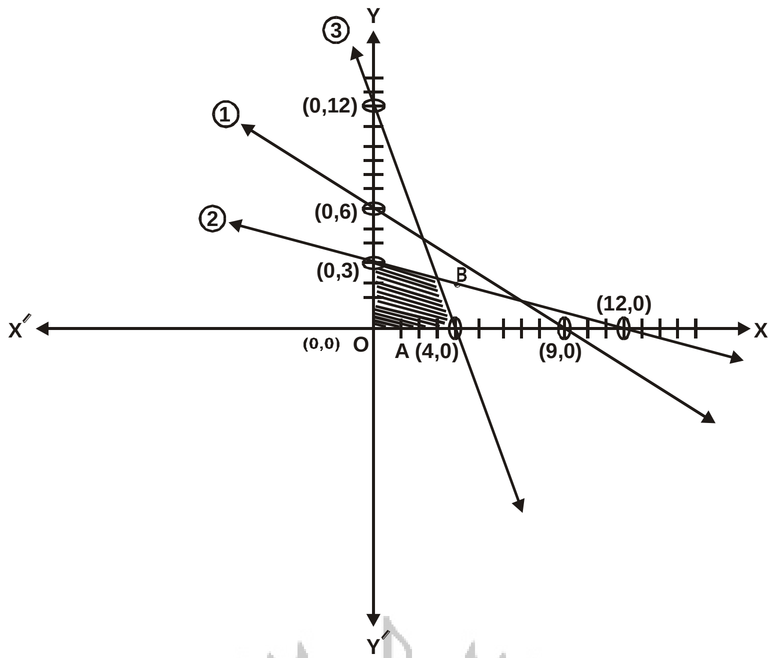
$3x + y < 12$

$3(0) + 0 < 12$

$0 < 12$

Which is true.

$\therefore$ Graph of an inequality $3x + y \leq 12$ will be towards the origin side.


Y
③
(0,12)
①
(0,6)
②
(0,3)
B
(12,0)
X′
X
(0,0)
O
A (4,0)
(9,0)
Y′

$\therefore$ OABC is the feasible solution region so the corner points are

O (0, 0), A (4, 0), C (0, 3)

To find B solving eq. (2) & eq. (3)

Eq. (2) × 3 – Eq. (3), we get

$$\begin{array}{rcl} 3x + 12y &=& 36 \\ \underline{-\ 3x \pm y} &=& \underline{-12} \\ 11y &=& 24 \\ y &=& \frac{24}{11} \end{array}$$

Put $y = \frac{24}{11}$ in eq. (3)

$$3x + \frac{24}{11} = 12$$

$$3x = 12 - \frac{24}{11}$$

$$3x = \frac{132 - 24}{11}$$

$$x = \frac{108}{33} = \frac{36}{11}$$

$$\therefore \quad B\left(\frac{36}{11}, \frac{24}{11}\right)$$

(iv) $\mathbf{x + 2y \le 14}$

$\mathbf{3x + 4y \le 36}$

$\mathbf{2x + y \le 10}$

$\mathbf{x \ge 0 \quad , \quad y \ge 0}$

The associated equations are

$x + 2y = 14$ .... (1)

$3x + 4y = 36$ .... (2)

$2x + y = 10$ .... (3)

<u>x-intercept</u>

Put y = 0 in eqs. (1), (2) and (3)

$x + 2(0) = 14$	$3x + 4(0) = 36$	$2x + 0 = 10$
$x = 14$	$3x = 36$	$2x = 10$
$\therefore$ Point is (14, 0)	$x = \frac{36}{3} = 12$	$x = \frac{10}{2} = 5$
	$\therefore$ Point is (12, 0)	$\therefore$ Point is (5, 0)

<u>y-intercept</u>

Put $x = 0$ in eqs. (1), (2) and (3)

$0 + 2y = 14$

$y = \frac{14}{2} = 7$

$\therefore$ Point is (0, 7)

$3(0) + 4y = 36$

$4y = 36$

$x = \frac{36}{4} = 9$

$\therefore$ Point is (0, 9)

$2(0) + y = 10$

$y = 10$

$\therefore$ Point is (0, 10)

<u>Test Point</u>

Put (0, 0) in

$x + 2y < 14$

$0 + 2(0) < 14$

$0 < 14$

Which is true.

$\therefore$ Graph of an inequality $x + 2y \leq 14$ will be towards the origin side.

Put (0, 0) in

$3x + 4y < 36$

$3(0) + 4(0) < 36$

$0 < 36$

Which is true.

$\therefore$ Graph of an inequality $3x + 4y \leq 36$ will be towards the origin side.

Put (0, 0) in

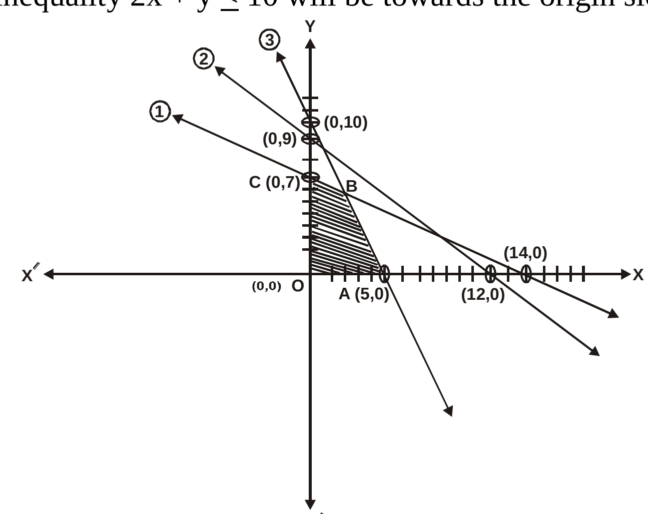
$2x + y < 10$

$2(0) + 0 < 10$

$0 < 10$

Which is true.

$\therefore$ Graph of an inequality $2x + y \leq 10$ will be towards the origin side.



$\therefore$ OABC is the feasible solution region so the corner points are

O (0, 0), A (5, 0), C (0, 7)

To find B solving eq. (1) & eq. (3)

Eq. (1) × 2 – Eq. (3), we get

$$\begin{array}{rcl} 2x + 4y &=& 28 \\ \underline{-\ 2x \pm\ y} &\underline{=}& \underline{-10} \\ 3y &=& 18 \\ y &=& \dfrac{18}{3} = 6 \end{array}$$

$$\begin{array}{rcl} \text{Put}\quad y &=& 6 \text{ in eq. (1)} \\ x + 2(6) &=& 14 \\ x + 12 &=& 14 \\ x &=& 14 - 12 \\ x &=& 2 \end{array}$$

$\therefore$ B (2, 6)

(v)
$$\begin{aligned} \mathbf{x + 3y} &\le \mathbf{15} \\ \mathbf{2x + y} &\le \mathbf{12} \\ \mathbf{4x + 3y} &\le \mathbf{24} \\ \mathbf{x \ge 0}\ ,\quad \mathbf{y} &\ge \mathbf{0} \end{aligned}$$

The associated equations are

$$\begin{array}{rclr} x + 3y &=& 15 & \text{.......... (1)} \\ 2x + y &=& 12 & \text{.......... (2)} \\ 4x + 3y &=& 24 & \text{.......... (3)} \end{array}$$

<u>x-intercept</u>

Put y = 0 in eqs. (1), (2) and (3)

$$\begin{array}{rcl} x + 3(0) &=& 15 \\ x &=& 15 \end{array}$$

$\therefore$ Point is (15, 0)

$$\begin{array}{rcl} 2x + 0 &=& 12 \\ 2x &=& 12 \\ x &=& \dfrac{12}{2} = 6 \end{array}$$

$\therefore$ Point is (6, 0)

$$\begin{array}{rcl} 4x + 3(0) &=& 24 \\ 4x &=& 24 \\ x &=& \dfrac{24}{4} = 6 \end{array}$$

$\therefore$ Point is (6, 0)

<u>y-intercept</u>

Put x = 0 in eqs. (1), (2) and (3)

$$\begin{array}{rcl} 0 + 3y &=& 15 \\ y &=& \dfrac{15}{3} = 5 \end{array}$$

$\therefore$ Point is (0, 5)

$$\begin{array}{rcl} 2(0) + y &=& 12 \\ y &=& 12 \end{array}$$

$\therefore$ Point is (0, 12)

$$\begin{array}{rcl} 4(0) + 3y &=& 24 \\ 3y &=& 24 \\ y &=& \dfrac{24}{3} = 8 \end{array}$$

$\therefore$ Point is (0, 8)

<u>Test Point</u>

Put (0, 0) in

$x + 3y < 15$

$0 + 3(0) < 15$

$0 < 15$

Which is true.

$\therefore$ Graph of an inequality $x + 3y \leq 15$ will be towards the origin side.

Put (0, 0) in

$2x + y < 12$

$2(0) + 0 < 12$

$0 < 12$

Which is true.

$\therefore$ Graph of an inequality $2x + y \leq 12$ will be towards the origin side.

Put (0, 0) in

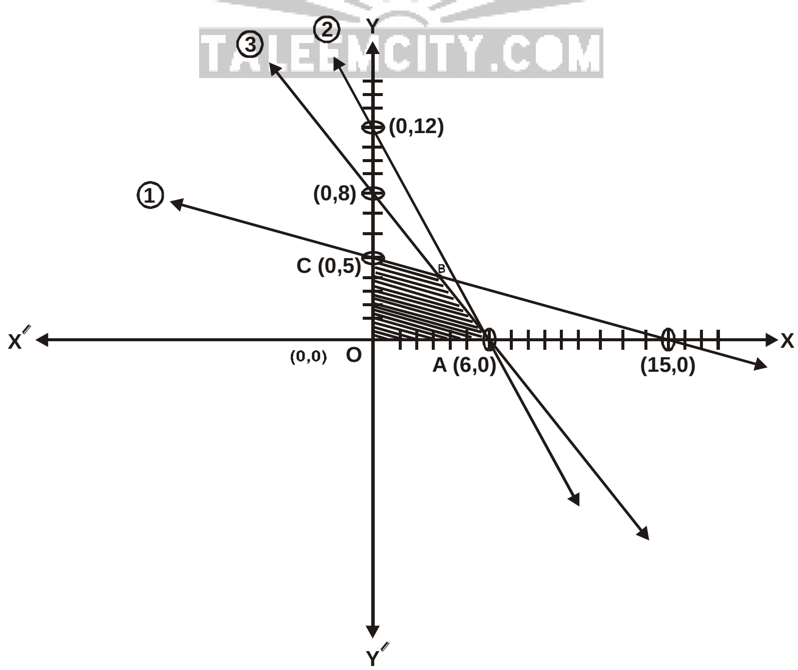
$4x + 3y < 24$

$4(0) + 3(0) < 24$

$0 < 24$

Which is true.

$\therefore$ Graph of an inequality $4x + 3y \leq 24$ will be towards the origin side.



$\therefore$ OABC is the feasible solution region so the corner points are

O (0, 0), A (6, 0), C (0, 5)

To find B solving eq. (1) & eq. (3)

Eq. (1) – Eq. (3), we get

$$\begin{aligned} x + 3y &= 15 \\ \underline{-\ 4x \pm 3y} &\underline{= -24} \\ -3x &= -9 \\ y &= \frac{-9}{-3} = 3 \end{aligned}$$

$$\begin{aligned} \text{Put } x &= 3 \text{ in eq. (1)} \\ 3 + 3y &= 15 \\ 3y &= 15 - 3 \\ 3y &= 12 \\ y &= \frac{12}{3} = 4 \end{aligned}$$

$\therefore$ B (3, 4)

(vi) $\mathbf{2x + y \le 20}$

$\mathbf{8x + 15y \le 120}$

$\mathbf{x + y \le 11}$

$\mathbf{x \ge 0\ ,\quad y \ge 0}$

The associated equations are

$$\begin{aligned} 2x + y &= 20 \quad \text{.......... (1)} \\ 8x + 15y &= 120 \quad \text{.......... (2)} \\ x + y &= 11 \quad \text{.......... (3)} \end{aligned}$$

<u>x-intercept</u>

Put y = 0 in eqs. (1), (2) and (3)

$$\begin{aligned} 2x + 0 &= 20 \\ 2x &= 20 \\ x &= \frac{20}{2} = 10 \end{aligned}$$

$\therefore$ Point is (10, 0)

$$\begin{aligned} 8x + 15(0) &= 120 \\ 8x &= 120 \\ x &= \frac{120}{8} = 15 \end{aligned}$$

$\therefore$ Point is (15, 0)

$$\begin{aligned} x + 0 &= 11 \\ x &= 11 \end{aligned}$$

$\therefore$ Point is (11, 0)

<u>y-intercept</u>

Put x = 0 in eqs. (1), (2) and (3)

$$\begin{aligned} 2(0) + y &= 20 \\ y &= 20 \end{aligned}$$

$\therefore$ Point is (0, 20)

$$\begin{aligned} 8(0) + 15y &= 120 \\ 15\,y &= 120 \end{aligned}$$

$$\begin{aligned} 0 + y &= 11 \\ y &= 11 \end{aligned}$$

$\therefore$ Point is (0, 11)

$$y = \frac{120}{15} = 8$$

$\therefore$ Point is (0, 8)

Test Point

Put (0, 0) in

$2x + y < 20$

$2(0) + 0 < 20$

$0 < 20$

Which is true.

$\therefore$ Graph of an inequality $2x + y \leq 20$ will be towards the origin side.

Put (0, 0) in

$8x + 15y < 120$

$8(0) + 15(0) < 120$

$0 < 120$

Which is true.

$\therefore$ Graph of an inequality $8x + 15y \leq 120$ will be towards the origin side.

Put (0, 0) in

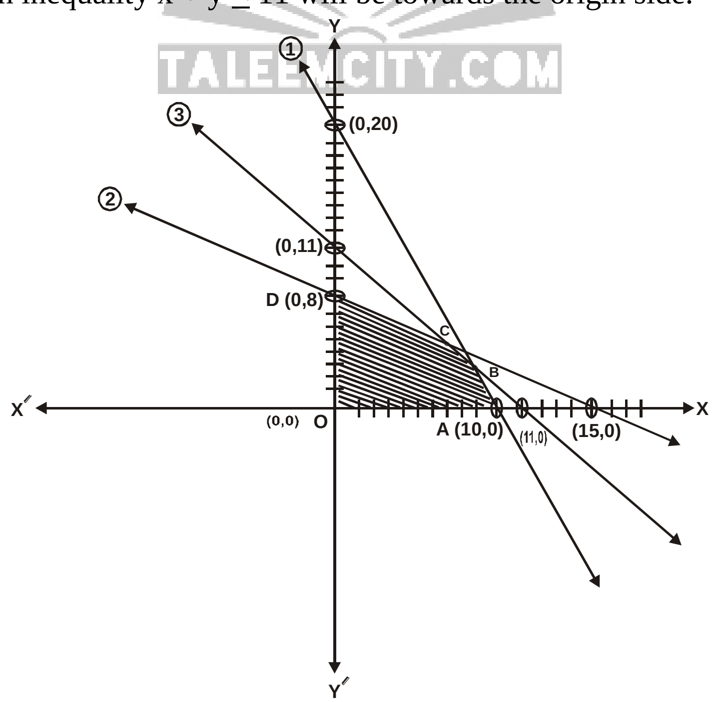
$x + y < 11$

$0 + 0 < 11$

$0 < 11$

Which is true.

$\therefore$ Graph of an inequality $x + y \leq 11$ will be towards the origin side.



$\therefore$ OABCD is the feasible solution region so the corner points are

O (0, 0), A (10, 0), D (0, 8)

To find B solving eq. (1) & eq. (3)

Eq. (1) – Eq. (3), we get

$$\begin{array}{rcl} 2x + y &=& 20 \\ \underline{-\ x \pm y} &=& \underline{-11} \\ x &=& 9 \end{array}$$

Put $x = 9$ in eq. (3)

$$\begin{aligned} 9 + y &= 11 \\ y &= 11 - 9 \\ y &= 2 \end{aligned}$$

$\therefore$ B (9, 2)

To find C solving eq. (2) & eq. (3)

Eq. (2) – Eq. (3) × 8, we get

$$\begin{array}{rcl} 8x + 15y &=& 120 \\ \underline{-\ 8x \pm 8y} &=& \underline{-88} \\ 7y &=& 32 \\ y &=& \dfrac{32}{7} \end{array}$$

Put $y = \dfrac{32}{7}$ in eq. (3)

$$\begin{aligned} x + \frac{32}{7} &= 11 \\ x &= 11 - \frac{32}{7} \\ &= \frac{77 - 32}{7} \\ &= \frac{45}{7} \end{aligned}$$

$\therefore$ $C\left(\dfrac{45}{7}, \dfrac{32}{7}\right)$