EXERCISE 6.2

Q.1: Write down equations of the tangent and normal to the circle.

(i) $\mathbf{x^2 + y^2 = 25}$ **at (4, 3)**

(Lahore Board 2011)

Solution:

$$x^2 + y^2 = 25$$
$$x^2 + y^2 - 25 = 0$$

Compare it with

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$\Rightarrow$ $g = 0$, $f = 0$, $c = -25$

Equation of tangent line is

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$$
$$4x + 3y + 0 + 0 - 25 = 0$$
$$\boxed{4x + 3y - 25 = 0}$$

Equation of normal line is

$$(y - y_1)(x_1 + g) = (x - x_1)(y_1 + f)$$
$$(y - 3)(4 + 0) = (x - 4)(3 + 0)$$
$$4y - 12 = 3x - 12$$
$$3x - 4y - 12 + 12 = 0$$
$$\boxed{3x - 4y = 0}$$

(b) $\mathbf{x^2 + y^2 = 25}$ **at** $(5\cos\theta, 5\sin\theta)$

Solution:

Equation of tangent line is

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$$
$$5\cos\theta\, x + 5\sin\theta\, y + 0 + 0 - 25 = 0$$
$$5\cos\theta\, x + 5\sin\theta\, y - 25 = 0$$
$$5(\cos\theta\, x + \sin\theta\, y - 5) = 0$$
$$\boxed{x\cos\theta + y\sin\theta - 5 = 0}$$

Equation of normal line is

$$(y - y_1)(x_1 + g) = (x - x_1)(y_1 + f)$$
$$(y - 5\sin\theta)(5\cos\theta + 0) = (x - 5\cos\theta)(5\sin\theta + 0)$$

$5\cos\theta\, y - 25\sin\theta\cos\theta = 5\sin\theta x - 25\sin\theta\cos\theta$

$5\sin\theta x - 5\cos\theta y = 0$

$\boxed{x\sin\theta - y\cos\theta = 0}$

(ii) $\mathbf{3x^2 + 3y^2 + 5x - 13y + 2 = 0}$ **at** $\mathbf{(1, \frac{10}{3})}$

Solution:

$3x^2 + 3y^2 + 5x - 13y + 2 = 0$

$3\left(x^2 + y^2 + \frac{5}{3}x - \frac{13}{3}y + \frac{2}{3}\right) = 0$

$x^2 + y^2 + \frac{5}{3}x - \frac{13}{3}y + \frac{2}{3} = 0$

Compare it with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$\Rightarrow \quad 2g = \frac{5}{3}, \quad 2f = \frac{-13}{3}, \quad C = \frac{2}{3}$

$g = \frac{5}{6}, \quad f = \frac{-13}{6}$

Equation of tangent at $(1, \frac{10}{3})$

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

$x + \frac{10}{3}y + \frac{5}{6}(x + 1) - \frac{13}{6}(y + \frac{10}{3}) + \frac{2}{3} = 0$

$x + \frac{10}{3}y + \frac{5}{6}x + \frac{5}{6} - \frac{13}{6}y - \frac{65}{9} + \frac{2}{3} = 0$

$\frac{18x + 60y + 15x + 15 - 39y - 130 + 12}{18} = 0$

$\boxed{33x + 21y - 103 = 0}$

Equation of normal

$(y - y_1)(x_1 + g) = (x - x_1)(y_1 + f)$

$\left(y - \frac{10}{3}\right)\left(1 + \frac{5}{6}\right) = (x - 1)\left(\frac{10}{3} + \frac{13}{6}\right)$

$\left(y - \frac{10}{3}\right)\left(\frac{11}{6}\right) = (x - 1)\left(\frac{20 - 13}{6}\right)$

$11y - \frac{11}{3} = 7x - 7$

$\frac{33y - 110}{3} = 7x - 7$

$33y - 110 = 21x - 21$

$21x - 33y - 21 + 110 = 0$

$\boxed{21x - 33y + 89 = 0}$

Q.2: Write down equations of the tangent and normal to the circle $4x^2 + 4y^2 - 16x + 24y - 117 = 0$ at the points on circle whose abscissa is –4.

Solution:

Given

$$4x^2 + 4y^2 - 16x + 24y - 117 = 0 \qquad \text{at} = -4$$

To find "y" put x = – 4 in (I)

$$4(-4)^2 + 4y^2 - 16(-4) + 24y - 117 = 0$$
$$64 + 4y^2 + 64 + 24y - 117 = 0$$
$$4y^2 + 24y + 11 = 0$$
$$4y^2 + 22y + 2y + 11 = 0$$
$$2y(2y + 11) + 1(2y + 11) = 0$$
$$(2y + 11)(2y + 1) = 0$$

=> $2y + 11 = 0 \qquad 2y + 1 = 0$

$$y = \frac{-11}{2} \qquad y = \frac{-1}{2}$$

Thus the points on the circle are $\left(-4, \frac{-11}{2}\right)$ & $\left(-4, \frac{-1}{2}\right)$

$$4\left(x^2 + y^2 - 4x + 6y - \frac{117}{4}\right) = 0$$
$$x^2 + y^2 - 4x + 6y - \frac{117}{4} = 0$$

Compare it with

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

=> $2g = -4 \quad , \quad 2f = 6 \quad , \quad c = \frac{-117}{4}$

$$g = -2 \quad , \quad f = 3$$

Equation of tangent at $\left(-4, \frac{-1}{2}\right)$

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$$
$$-4x - \frac{1}{2}y - 2(x - 4) + 3\left(y - \frac{1}{2}\right) - \frac{117}{4} = 0$$
$$-4x - \frac{y}{2} - 2x + 8 + 3y - \frac{3}{2} - \frac{117}{4} = 0$$
$$\frac{-16x - 2y - 8x + 32 + 12y - 6 - 117}{4} = 0$$
$$-24x + 10y - 91 = 0$$
$$-(24x - 10y + 91) = 0$$
$$\boxed{24x - 10y + 91 = 0}$$

Equation of normal at $\left(-4, \frac{-1}{2}\right)$

$$(y - y_1)(x_1 + g) = (x - x_1)(y_1 + f)$$
$$\left(y + \frac{1}{2}\right)(-4 - 2) = (x + 4)\left(\frac{-1}{2} + 3\right)$$
$$\left(y + \frac{1}{2}\right)(-6) = (x + 4)\left(\frac{5}{2}\right)$$
$$-6y - 3 = \frac{5x + 20}{2}$$
$$-12y - 6 = 5x + 20$$
$$5x + 12y + 20 + 6 = 0$$
$$\boxed{5x + 12y + 26 = 0}$$

Equation of tangent at $(-4, \frac{-11}{2})$

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$$

$$-4x - \frac{11}{2}y - 2(x - 4) + 3\left(y - \frac{11}{2}\right) - \frac{117}{4} = 0$$

$$-4x - \frac{11}{2}y - 2x + 8 + 3y - \frac{33}{2} - \frac{117}{4} = 0$$

$$\frac{-16x - 22y - 8x + 32 + 12y - 66 - 117}{4} = 0$$

$$-24x - 10y - 151 = 0$$

$$-(24x + 10y + 151) = 0$$

$$\boxed{24x + 10y + 151 = 0}$$

Equation of normal at $(-4, \frac{-11}{2})$

$$(y - y_1)(x_1 + g) = (x - x_1)(y_1 + f)$$

$$\left(y + \frac{11}{2}\right)(-4-2) = (x + 4)\left(\frac{-11}{2} + 3\right)$$

$$\left(y + \frac{11}{2}\right)(-6) = (x + 4)\left(\frac{-5}{2}\right)$$

$$-6y - 33 = \frac{-5x - 20}{2}$$

$$-12y - 66 = -5x - 20$$

$$5x - 12y - 66 + 20 = 0$$

$$\boxed{5x - 12y - 46 = 0}$$

Q.3: Check the position of the point (5, 6) with respect to the circle.

(i) $\mathbf{x^2 + y^2 = 81}$

(Lahore Board 2009, 2010)

Solution:

Given $x^2 + y^2 - 81 = 0$ .... (I)

Put (5, 6) in L.H.S of (I)

$= (5)^2 + (6)^2 - 81$

$= 25 + 36 - 81$

$= -20 < 0$ (– ve)

then (5, 6) lies inside the circle.

(ii) $\mathbf{2x^2 + 2y^2 + 12x - 8y + 1 = 0}$

(Lahore Board 2011)

Solution:

Given $2x^2 + 2y^2 + 12x - 8y + 1 = 0$ .... (I)

Put (5, 6) in L.H.S of (i)

$= 2(5)^2 + (6)^2 + 12(5) - 8(6) + 1$

$= 50 + 72 + 60 - 48 + 1$

$= 135 > 0$ (+ ve)

Then (5, 6) lies outside the circle.

Q.4: Find length of the tangent drawn from the point (– 5, 4) to the circle $5x^2 + 5y^2 - 10x + 15y - 131 = 0$
(Lahore Board 2009)

Solution:

Given $5x^2 + 5y^2 - 10x + 15y - 131 = 0$

Dividing throughout by 5

$$x^2 + y^2 - 2x + 3y - \frac{131}{5} = 0$$

$$\text{Length of Tangent} = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + C\ )}$$

$$= \sqrt{(-5)^2 + (4)^2 - 2(-5) + 3(4) - \frac{131}{5}}$$

$$= \sqrt{25 + 16 + 10 + 12 - \frac{131}{5}}$$

$$= \sqrt{63 - \frac{131}{5}}$$

$$= \sqrt{\frac{315 - 131}{5}}$$

$$= \sqrt{\frac{184}{5}} \quad \text{Ans.}$$

Q.5: Find the length of the chord cut off from the line $2x + 3y = 13$ by the circle $x^2 + y^2 = 26$.

Solution:

Given line $2x + 3y = 13$ .... (i)

Circle is $x^2 + y^2 = 26$ .... (ii)

From (i) $y = \dfrac{13 - 2x}{3}$ .... (iii)

Put in (ii)

$$x^2 + \left(\frac{13 - 2x}{3}\right)^2 = 26$$

$$x^2 + \frac{169 + 4x^2 - 52x}{9} = 26$$

$$9x^2 + 169 + 4x^2 - 52x = 236$$

$$13x^2 - 52x - 65 = 0$$

$$x^2 - 4x - 5 = 0 \qquad \text{(Dividing throughout by 5)}$$

$$x^2 - 5x + x - 5 = 0$$

$$x(x - 5) + 1(x - 5) = 0$$

$(x-5)(x+1) = 0$

$x = 5 \qquad x = -1$

If $x = 5 = y = \dfrac{13-2(5)}{3}$

$y = \dfrac{13-10}{3}$

$y = \dfrac{3}{3} = 1$

and if $x = -1 \quad y = \dfrac{13-2(-1)}{3}$

$y = \dfrac{13+2}{3} = \dfrac{15}{3}$

$y = 5$

Hence points of intersection are A (5, 1) & B (–1, 5)

Required Length of chord $= |AB| = \sqrt{(-1-5)^2+(5-1)^2}$

$= \sqrt{36+16}$

$= \sqrt{52}$

$= 2\sqrt{13}$ Ans

Q.6: Find the coordinates of the points of intersection of line $x + 2y = 6$ with the circle $x^2 + y^2 - 2x - 2y - 39 = 0$

Solution:

Line is $x + 2y = 6$ .... (i)

Circle is $x^2 + y^2 - 2x - 2y - 39 = 0$ .... (ii)

From (i) $x = 6 - 2y$ .... (iii) Put in (ii)

$(6-2y)^2 + y^2 - 2(6-2y) - 2y - 39 = 0$

$36 + 4y^2 - 24y + y^2 - 12 + 4y - 2y - 39 = 0$

$5y^2 - 22y - 15 = 0$

$5y^2 - 25y + 3y - 15 = 0$

$5y(y-5) + 3(y-5) = 0$

$\Rightarrow \quad y - 5 = 0 \qquad 5y + 3 = 0$

$y = 5 \quad \& \quad y = \dfrac{-3}{5}$

if $y = 5 \qquad x = 6 - 2(5)$ (By iii)

$x = 6 - 10$

$x = -4$

if $y = \dfrac{-3}{5} \qquad x = 6 - 2\left(\dfrac{-3}{5}\right)$ (By iii)

$x = 6 + \dfrac{6}{5} \quad \Rightarrow \quad x = \dfrac{36}{5}$

Hence points of intersection are

$(-4, 5)$ & $\left(\frac{36}{5}, \frac{-3}{5}\right)$ Ans.

Q.7 Find equations of the tangents to the circle $x^2 + y^2 = 2$

(i) Parallel the $x - 2y + 1 = 0$

Solution:

Let required tangent $y = mx + c \rightarrow$ (i)

Given circle is $x^2 + y^2 = 2 \Rightarrow r^2 = 2$

Give line is $x - 2y + 1 = 0$

Slope of line $= m = -\frac{\text{cofficient of x}}{\text{coefficient of y}} = -\frac{1}{-2} = \frac{1}{2}$

Since the tangent line is parallel to this line so $m = \frac{1}{2}$

We know that condition of tangency for circle is

$$c^2 = r^2(1 + m^2)$$

$$c^2 = 2\left(1 + \frac{1}{4}\right)$$

$$= 2\left(\frac{5}{4}\right) = \frac{10}{4}$$

$\Rightarrow \quad c = \pm\frac{\sqrt{10}}{2}$ Substitute values in (i)

$$y = \frac{1}{2}x \pm \frac{\sqrt{10}}{2}$$

$$y = \frac{x \pm \sqrt{10}}{2}$$

$$2y = x \pm \sqrt{10}$$

$x - 2y \pm \sqrt{10} = 0$ Required equations of tangent.

(ii) Perpendicular to the line $3x + 2y = 6$

Solution:

Given circle $x^2 + y^2 = 2 \Rightarrow r^2 = 2$

Given line $3x + 2y = 6$

$\therefore$ Slope of line $= \frac{-\text{coeff of x}}{+\text{coeff of y}} = -\frac{3}{2}$

But since tangent line is perpendicular to this line so its slope will be $= \frac{-1}{m} = \frac{2}{3} = m$

We know that condition of tangency of circle is

$c^2 = r^2\ (1 + m^{1^2})$

$c^2 = 2\ (1 + \frac{4}{9})$

$c^2 = 2\left(\frac{13}{9}\right) = \frac{26}{9}$

$c = \pm\frac{\sqrt{26}}{3}$

$y = \frac{2x \pm \sqrt{26}}{3} \quad => \quad 2x - 3y \pm \sqrt{26} = 0$ Ans.

Required equations of tangents are

$y = mx + c$

$y = \frac{2}{3}x \pm \frac{\sqrt{26}}{3}$

$y = \frac{2x \pm \sqrt{26}}{3}$

$2x - 3y \pm \sqrt{26} = 0$ Ans

Q.8: Find equations of tangent drawn from

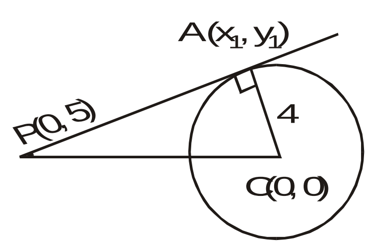
(i) (0, 5) to $x^2 + y^2 = 16$

Solution:

Given circle $x^2 + y^2 = 16$

$=> \quad r^2 = 16 \qquad r = 4$ & Center (0, 0)

Let the tangent drawn from the P(0, 5) point (0, 5) to the circle touch circle at point (x_1, y_1)



$\therefore$ Given circle becomes

$x_1^2 + y_1^2 = 16 \qquad (1)$

Now m_1 = Slope of PA $= \frac{y_1 - 5}{x_1 - 0} = \frac{y_1 - 5}{x_1}$

m_2 = Slope of CA $= \frac{y_1 - 0}{x_1 - 0} = \frac{y_1}{x_1}$

Since two lines are perpendicular $\because \quad m_1 \times m_2 = -1$

$\frac{y_1 - 5}{x_1} \times \frac{y_1}{x_1} = -1$

$y_1^2 - 5y_1 = -x_1^2$

$x_1^2 + y_1^2 = 5y_1 \qquad (2)$

$16 = 5y_1$ (using 1)

$\Rightarrow y_1 = \frac{16}{5}$ Put in (2)

$$x_1^2 + \frac{256}{25} = 5\left(\frac{16}{5}\right)$$

$$x_1^2 = 16 - \frac{256}{25}$$

$$x_1^2 = \frac{400 - 256}{25}$$

$$= \frac{144}{25} \Rightarrow x_1 = \pm\frac{12}{5}$$

We have two points $\left(\frac{12}{5}, \frac{16}{5}\right)$ & $\left(\frac{-12}{5}, \frac{16}{5}\right)$

Now $m_1 =$ slope of line PA $= \frac{y_1 - 5}{x_1}$ at $\left(\frac{12}{5}, \frac{16}{5}\right)$

$$\text{Now } m_1 = \frac{\frac{16}{5} - 5}{\frac{12}{5}}$$

$$= \frac{\frac{16-25}{5}}{\frac{12}{5}} = \frac{-9}{12} = \frac{-3}{4}$$

Equation of tangent at point $\left(\frac{12}{5}, \frac{16}{5}\right)$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{16}{5} = \frac{-3}{4}\left(x - \frac{12}{5}\right)$$

$$\frac{5y - 16}{5} = \frac{-3}{4}\left(\frac{5x - 12}{5}\right)$$

$$20y - 64 = -15x + 36$$

$$15x + 20y = 100$$ Ans.

Next, $m_1 =$ Slope of line PA at point $\left(\frac{-12}{5}, \frac{16}{5}\right)$

$$m_1 = \frac{\frac{16}{5} - 5}{\frac{-12}{5}} = \frac{16 - 25}{-12} = \frac{-9}{-12} = \frac{3}{4}$$

Equation of tangent at point $\left(\frac{-12}{5}, \frac{16}{5}\right)$ is given by

$$y - \frac{16}{5} = \frac{3}{4}\left(x + \frac{12}{5}\right)$$

$$\frac{5y - 6}{5} = \frac{3}{4}\left(\frac{5x + 12}{5}\right)$$

$$20\,y - 64 = 15x + 36$$

$$15x - 20y + 100 = 0 \qquad \text{Ans}$$

(ii) Find equation of tangents drawn from (–1, 2) to the circle $x^2 + y^2 + 4x + 2y = 0$.

Solution:

Given $x^2 + y^2 + 4x + 2y = 0$ .... (1)

Center $= \left(-\frac{4}{2}, \frac{2}{-2}\right) = (-2, -1)$

Let tangent drawn from (–1, 2) to the circle touch the circle at the point (x_1, y_1) then

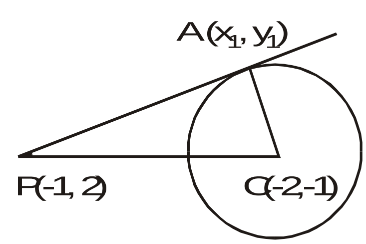
(I) becomes

$x_1^2 + y_1^2 + 4x_1 + 2y_1 = 0$ .... (2)

m_1 = Slope of PA $= \dfrac{y_1 - 2}{x_1 + 1}$

m_2 = Slope of CA $= \dfrac{y_1 + 1}{x_1 + 2}$

Since two lines are perpendicular so



$$m_1 m_2 = -1$$

$$\frac{y_1 - 2}{x_1 + 1} \times \frac{y_1 + 1}{x_1 + 2} = -1$$

$$y_1^2 + y_1 - 2y_1 - 2 = -(x_1^2 + 3x_1 + 2)$$

$$y_1^2 - y_1 - 2 + x_1^2 + 3x_1 + 2 = 0$$

$x_1^2 + y_1^2 - 3x_1 - y_1 = 0$ .... (3)

$-x_1^2 \pm y_1^2 \pm 4x_1 \pm 2y_1 = 0$ (By using - 2)

$-x_1 - 3y_1 = 0$

=> $x_1 = -3y_1$ (4) Put in (3)

$$9y_1^2 + y_1^2 - 9y_1 - y_1 = 0$$

$$10y_1^2 - 10y_1 = 0$$

$$10y_1(y_1-1) = 0$$

$\Rightarrow \quad y_1 = 0 \quad , \quad y_1 = 1$

If $y_1 = 0 \quad x_1 = 0$ (Using - 4) if $y_1 = 1, x_1 = -3$

Required points of tangency are (0, 0) & (–3, 1)

At Point (0, 0)

$m_1 = \text{Slope of (PA)} = \dfrac{-2}{1} = -2$

Equation of tangent at point (0, 0)

$y - y_1 = m(x - x_1)$

$y - 0 = -2(x - 0)$

$y = -2x$

$2x + y = 0$ Ans

At Point (– 3, 1)

$m_1 = \text{Slope of (PA)} = \dfrac{1-2}{-3+1} = \dfrac{-1}{-2}$

Equation of tangent at point (– 3, 1)

$y - y_1 = m(x - x_1)$

$y - 1 = \dfrac{1}{2}(x + 3)$

$2y - 2 = x + 3$

$x - 2y + 5 = 0$ Ans

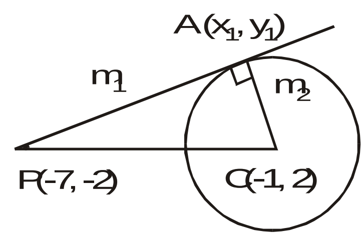
Q.8 (iii): $\mathbf{(-7, -2)}$ **to** $\mathbf{(x+1)^2 + (y-2)^2 = 26}$

Solution:

Given circle $(x+1)^2 + (y-2)^2 = 26$

Center $= (-1, 2)$

Let tangent drawn from point (– 7, – 2) to the circle touch it at point (x_1, y_1).



Given circle becomes

$(x_1+1)^2 + (y_1-2)^2 = 26$

$x_1^2 + 1 + 2x_1 + y_1^2 + 4 - 4y_1 - 26 = 0$

$x_1^2 + y_1^2 + 2x_1 - 4y_1 - 21 = 0$ .... (i)

Now $m_1 = \text{Slope of PA} = \dfrac{y_1+2}{x_1+7}$ $\qquad m_2 = \text{Slope of CA} = \dfrac{y_1-2}{x_1+1}$

Since lines are perpendicular

So $m_1 m_2 = -1$

$$\frac{y_1+2}{x_1+7} \times \frac{y_1-2}{x_1+1} = -1$$

$$y_1^2 - 4 = -x_1^2 - 8x_1 - 7$$

$$x_1^2 + y_1^2 + 8x_1 + 3 = 0 \quad \text{................ (ii)}$$

Subtracting $\quad -x_1^2 \pm y_1^2 \pm 2x_1 \mp 21 \mp 4y_1 = 0$

$$6x_1 + 4y_1 + 24 = 0$$

$$4y_1 = -24 - 6x_1$$

$$y_1 = \frac{-2(12 + 3x_1)}{4}$$
$$= \frac{-(12 + 3x_1)}{2}$$

Put in (ii)

$$x_1^2 + \frac{(12 + 3x_1)^2}{4} + 8x_1 + 3 = 0$$
$$x_1^2 + \frac{144 + 9x_1^2 + 72x_1}{4} + 8x_1 + 3 = 0$$
$$4x_1^2 + 144 + 9x_1^2 + 72x_1 + 32x_1 + 12 = 0$$
$$13x_1^2 + 104x_1 + 156 = 0$$
$$13(x_1^2 + 8x_1 + 12) = 0$$
$$x_1^2 + 8x_1 + 12 = 0$$
$$x_1^2 + 6x_1 + 2x_1 + 12 = 0$$
$$(x_1 + 2)(x_1 + 6) = 0 \quad \Rightarrow \quad x_1 = 2 \quad \& \quad x_1 = -6$$

if $x_1 = -2$; $y_1 = -\left(\frac{3(-2) + 12}{2}\right) = -\left(\frac{-6 + 12}{2}\right) = -3$

if $x_1 = -6$; $y_1 = -\left(\frac{3(-6) + 12}{2}\right) = -\left(\frac{-18 + 12}{2}\right) = 3$

Then points of tangency are $(-2, -3)$ & $(-6, 3)$

At Point $(-2, -3)$

m_1 = Slope of PA $= \frac{-3 + 2}{-2 + 7} = \frac{-1}{5}$

Equation of tangent at point $(-2, -3)$ is

$$y - y_1 = m(x - x_1)$$
$$y + 3 = \frac{-1}{5}(x + 2)$$
$$5(y + 3) = -(x + 2)$$
$$5y + 15 + x + 2 = 0$$
$$x + 5y + 17 = 0$$

At Point $(-6, 3)$

m_1 = Slope of (PA) $= \frac{3 + 2}{-6 + 7} = \frac{5}{1} = 5$

Equation of tangent at point $(-6, 3)$

$$y - y_1 = m(x - x_1)$$
$$y - 3 = 5(x + 6)$$
$$y - 3 = 5x + 30$$
$$5x - y + 33 = 0 \qquad \text{Ans}$$

Q.9: Find an equation of the chord of contact of the tangents drawn from (4, 5) to the circle $2x^2 + 2y^2 - 8x + 12y + 21 = 0$

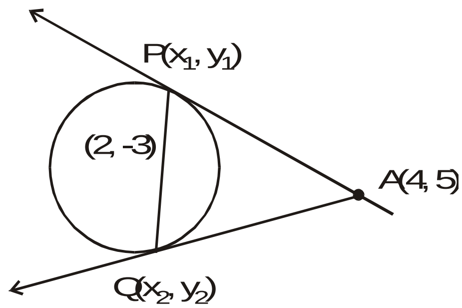
Solution:

Given $2x^2 + 2y^2 - 8x + 12y + 21 = 0$

Dividing throughout by 2

$$x^2 + y^2 - 4x + 6y + \frac{21}{2} = 0$$

Now Let points of contact of the two tangents be p (x, y_1) Q, x_2, y_2) An equation of the tangent at p(x_1, y_1) is



$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0 \quad \text{............ (1)}$$

Since $(-g, -f) = (2, -3)$

$$g = -2 \qquad f = 3 \qquad \text{Put in I}$$

$$xx_1 + yy_1 - 2(x + x_1) + 3(y + y_1) + \frac{21}{2} = 0 \quad \text{............ (2)}$$

Since it passes through (4, 5)

$$4x_1 + 5y_1 - 2(4 + x_1) + 3(5 + y_1) + \frac{21}{2} = 0$$

$$4x_1 + 5y_1 - 8 - 2x_1 + 15 + 3y_1 + \frac{21}{2} = 0$$

$$2x_1 + 8y_1 + 7 + \frac{21}{2} = 0$$

$$4x_1 + 16y_1 + 14 + 21 = 0$$

$$4x_1 + 16y_1 + 35 = 0 \quad \text{................ (i)}$$

Similarly for point Q (x_2, y_2), we have

$$4x_2 + 16y_2 + 35 = 0 \quad \text{................ (ii)}$$

(i) & (ii) Show that both the points P(x_1, y_1) & Q (x_2, y_2) lie on $4x + 16y + 35 = 0$ and so it is the required equation of the chord of contact.

EXERCISE 6.3

Q.1: Prove that normal lines of a circle pass through the center of the circle. (Lahore Board 2009)

Solution:

Let us consider a circle with center (0, 0) and radius r.

Therefore equation of circle is