

$$\text{(iv)} \quad 3x^2 + 5y^2 = 60 \quad \text{and} \quad 9x^2 + y^2 = 124$$

Solution:

$$3x^2 + 5y^2 = 60 \quad \dots \quad (1) \quad 9x^2 + y^2 = 124 \quad \dots \quad (2)$$

Multiplying (1) by (3) & Subtracting from (2)

$$\begin{array}{r} 9x^2 + y^2 = 124 \\ - 9x^2 + 15y^2 = -180 \\ \hline -14y^2 = -56 \\ y^2 = 4 \end{array} \Rightarrow y = \pm 2$$

Put in (1)

$$9x^2 + 4 = 124$$

$$9x^2 = 120$$

$$x^2 = \frac{120}{9} = \frac{40}{3} \quad x = \pm \sqrt{\frac{40}{3}}$$

Hence points of intersection are $\left(\pm \sqrt{\frac{40}{3}}, \pm 2 \right)$

EXERCISE 6.8

Q.1: Find an equation of each of the following with respect to new parallel axes obtained by shifting the origin to the indicated point.

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Solution:

$$\text{(i)} \quad x^2 + 16y - 16 = 0 \quad \dots \quad (1) \quad O'(0, 1) \Rightarrow h = 0, k = 1$$

We know that equations of transformation are

$$x = X + h, \quad y = Y + k$$

$$x = X + 0, \quad y = Y + 1 \quad \text{Put in (1)}$$

$$X^2 + 16(Y + 1) - 16 = 0$$

$$X^2 + 16Y + 16 - 16 = 0$$

$$X^2 + 16Y = 0 \quad \text{Ans}$$

$$\text{(ii)} \quad 4x^2 + y^2 + 16x - 10y + 37 = 0 \quad O'(-2, 5)$$

Solution:

$$4x^2 + y^2 + 16x - 10y + 37 = 0 \quad \dots \quad (i), \quad O'(-2, 5) \Rightarrow h = -2, k = 5$$

We know that equations of transformation are

$$x = X + h, \quad y = Y + k$$

$$x = X - 2, \quad y = Y + 5$$

Put in (1)

$$4(X - 2)^2 + (Y + 5)^2 + 16(X - 2) - 10(Y + 5) + 37 = 0$$

$$4(X^2 + 4 - 4X) + Y^2 + 25 + 10Y + 16X - 32 - 10Y - 50 + 37 = 0$$

$$4X^2 + 16 - 16X + Y^2 + 16X - 20 = 0$$

$$4X^2 + Y^2 - 4 = 0 \quad \text{Ans}$$

$$(iii) \quad 9x^2 + 4y^2 + 18x - 16y - 11 = 0 \quad O'(-1, 2)$$

Solution:

$$9x^2 + 4y^2 + 18x - 16y - 11 = 0 \dots (1), \quad O'(-1, 2) \Rightarrow h = -1, k = 2$$

We know that equations of transformation are

$$x = X + h, \quad y = Y + k$$

$$x = X - 1, \quad y = Y + 2$$

Put in (1)

$$9(X - 1)^2 + 4(Y + 2)^2 + 18(X - 1) - 16(Y + 2) - 11 = 0$$

$$9(X^2 + 1 - 2X) + 4(Y^2 + 4 + 4Y) + 18X - 18 - 16Y - 32 - 11 = 0$$

$$9X^2 + 9 - 18X + 4Y^2 + 16 + 16Y + 18X - 18 - 16Y - 32 - 11 = 0$$

$$9X^2 + 4Y^2 - 36 = 0 \quad \text{Ans}$$

$$(iv) \quad x^2 - y^2 + 4x + 8y - 11 = 0 \quad O'(-2, 4)$$

Solution:

$$x^2 - y^2 + 4x + 8y - 11 = 0 \dots (1), \quad O'(-2, 4) \Rightarrow h = -2, k = 4$$

We know that equations of transformations of transformation

$$x = X + h, \quad y = Y + k$$

$$x = X - 2, \quad y = Y + 4$$

Put in (1)

$$(X - 2)^2 - (Y + 4)^2 + 4(X - 2) + 8(Y + 4) - 11 = 0$$

$$X^2 + 4 - 4X - Y^2 - 16 - 8Y + 4X - 8 + 8Y + 32 - 11 = 0$$

$$X^2 - Y^2 + 1 = 0 \quad \text{Ans.}$$

$$(v) \quad 9x^2 - 4y^2 + 36x + 8y - 4 = 0 \quad O'(-2, 1)$$

Solution:

$$9x^2 - 4y^2 + 36x + 8y - 4 = 0 \dots (1), \quad O'(-2, 1) \Rightarrow h = -2, k = 1$$

We know that equations of transformation are

$$x = X + h, \quad y = Y + k$$

$$x = X - 2, \quad y = Y + 1$$

Put in (1)

$$9(X - 2)^2 - 4(Y + 1)^2 + 36(X - 2) + 8(Y + 1) - 4 = 0$$

$$\begin{aligned} 9(X^2 + 4 - 4X) - 4(Y^2 + 1 + 2Y) + 36X - 72 + 8Y + 8 - 4 &= 0 \\ 9X^2 + 36 - 36X - 4Y^2 - 4 - 8Y + 36X - 72 + 8Y + 4 &= 0 \\ 9X^2 - 4Y^2 - 36 &= 0 \quad \text{Ans.} \end{aligned}$$

Q.2: Find coordinates of the new origin so that first-degree terms are removed from the transformed equation of each of the following. Also find the transformed equation.

(i) $3x^2 - 2y^2 + 24x + 12y + 24 = 0$

Solution:

$$3x^2 - 2y^2 + 24x + 12y + 24 = 0 \quad \dots(1)$$

Let the coordinates of the new origin be (h, k)

Equations of transformed axes are $x = X + h$, $y = Y + K$ Put in (i)

$$\begin{aligned} 3(X + h)^2 - 2(Y + K)^2 + 24(X + h) + 12(Y + k) + 24 &= 0 \\ 3(X^2 + h^2 + 2Xh) - 2(Y^2 + k^2 + 2Yk) + 24X + 24h + 12Y + 12k + 24 &= 0 \\ 3X^2 + 3h^2 + 6Xh - 2Y^2 - 2k^2 - 4Yk + 24X + 24h + 12Y + 12k + 24 &= 0 \\ 3X^2 - 2Y^2 + X(6h + 24) - Y(4k - 12) + 3h^2 - 2k^2 + 24h + 12k + 24 &= 0 \quad \dots(2) \end{aligned}$$

Now, we remove first-degree terms from the transformed equation

$$6h + 24 = 0, \quad 4k - 12 = 0$$

$\Rightarrow h = -4, \quad k = 3$

Thus new origin O' (h, k) = O' (-4, 3)

Put $h = -4, \quad k = 3$ in (2)

$$3X^2 - 2Y^2 + 0 - 0 + 3(16) - 2(9) + 24(-4) + 12(3) + 24 = 0$$

$$3X^2 - 2Y^2 + 48 - 18 - 96 + 36 + 24 = 0$$

$$3X^2 - 2Y^2 - 6 = 0$$

(ii) $25x^2 + 9y^2 + 50x - 36y - 164 = 0$

Solution:

$$25x^2 + 9y^2 + 50x - 36y - 164 = 0 \quad \dots(1)$$

Let the coordinates of the new origin be (h, k) equation of transformed axes are $x = X + h, \quad y = Y + k$

Put in (1)

$$25(X + h)^2 + 9(Y + k)^2 + 50(X + h) - 36(Y + k) - 164 = 0$$

$$25(X^2 + h^2 + 2Xh) + 9(Y^2 + k^2 + 2Yk) + 50X + 50h - 36Y - 36k - 164 = 0$$

$$25X^2 + 25h^2 + 50Xh + 9Y^2 + 9k^2 + 18Yk + 50X + 50h - 36Y - 36k - 164 = 0$$

$$25X^2 + 9Y^2 + (50h + 50)X + (18k - 36)Y + 25h^2$$

$$+ 9k^2 + 50h - 36k - 164 = 0 \quad \dots(2)$$

Now we remove first – degree terms, from the transformed equation

$$50h + 50 = 0 \quad 18k - 36 = 0$$

$$50h = -50$$

$$\boxed{h = -1}$$

$$18k = 36$$

$$\boxed{k = 2}$$

New origin is $O'(h, k) = O'(-1, 2)$

Put $h = -1$, $k = 2$ in (2)

$$25X^2 + 9Y^2 + 0 + 0 + 25(1) + 9(4) + 50(-1) - 36(2) - 164 = 0$$

$$25X^2 + 9Y^2 + 25 + 36 - 50 - 72 - 164 = 0$$

$$25X^2 + 9Y^2 - 225 = 0$$

(iii) $x^2 - y^2 - 6x + 2y + 7 = 0$

Solution:

$$x^2 - y^2 - 6x + 2y + 7 = 0 \quad (1)$$

Let the coordinates of the new origin O' be (h, k) . The equations of transformation are $x = X + h$, $y = Y + k$.

Put in (1)

$$(X + h)^2 - (Y + k)^2 - 6(X + h) + 2(Y + k) + 7 = 0$$

$$X^2 + h^2 + 2Xh - Y^2 - k^2 - 2Yk - 6X - 6h + 2Y + 2k + 7 = 0$$

$$X^2 - Y^2 + X(2h - 6) - Y(2k - 2) + h^2 - k^2 - 6h + 2k + 7 = 0 \quad \dots\dots (2)$$

Now we remove first degree terms from the transformed equation

$$2h - 6 = 0 \quad 2k - 2 = 0$$

$$h = \frac{6}{2} \quad 2k = 2$$

$$h = 3 \quad k = 1$$

New origin is $O'(h, k) = O'(3, 1)$

Put $h = 3$, $k = 1$ in (2)

$$X^2 - Y^2 + 0 - 0 + 9 - 1 - 18 + 2 + 7 = 0$$

$$X^2 - Y^2 - 1 = 0$$

Q.3: In each of the following, find an equation referred to the new axes obtained by rotation of axes about the origin through the given angle.

(i) $xy = 1 \dots (1), \theta = 45^\circ$

Solution:

We know that equations of rotation are

$$x = X \cos\theta - Y \sin\theta \Rightarrow x = X \cos 45^\circ - Y \sin 45^\circ \Rightarrow x = \frac{X}{\sqrt{2}} - \frac{Y}{\sqrt{2}}$$

$$y = X \sin\theta + Y \cos\theta \Rightarrow y = X \sin 45^\circ + Y \cos 45^\circ \Rightarrow y = \frac{X}{\sqrt{2}} + \frac{Y}{\sqrt{2}}$$

Putting these values in (1) we get

$$\begin{aligned} \left(\frac{X-Y}{\sqrt{2}}\right)\left(\frac{X+Y}{\sqrt{2}}\right) &= 1 \\ \frac{X^2 - Y^2}{2} = 1 &\Rightarrow X^2 - Y^2 = 2 \quad \text{Ans.} \end{aligned}$$

(ii) $7x^2 - 8xy + y^2 - 9 = 0$, $\theta = \arctan 2$

Solution:

$$\begin{aligned} 7x^2 - 8xy + y^2 - 9 &= 0 \quad (1) \quad \theta = \tan^{-1} 2 \\ \Rightarrow \tan \theta &= 2 \Rightarrow \cot \theta = \frac{1}{2} \\ \sec \theta &= \sqrt{1 + \tan^2 \theta}, \quad \cosec \theta = \sqrt{1 + \cot^2 \theta} \\ &= \sqrt{1+4}, \quad = \sqrt{1+\frac{1}{4}} \end{aligned}$$

$$\sec \theta = \sqrt{5}, \quad \cosec \theta = \frac{\sqrt{5}}{2}$$

$$\cos \theta = \frac{1}{\sqrt{5}}, \quad \sin \theta = \frac{2}{\sqrt{5}}$$

$$x = X \cos \theta - Y \sin \theta, \quad y = X \sin \theta + Y \cos \theta$$

$$= X \frac{1}{\sqrt{5}} - Y \frac{2}{\sqrt{5}}, \quad y = X \frac{2}{\sqrt{5}} + Y \frac{1}{\sqrt{5}}$$

$$x = \frac{X-2Y}{\sqrt{5}}, \quad y = \frac{2X+Y}{\sqrt{5}}$$

Putting these values in (1)

$$7\left(\frac{X-2Y}{\sqrt{5}}\right)^2 - 8\left(\frac{X-2Y}{\sqrt{5}}\right)\left(\frac{2X+Y}{\sqrt{5}}\right) + \left(\frac{2X+Y}{\sqrt{5}}\right)^2 - 9 = 0$$

$$7\left(\frac{(X^2 + 4Y^2 - 4XY)}{5}\right) - 8\left(\frac{(2X^2 + XY - 4XY - 2Y^2)}{5}\right) + \frac{4X^2 + Y^2 + 4XY}{5} - 9 = 0$$

$$7X^2 - 28XY + 28Y^2 - 16X^2 + 24XY + 16Y^2 + 4X^2 + 4XY + Y^2 - 45 = 0$$

$$-5X^2 + 45Y^2 - 45 = 0$$

$$-5(X^2 - 9Y^2 + 9) = 0$$

$$X^2 - 9Y^2 + 9 = 0$$

$$(iii) \quad 9x^2 + 12xy + 4y^2 - x - y = 0 \quad \theta = \operatorname{arc tan} \frac{2}{3}$$

Solution:

$$9x^2 + 12xy + 4y^2 - x - y = 0 \quad (1)$$

$$\theta = \tan^{-1} \frac{2}{3}$$

$$\tan \theta = \frac{2}{3} \Rightarrow \cot \theta = \frac{3}{2}$$

$$\begin{aligned} \sec \theta &= \sqrt{1 + \tan^2 \theta} \\ &= \sqrt{1 + \frac{4}{9}} = \frac{\sqrt{13}}{3} \end{aligned}$$

$$\sec \theta = \frac{\sqrt{13}}{3} \Rightarrow \cos \theta = \frac{3}{\sqrt{13}}$$

$$\sin \theta = \sqrt{1 - \cos^2 \theta} = \sqrt{1 - \frac{9}{13}} = \sqrt{\frac{13-9}{13}} = \sqrt{\frac{4}{13}} = \frac{2}{\sqrt{13}}$$

$$x = X \cos \theta - Y \sin \theta \Rightarrow x = X \frac{3}{\sqrt{13}} - Y \frac{2}{\sqrt{13}} = \frac{3X - 2Y}{\sqrt{13}}$$

$$y = X \sin \theta + Y \cos \theta \Rightarrow y = X \frac{2}{\sqrt{13}} + Y \frac{3}{\sqrt{13}} \quad y = \frac{2X + 3Y}{\sqrt{13}}$$

Putting the values in (1)

$$\begin{aligned} 9 \left(\frac{3X - 2Y}{\sqrt{13}} \right)^2 + 12 \left(\frac{3X - 2Y}{\sqrt{13}} \right) \left(\frac{2X + 3Y}{\sqrt{13}} \right) + 4 \left(\frac{2X + 3Y}{\sqrt{13}} \right)^2 - \left(\frac{3X - 2Y}{\sqrt{13}} \right) \\ - \left(\frac{2X + 3Y}{\sqrt{13}} \right) = 0 \end{aligned}$$

$$\begin{aligned} 9 \frac{(9X^2 + 4Y^2 - 12XY)}{13} + \frac{12}{13} (6X^2 + 9XY - 4XY - 6Y^2) + \frac{4}{13} (4X^2 + 9Y^2 + \\ 12XY) - \frac{3X - 2Y}{\sqrt{13}} - \frac{2X + 3Y}{\sqrt{13}} = 0 \end{aligned}$$

$$81X^2 + 36Y^2 - 108XY + 72X^2 + 108XY - 48XY - 72Y^2 + 16X^2 + 36Y^2 + 48XY - 3\sqrt{13}X + 2\sqrt{13}Y - 2\sqrt{13}X - 3\sqrt{13}Y = 0$$

$$169X^2 - 5\sqrt{13}X - \sqrt{13}Y = 0$$

Dividing throughout by $\sqrt{13}$

$$13\sqrt{13}X^2 - 5X - Y = 0 \quad \text{Ans}$$

Q.4: Find the measure of angle through which the axes be rotated, so that the product term XY is removed from transformed equation. Also find transformation.

$$2x^2 + 6xy + 10y^2 - 11 = 0 \quad \dots \quad (1)$$

Solution:

$$2x^2 + 6xy + 10y^2 - 11 = 0 \quad \dots \quad (1)$$

The transformed equations for rotated axes are

$$x = X \cos \theta - Y \sin \theta, \quad y = X \sin \theta + Y \cos \theta \quad \text{Put in (1)}$$

$$2(X \cos \theta - Y \sin \theta)^2 + 6(X \cos \theta - Y \sin \theta)(X \sin \theta + Y \cos \theta) + 10(X \sin \theta + Y \cos \theta)^2 - 11 = 0$$

$$\begin{aligned} & 2X^2 \cos^2 \theta - 2Y^2 \sin^2 \theta - 4XY \sin \theta \cos \theta + 6(X^2 \cos^2 \theta \sin \theta + XY \cos^2 \theta \\ & - XY \sin^2 \theta - Y^2 \sin \theta \cos \theta) + 10X^2 \sin^2 \theta + 10Y^2 \cos^2 \theta \\ & + 20XY \sin \theta \cos \theta - 11 = 0 \end{aligned}$$

$$\begin{aligned} & 2X^2 \cos^2 \theta + 10X^2 \sin^2 \theta + 2Y^2 \sin^2 \theta + 10Y^2 \cos^2 \theta + XY(6 \cos^2 \theta \\ & - 6 \sin^2 \theta + 20 \sin \theta \cos \theta - 4 \cos \theta \sin \theta) + 6X^2 \cos \theta \sin \theta - 6Y^2 \sin^2 \theta = 0 \\ & X^2(2\cos^2 \theta + 10\sin^2 \theta + 6 \sin \theta \cos \theta) + XY(6 \cos^2 \theta - 6 \sin^2 \theta + 20 \sin \theta \cos \theta \\ & - 4 \cos \theta \sin \theta) + Y^2(2 \sin^2 \theta - 6 \sin \theta \cos \theta + 10 \cos^2 \theta) - 11 = 0 \quad \dots \quad (2) \end{aligned}$$

To remove XY term, we put

$$6\cos^2 \theta - 6\sin^2 \theta + 20 \sin \theta \cos \theta - 4 \sin \theta \cos \theta = 0$$

$$3\cos^2 \theta - 3\sin^2 \theta + 8 \sin \theta \cos \theta = 0$$

$$3 - 3 \tan^2 \theta - 8 \tan \theta = 0 \quad (\text{Dividing throughout by } \cos^2 \theta)$$

$$3 \tan^2 \theta + 8 \tan \theta - 3 = 0$$

$$\begin{aligned} \tan \theta &= \frac{-8 \pm \sqrt{(8)^2 - 4(3)(-3)}}{2(3)} &= \frac{8 \pm \sqrt{64 + 36}}{6} \\ &= \frac{8 \pm \sqrt{100}}{6} &= \frac{8 \pm 10}{6} \end{aligned}$$

Either

$$\tan \theta = \frac{8 + 10}{6} = \frac{18}{6} \quad \text{or} \quad \tan \theta = \frac{8 - 10}{6} = \frac{-2}{6}$$

$$\tan \theta = 3, \quad \tan \theta = -\frac{1}{3}$$

As θ is taken in 1st quadrant so $\tan \theta = 3$ is only admissible value

$$\sec \theta = \sqrt{1 + \tan^2 \theta} = \sqrt{1 + 9} = \sqrt{10}$$

$$\cos \theta = \frac{1}{\sqrt{10}}$$

$$\begin{aligned}\sin \theta &= \sqrt{1 - \cos^2 \theta} = \sqrt{1 - \frac{1}{10}} = \sqrt{\frac{10-1}{10}} \\ &= \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}\end{aligned}$$

Put in (2)

$$x^2 \left[2 \left(\frac{1}{\sqrt{10}} \right)^2 + 6 \frac{1}{\sqrt{10}} \frac{3}{\sqrt{10}} + 10 \left(\frac{3}{\sqrt{10}} \right)^2 \right] + XY(0) + Y^2$$

$$\left[2 \left(\frac{3}{\sqrt{10}} \right)^2 - 6 \frac{3}{\sqrt{10}} \frac{1}{\sqrt{10}} + 10 \left(\frac{1}{\sqrt{10}} \right)^2 \right] - 11 = 0$$

$$x^2 \left[\frac{2}{10} + \frac{18}{10} + \frac{90}{10} \right] + Y^2 \left[\frac{18}{10} - \frac{18}{10} + \frac{10}{10} \right] - 11 = 0$$

$$x^2 \left[\frac{2+18+90}{10} \right] + Y^2 \left[\frac{18-18+10}{10} \right] - 11 = 0$$

$$11X^2 + Y^2 - 11 = 0 \quad \text{Ans.}$$

(ii) $xy + 4x - 3y - 10 = 0 \quad \dots\dots (1)$

We know that for rotation at axes

$$x = X \cos \theta - Y \sin \theta, \quad y = X \sin \theta + Y \cos \theta \quad \text{Put in (1)}$$

$$(X \cos \theta - Y \sin \theta)(X \sin \theta + Y \cos \theta) + 4(X \cos \theta - Y \sin \theta) - 3(X \sin \theta + Y \cos \theta) - 10 = 0$$

$$\begin{aligned}X^2 \cos \theta \sin \theta + XY \cos^2 \theta - XY \sin^2 \theta - Y^2 \sin \theta \cos \theta + 4X \cos \theta - 4Y \sin \theta \\ - 3X \sin \theta - 3Y \cos \theta - 10 = 0\end{aligned}$$

$$\begin{aligned}X^2 \cos \theta \sin \theta - Y^2 \sin \theta \cos \theta + XY (\cos^2 \theta - \sin^2 \theta) + X(4 \cos \theta - 3 \sin \theta) \\ + Y(-4 \sin \theta - 3 \cos \theta) - 10 = 0 \quad (2)\end{aligned}$$

To remove XY terms we put

$$\cos^2 \theta - \sin^2 \theta = 0 \Rightarrow \cos 2\theta = 0$$

$$\Rightarrow 2\theta = \cos^{-1}(0) = 90^\circ$$

$$\Rightarrow \theta = 45^\circ \quad \text{Put in (2)}$$

$$\begin{aligned}X^2 \cos 45^\circ \sin 45^\circ - Y^2 \sin 45^\circ \cos 45^\circ + XY(0) + X(4 \cos 45^\circ - 3 \sin 45^\circ) + Y \\ (-4 \sin 45^\circ - 3 \cos 45^\circ) - 10 = 0\end{aligned}$$

$$X^2 \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} - Y^2 \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} + X(4 \times \frac{1}{\sqrt{2}} - 3 \times \frac{1}{\sqrt{2}}) + Y(-4 \frac{1}{\sqrt{2}} - 3 \frac{1}{\sqrt{2}}) - 10 = 0$$

$$\frac{1}{2}X^2 - \frac{1}{2}Y^2 + X\left(\frac{1}{\sqrt{2}}\right) + Y\left(\frac{-7}{\sqrt{2}}\right) - 10 = 0$$

$$X^2 - Y^2 + \frac{2}{\sqrt{2}} + X - \frac{14Y}{\sqrt{2}} - 20 = 0$$

$$\begin{aligned} X^2 - Y^2 + \sqrt{2}X - 7\sqrt{2} - 20 &= 0 && \text{Ans.} \\ (\text{iii}) \quad 5x^2 - 6xy + 5y^2 - 8 &= 0 \end{aligned}$$

Solution:

$$5x^2 - 6xy + 5y^2 - 8 = 0 \quad (1)$$

We know that for rotation of axes

$$x = X \cos \theta - Y \sin \theta, \quad y = X \sin \theta + Y \cos \theta \quad \text{Put in (1)}$$

$$5(X \cos \theta - Y \sin \theta)^2 - 6(X \cos \theta - Y \sin \theta)(X \sin \theta + Y \cos \theta) + 5(X \sin \theta + Y \cos \theta)^2 - 8 = 0$$

$$5(X^2 \cos^2 \theta + Y^2 \sin^2 \theta - 2XY \sin \theta \cos \theta) - 6(X^2 \cos \theta \sin \theta + XY \cos^2 \theta - XY \sin^2 \theta - Y^2 \sin \theta \cos \theta) + 5(X^2 \sin^2 \theta + Y^2 \cos^2 \theta + 2XY \sin \theta \cos \theta) - 8 = 0$$

$$5X^2 \cos^2 \theta + 5Y^2 \sin^2 \theta - 10XY \sin \theta \cos \theta - 6X^2 \cos \theta \sin \theta - 6XY \cos^2 \theta + 6XY \sin^2 \theta + 6Y^2 \sin \theta \cos \theta + 5X^2 \sin^2 \theta + 5Y^2 \cos^2 \theta + 10XY \sin \theta \cos \theta - 8 = 0$$

$$X^2(5\cos^2 \theta - 6 \cos \theta \sin \theta + 5\sin^2 \theta) + XY(-10 \cos \theta \sin \theta - 6\cos^2 \theta + 6\sin^2 \theta + 10 \sin \theta \cos \theta) + Y^2(5 \sin^2 \theta + 6 \sin \theta \cos \theta + 5 \cos^2 \theta) - 8 = 0 \quad \dots\dots (2)$$

To remove XY terms put (2)

$$-6 \cos^2 \theta + 6 \sin^2 \theta = 0$$

$$\cos^2 \theta = \sin^2 \theta \Rightarrow \tan^2 \theta = 1$$

$$\tan \theta = 1 \quad (\theta \text{ is taken in } 1^{\text{st}} \text{ Quadrant})$$

$$\theta = 45^\circ \quad \text{Put in (2)}$$

$$X^2(5(\cos 45^\circ)^2 - 6\cos 45^\circ \sin 45^\circ + 5(\sin 45^\circ)^2) + 0 + Y^2(5(\sin 45^\circ)^2 + 6 \sin 45^\circ \cos 45^\circ + 5(\cos 45^\circ)^2) - 8 = 0$$

$$X^2 \left(5 + \left(\frac{1}{\sqrt{2}} \right)^2 - 6 \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} + 5 \left(\frac{1}{\sqrt{2}} \right)^2 \right) + Y^2 \left(5 + \left(\frac{1}{\sqrt{2}} \right)^2 + 6 \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} + 5 \left(\frac{1}{\sqrt{2}} \right)^2 \right)$$

$$X^2 \left(\frac{5}{2} - \frac{6}{2} + \frac{5}{2} \right) + Y^2 \left(\frac{5}{2} + \frac{6}{2} + \frac{5}{2} \right) - 8 = 0$$

$$2X^2 + 8Y^2 - 8 = 0$$

$$\Rightarrow X^2 + 4Y^2 - 4 = 0 \quad \text{Ans}$$

EXERCISE 6 . 9

Q.1: By rotation of axes, eliminates the xy -term in each of the following equations. Identify the conic & find its elements.

$$(\text{i}) \quad 4x^2 - 4xy + y^2 - 6 = 0$$

Solution:

$$4x^2 - 4xy + y^2 - 6 = 0 \quad (1)$$

$$\text{Here } a = 4 \quad b = 1 \quad 2h = -4$$