

## 14 Solutions of Trigonometric Equations

### Trigonometric Equations

The equations involving atleast one trigonometric function, are called Trigonometric equations. For example  $\sin x = \frac{1}{4}$ ,  $\sec x = \tan x$ ;  $\sin^2 x - \sec x + 1 = 0$  etc.

### EXERCISE 14

① Find the solutions of the following equation which lie in  $[0, 2\pi]$

(i)  $\sin x = -\frac{\sqrt{3}}{2}$

Sol:- Given that  $\sin x = -\frac{\sqrt{3}}{2}$

$\because \sin x$  is -ve in III and IV quad. with reference angle  $= \frac{\pi}{3}$

$$\therefore x = \pi + \frac{\pi}{3} \text{ and } x = 2\pi - \frac{\pi}{3} \quad x \in [0, 2\pi]$$

$$\Rightarrow x = \frac{4\pi}{3}, \quad x = \frac{5\pi}{3}$$

$$\therefore B.S. = \left\{ \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$$

(ii)  $\csc x = 2$

Sol:- Given that  $\csc x = 2$

$$\Rightarrow \sin x = \frac{1}{\csc x} = \frac{1}{2}$$

$\because \sin x$  is +ve in I and II quad. with reference angle  $= \frac{\pi}{6}$

$$\therefore \theta = \frac{\pi}{6} \text{ and } \theta = \pi - \frac{\pi}{6} \quad \theta \in [0, 2\pi]$$

$$\Rightarrow \theta = \frac{\pi}{6} \text{ and } \theta = \frac{5\pi}{6}$$

$$\therefore B.S. = \left\{ \frac{\pi}{6}, \frac{5\pi}{6} \right\}$$

(iii)  $\sec x = -2$

Sol:- Given that  $\sec x = -2$

$$\Rightarrow \cos x = \frac{1}{\sec x} = \frac{1}{-2} = -\frac{1}{2}$$

$\because \cos x$  is -ve in II and III quad. with reference angle  $= \frac{\pi}{3}$

$$\therefore x = \pi - \frac{\pi}{3} \text{ and } x = \pi + \frac{\pi}{3} \quad x \in [0, 2\pi]$$

$$\Rightarrow x = \frac{2\pi}{3} \text{ and } x = \frac{4\pi}{3}$$

$$\therefore B.S. = \left\{ \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$$

(iv)  $\cot x = \frac{1}{\sqrt{3}}$

Sol:-  $\cot x = \frac{1}{\sqrt{3}} \Rightarrow \tan x = \frac{1}{\cot x} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$

$$\Rightarrow \tan x = \sqrt{3}$$

$\tan x$  is +ve in I and III quad. with reference angle  $= \frac{\pi}{3}$

$$\therefore \theta = \frac{\pi}{3} \text{ and } \theta = \pi + \frac{\pi}{3}, \quad \theta \in [0, 2\pi]$$

$$\therefore B.S. = \left\{ \frac{\pi}{3}, \frac{4\pi}{3} \right\}$$

② Solve the following trigonometric equations.

(i)  $\tan^2 \theta = \frac{1}{3}$

Sol:- Given that  $\tan^2 \theta = \frac{1}{3}$

$$\Rightarrow \tan \theta = \pm \sqrt{\frac{1}{3}}$$

$$\Rightarrow \tan \theta = \pm \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}}$$

$$\therefore \tan \theta \text{ is +ve in I and III quad. with reference angle } = \frac{\pi}{6}$$

$$\therefore \theta = \frac{\pi}{6} \text{ and } \theta = \pi + \frac{\pi}{6}$$

$$\Rightarrow \theta = \frac{\pi}{6} + \theta = \frac{7\pi}{6}$$

$$\text{where } \theta \in [0, 2\pi]$$

$$\therefore \theta \text{ is the period of } \tan \theta$$

$$\therefore \text{general value of } \theta \text{ are } \frac{\pi}{6} + n\pi$$

$$\theta \text{ are } \frac{\pi}{6} + n\pi$$

$$\therefore B.S. = \left\{ \frac{\pi}{6} + n\pi \right\} \cup \left\{ \frac{7\pi}{6} + n\pi \right\} \text{ where } n \in \mathbb{Z}$$

$$\therefore \text{general value of } \theta \text{ are } \frac{\pi}{6} + n\pi$$

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$$\therefore \text{general value of } \theta \text{ are } \frac{\pi}{6} + n\pi$$

$$\text{and } \frac{2\pi}{3} + 2n\pi, \quad n \in \mathbb{Z}$$

$$\text{odd } \frac{4\pi}{3} + 2n\pi \text{ and } \Rightarrow \tan \theta = -\frac{1}{\sqrt{3}} (\text{ve})$$

$$\therefore \tan \theta \text{ is -ve in II and IV}$$

$$\therefore S.S. = \left\{ \frac{2\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{2\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{4\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{3} + 2n\pi \right\}, \quad n \in \mathbb{Z}$$

$$\text{quad. with reference angle } = \frac{\pi}{6}$$

$$\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \text{ & } \theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

$$\text{where } \theta \in [0, 2\pi]$$

$$\therefore \pi \text{ is the period of } \tan \theta$$

$$\therefore \text{general values of } \theta \text{ are } \frac{5\pi}{6} + n\pi, \quad n \in \mathbb{Z}$$

$$\therefore S.S. = \left\{ \frac{5\pi}{6} + n\pi \right\}, \quad n \in \mathbb{Z}$$

$$(iii) \text{ Given that } \sec^2 \theta = \frac{4}{3}$$

$$\Rightarrow \sec \theta = \pm \frac{2}{\sqrt{3}}$$

$$\Rightarrow \sec \theta = \frac{2}{\sqrt{3}} \quad \sec \theta = -\frac{2}{\sqrt{3}}$$

$$\therefore \cos \theta = \frac{1}{\sec \theta} = \frac{1}{2} = \frac{\sqrt{3}}{2}$$

$$\therefore \cos \theta \text{ is +ve in I & IV quad. with ref. angle } = \frac{\pi}{6}$$

$$\therefore \theta = \frac{\pi}{6} \text{ & } \theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

$$\text{where } \theta \in [0, 2\pi]$$

$$\therefore 2\pi \text{ is the period of } \cos \theta$$

$$\therefore \text{general values of } \theta \text{ are } \frac{\pi}{6} + 2n\pi$$

$$\text{and } \frac{11\pi}{6} + 2n\pi, \quad n \in \mathbb{Z}$$

$$\therefore S.S. = \left\{ \frac{\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{11\pi}{6} + 2n\pi \right\}$$

$$\cup \left\{ \frac{5\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{7\pi}{6} + 2n\pi \right\}, \quad n \in \mathbb{Z}$$

$$(iv) \text{ Given that } \cot^2 \theta = \frac{1}{3}$$

$$\Rightarrow \cot \theta = \pm \frac{1}{\sqrt{3}}$$

$$\Rightarrow \cot \theta = \frac{1}{\sqrt{3}}$$

$$\therefore \tan \theta = \frac{1}{\cot \theta} = \sqrt{3}$$

$$\therefore \tan \theta \text{ is +ve in I & III quad.}$$

$$\text{with ref. angle } = \frac{\pi}{3}$$

$$\therefore \theta = \frac{\pi}{3} \text{ & } \theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$\text{where } \theta \in [0, 2\pi]$$

$$\therefore \pi \text{ is the period of } \tan \theta$$

$$\therefore \text{general values of } \theta \text{ are } \frac{\pi}{3} + n\pi$$

$$\text{and } \frac{4\pi}{3} + n\pi, \quad n \in \mathbb{Z}$$

$$\therefore S.S. = \left\{ \frac{\pi}{3} + n\pi \right\} \cup \left\{ \frac{4\pi}{3} + n\pi \right\}$$

$$(5) 3\tan^2 \theta + 2\sqrt{3}\tan \theta + 1 = 0$$

$$\Rightarrow (\sqrt{3}\tan \theta + 1)^2 = 0$$

$$\Rightarrow \sqrt{3}\tan \theta + 1 = 0$$

$$\Rightarrow \sqrt{3}\tan \theta = -1$$

$$\text{general value of } \tan \theta$$

$$\text{odd } \frac{4\pi}{3} + 2n\pi \text{ and } \tan \theta = -\frac{1}{\sqrt{3}} (\text{ve})$$

$$\therefore \tan \theta \text{ is -ve in II and IV}$$

$$\text{quad. with reference angle } = \frac{\pi}{6}$$

$$\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \text{ & } \theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

$$\text{where } \theta \in [0, 2\pi]$$

$$\therefore \pi \text{ is the period of } \tan \theta$$

$$\therefore \text{general values of } \theta \text{ are } \frac{5\pi}{6} + n\pi, \quad n \in \mathbb{Z}$$

$$\therefore S.S. = \left\{ \frac{5\pi}{6} + n\pi \right\}, \quad n \in \mathbb{Z}$$

$$(6) \tan^2 \theta - \sec \theta - 1 = 0$$

$$\Rightarrow \sec \theta - 1 - \sec \theta - 1 = 0$$

$$\Rightarrow \sec^2 \theta - \sec \theta - 2 = 0$$

$$\Rightarrow \sec^2 \theta - 2\sec \theta + \sec \theta - 2 = 0$$

$$\Rightarrow \sec(\sec \theta - 2) + 1(\sec \theta - 2) = 0$$

$$\Rightarrow (\sec \theta - 2)(\sec \theta + 1) = 0$$

$$\Rightarrow \sec \theta - 2 = 0$$

$$\Rightarrow \sec \theta + 1 = 0$$

$$\Rightarrow \sec \theta = -1$$

$$\Rightarrow \cos \theta = \frac{1}{\sec \theta} = -1$$

$$\therefore \cos \theta \text{ is -ve in II & III quad. with ref. angle } = 0$$

$$\therefore \theta = \pi - 0 \text{ & } \theta = \pi + 0$$

$$\Rightarrow \theta = \pi, \quad \theta \in [0, 2\pi]$$

$$\therefore 2\pi \text{ is the period of } \cos \theta$$

$$\therefore \text{general value of } \theta \text{ is } \pi + 2n\pi$$

$$\text{where } n \in \mathbb{Z}$$

$$(7) \sec^2 \theta - \sec \theta - 2 = 0$$

$$\Rightarrow \sec \theta - 1 - \sec \theta - 2 = 0$$

$$\Rightarrow \sec^2 \theta - 2\sec \theta + \sec \theta - 2 = 0$$

$$\Rightarrow \sec(\sec \theta - 2) + 1(\sec \theta - 2) = 0$$

$$\Rightarrow (\sec \theta - 2)(\sec \theta + 1) = 0$$

$$\Rightarrow \sec \theta - 2 = 0$$

$$\Rightarrow \sec \theta + 1 = 0$$

$$\Rightarrow \sec \theta = -1$$

$$\therefore \cos \theta = \frac{1}{\sec \theta} = -1$$

$$\therefore \cos \theta \text{ is -ve in II & III quad. with ref. angle } = 0$$

$$\therefore \theta = \pi - 0 \text{ & } \theta = \pi + 0$$

$$\Rightarrow \theta = \pi, \quad \theta \in [0, 2\pi]$$

$$\therefore 2\pi \text{ is the period of } \cos \theta$$

$$\therefore \text{general value of } \theta \text{ is } \pi + 2n\pi$$

$$\text{where } n \in \mathbb{Z}$$

$$(8) \tan^2 \theta + \tan \theta - 2 = 0$$

$$\Rightarrow \tan \theta - 1 - \tan \theta - 2 = 0$$

$$\Rightarrow \tan^2 \theta - 2\tan \theta + \tan \theta - 2 = 0$$

$$\Rightarrow \tan(\tan \theta - 2) + 1(\tan \theta - 2) = 0$$

$$\Rightarrow (\tan \theta - 2)(\tan \theta + 1) = 0$$

$$\Rightarrow \tan \theta - 2 = 0$$

$$\Rightarrow \tan \theta + 1 = 0$$

$$\Rightarrow \tan \theta = -1$$

$$\text{which is impossible}$$

$$\therefore \tan \theta \in [-1, 1]$$

$$\therefore S.S. = \left\{ n\pi \right\}, \quad n \in \mathbb{Z}$$

$$(9) 2\sin^2 \theta + \cos^2 \theta - 1 = 0$$

$$\Rightarrow 2\sin^2 \theta + 1 - \sin^2 \theta - 1 = 0$$

$$\Rightarrow 2\sin^2 \theta - \sin^2 \theta = 0$$

$$\Rightarrow \sin^2 \theta (2 - \sin^2 \theta) = 0$$

$$\Rightarrow \sin \theta = 0$$

$$\Rightarrow 2 - \sin^2 \theta = 0$$

$$\Rightarrow \sin^2 \theta = 2$$

$$\Rightarrow \sin \theta = \pm \sqrt{2}$$

$$\text{which is impossible}$$

$$\therefore \sin \theta \in [-1, 1]$$

$$(10) 2\sin^2 \theta - \sin \theta = 0$$

$$\Rightarrow \sin \theta (2\sin \theta - 1) = 0$$

$$\Rightarrow \sin \theta = 0$$

$$\Rightarrow 2\sin \theta - 1 = 0$$

$$\Rightarrow 2\sin \theta = 1$$

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$$\Rightarrow \sin \theta = \frac{1}{2}$$

$\because \sin \theta$  is +ve in I & II quad. with ref. angle  $= \frac{\pi}{6}$

$$\therefore \theta = \frac{\pi}{6} + \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

where  $\theta \in [0, 2\pi]$

$\because 2\pi$  is the period of  $\sin \theta$

$\therefore$  general values of  $\theta$  are

$$\frac{\pi}{6} + 2n\pi \text{ and } \frac{5\pi}{6} + 2n\pi, n \in \mathbb{Z}$$

$$\therefore S.S. = \{n\pi\} \cup \left\{ \frac{\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{6} + 2n\pi \right\}, n \in \mathbb{Z}$$

$$⑦ 3\cos^2 \theta - 2\sqrt{3} \cos \theta \sin \theta - 3 \sin^2 \theta = 0$$

Dividing by  $\sin^2 \theta$ , we get

$$3\tan^2 \theta - 2\sqrt{3} \cot \theta - 3 = 0$$

Using quadratic formula,

$$\cot \theta = \frac{-(-2\sqrt{3}) \pm \sqrt{(-2\sqrt{3})^2 - 4(3)(-3)}}{2(3)}$$

$$= \frac{2\sqrt{3} \pm \sqrt{12+36}}{6} = \frac{2\sqrt{3} \pm \sqrt{48}}{6}$$

$$= \frac{2\sqrt{3} \pm 4\sqrt{3}}{6} = \frac{2\sqrt{3}+4\sqrt{3}}{6}, \frac{2\sqrt{3}-4\sqrt{3}}{6}$$

$$= \frac{6\sqrt{3}}{6}, \frac{-2\sqrt{3}}{6} = \sqrt{3}, -\frac{\sqrt{3}}{3}$$

$$\cot \theta = \sqrt{3}, -\frac{1}{\sqrt{3}}$$

$$\Rightarrow \cot \theta = \sqrt{3}$$

$$\Rightarrow \tan \theta = \frac{1}{\cot \theta} = \frac{1}{\sqrt{3}}$$

$\because \tan \theta$  is +ve in I & III quad. with ref. angle  $= \frac{\pi}{6}$

$$\therefore \theta = \frac{\pi}{6} + \theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

where  $\theta \in [0, 2\pi]$

$\because \pi$  is the period of  $\tan \theta$

$\therefore$  general values of  $\theta$  are  $\frac{\pi}{6} + n\pi$

$$\theta = \frac{\pi}{6} + n\pi, n \in \mathbb{Z}$$

$$\therefore S.S. = \left\{ \frac{\pi}{6} + n\pi \right\} \cup \left\{ \frac{7\pi}{6} + n\pi \right\}$$

where  $n \in \mathbb{Z}$ .

$$⑧ 4\sin^2 \theta - 8\cos \theta + 1 = 0$$

$$\Rightarrow 4(1 - \cos^2 \theta) - 8\cos \theta + 1 = 0$$

$$\Rightarrow 4 - 4\cos^2 \theta - 8\cos \theta + 1 = 0$$

$$\Rightarrow -4\cos^2 \theta - 8\cos \theta + 5 = 0$$

$$\Rightarrow -1(4\cos^2 \theta + 8\cos \theta - 5) = 0$$

$$\Rightarrow 4\cos^2 \theta + 8\cos \theta - 5 = 0 \quad \checkmark -1 \neq 0$$

$$\Rightarrow 4\cos^2 \theta + 10\cos \theta - 2\cos \theta - 5 = 0$$

$$\Rightarrow 2\cos \theta (2\cos \theta + 5) - 1(2\cos \theta + 5) = 0$$

$$\Rightarrow (2\cos \theta + 5)(2\cos \theta - 1) = 0$$

$$\Rightarrow 2\cos \theta + 5 = 0$$

$$\Rightarrow 2\cos \theta = -5$$

$$\Rightarrow \cos \theta = -\frac{5}{2}$$

$$\Rightarrow \cos \theta = -2.5$$

$$\text{which is impossible}$$

$$\therefore \cos \theta \in [-1, 1]$$

$$\Rightarrow 2\cos \theta - 1 = 0$$

$$\Rightarrow 2\cos \theta = 1$$

$$\Rightarrow \cos \theta = \frac{1}{2}$$

$$\therefore \cos \theta$$
 is +ve so I &

$$\text{II quad. with ref. angle} = \frac{\pi}{3}$$

$$\therefore \theta = \frac{\pi}{3} \text{ and } \theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

$$\text{where } \theta \in [0, 2\pi]$$

$$\therefore 2\pi$$
 is the period of  $\cos \theta$

$$\therefore$$
 general values of  $\theta$  are  $\frac{\pi}{3} + 2n\pi$  and

$$\frac{5\pi}{3} + 2n\pi, n \in \mathbb{Z}$$

$$\therefore S.S. = \left\{ \frac{\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{3} + 2n\pi \right\}, n \in \mathbb{Z}$$

$$⑨ \sqrt{3} \tan x - \sec x - 1 = 0 \quad \text{--- (1)}$$

$$\Rightarrow \sqrt{3} \tan x = \sec x + 1$$

Dividing both sides

$$\Rightarrow (\sqrt{3} \tan x)^2 = (\sec x + 1)^2$$

$$\Rightarrow 3 \tan^2 x = \sec^2 x + 2 \sec x + 1$$

$$\Rightarrow 3 \tan^2 x - \sec^2 x - 2 \sec x - 1 = 0$$

$$\Rightarrow 3(\sec^2 x - 1) - 3\sec^2 x - 2 \sec x - 1 = 0$$

$$\Rightarrow 3\sec^2 x - 3 - 3\sec^2 x - 2 \sec x - 1 = 0$$

$$\Rightarrow -2 \sec^2 x - 2 \sec x - 4 = 0$$

$$\Rightarrow 2(\sec^2 x - \sec x - 2) = 0$$

$$\Rightarrow \sec^2 x - \sec x - 2 = 0 \quad \checkmark 2 \neq 0$$

$$\Rightarrow \sec^2 x - 2 \sec x + 8 \sec x - 2 = 0$$

$$\Rightarrow 8 \sec x(\sec x - 2) + 1(8 \sec x - 2) = 0$$

$$\Rightarrow (8 \sec x - 2)(8 \sec x + 1) = 0$$

$$\Rightarrow 8 \sec x - 2 = 0 \quad \sec x + 1 = 0$$

$$\Rightarrow 8 \sec x = 2 \quad \Rightarrow \sec x = -1$$

$$\Rightarrow \sec x = \frac{1}{4} \quad \Rightarrow \sec x = \frac{1}{8}$$

$$\Rightarrow \sec x = -1 \quad \Rightarrow \sec x = -\frac{1}{8}$$

$$\therefore \sec x$$
 is -ve in I & IV quad. with

$$\text{ref. angle} = \frac{\pi}{3}$$

$$\therefore \sec x = -1 \quad \text{ref. angle} = 0$$

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$$\begin{aligned} & x = \frac{\pi}{3} \text{ & } x = 2\pi - \frac{\pi}{3} \\ \Rightarrow & x = \frac{\pi}{3} \text{ & } x = \frac{5\pi}{3} \quad \left| \begin{array}{l} x = \pi - \alpha \Rightarrow x = \pi + \alpha \\ \text{where } x \in [0, 2\pi] \end{array} \right. \\ & \Rightarrow x = \pi \text{ where } x \in [0, 2\pi] \end{aligned}$$

Putting  $x = \frac{\pi}{3}$  in ①, we get

$$\begin{aligned} \sqrt{3} \tan \frac{\pi}{3} - \sec \frac{\pi}{3} - 1 &= 0 \\ \Rightarrow \sqrt{3} \cdot \sqrt{3} - 2 - 1 &= 0 \\ \Rightarrow 3 - 2 - 1 &= 0 \\ \Rightarrow 3 - 3 &= 0 \\ \Rightarrow 0 &= 0 \quad (\text{satisfied}) \end{aligned}$$

$\therefore x = \frac{\pi}{3}$  is a solution of ①

$\because 2\pi$  is the period of  $\cos x$ .

$\therefore$  general values of  $x$  are

$$\frac{\pi}{3} + 2n\pi, n \in \mathbb{Z}$$

Putting  $x = \frac{5\pi}{3}$  in ①, we get

$$\begin{aligned} \sqrt{3} \tan \frac{5\pi}{3} - \sec \frac{5\pi}{3} - 1 &= 0 \\ \Rightarrow \sqrt{3}(-\sqrt{3}) - 2 - 1 &= 0 \\ \Rightarrow -3 - 2 - 1 &= 0 \\ \Rightarrow -6 &= 0 \quad (\text{not satisfied}) \end{aligned}$$

$\therefore x = \frac{5\pi}{3}$  is not a solution of ①

Putting  $x = \pi$  in ①, we get

$$\begin{aligned} \sqrt{3} \tan \pi - \sec \pi - 1 &= 0 \\ \Rightarrow \sqrt{3}(0) - (-1) - 1 &= 0 \\ \Rightarrow 0 + 1 - 1 &= 0 \\ \Rightarrow 0 &= 0 \quad (\text{satisfied}) \end{aligned}$$

$\therefore x = \pi$  is a solution of ①

$\because 2\pi$  is the period of  $\cos x$ .

$\therefore$  general values of  $x$  are  $\pi + 2n\pi, n \in \mathbb{Z}$

Thus S.S. =  $\left\{ \frac{\pi}{3} + 2n\pi \right\} \cup \left\{ \pi + 2n\pi \right\}, n \in \mathbb{Z}$

⑥  $\cos 2x = \sin 3x$

$$\begin{aligned} \Rightarrow \cos^2 x - \sin^2 x &= 3 \sin x - 4 \sin^3 x \\ \Rightarrow \cos^2 x - \sin^2 x - 3 \sin x + 4 \sin^3 x &= 0 \\ \Rightarrow 1 - \sin^2 x - \sin^2 x - 3 \sin x + 4 \sin^3 x &= 0 \\ \Rightarrow 4 \sin^3 x - 2 \sin^2 x - 3 \sin x + 1 &= 0 \quad \text{--- ①} \end{aligned}$$

$\therefore \sin x = 1$  satisfies eq. ①

$\therefore \sin x = 1$  is a root of ①

To find other roots

Using Synthetic Division

$$\begin{array}{r} | 1 & 4 & -2 & -3 & 1 \\ \downarrow & 4 & 2 & -1 & \boxed{2} \end{array}$$

The depressed equation is

$$4 \sin^2 x + 2 \sin x - 1 = 0$$

using quadratic formula

$$\sin x = \frac{-2 \pm \sqrt{(2)^2 - 4(4)(-1)}}{2(4)}$$

$$\begin{aligned} \Rightarrow \sin x &= \frac{-2 \pm \sqrt{4+16}}{8} = \frac{-2 \pm \sqrt{20}}{8} \\ \Rightarrow \sin x &= \frac{-2 \pm 2\sqrt{5}}{8} = \frac{[-1 \pm \sqrt{5}]}{4} \\ \Rightarrow \sin x &= \frac{-1 \pm \sqrt{5}}{4} \\ \Rightarrow \sin x &= \frac{-1 + \sqrt{5}}{4} \\ \Rightarrow \sin x &= \frac{-1 + \sqrt{5}}{4} \quad \& \sin x = \frac{-1 - \sqrt{5}}{4} \\ \Rightarrow \sin x &= 0.3090 \quad \& \sin x = -0.8090 \\ \therefore \text{roots of ① are} \\ \sin x = 1, \sin x = 0.3090, \sin x = -0.8090 \\ \text{Now } \sin x = 1 \\ \because \sin x \text{ is } +ve \text{ in I \& II quad.} \\ \text{with ref. angle } = \pi/2 \\ \therefore x = \frac{\pi}{2}, \pi - \frac{\pi}{2} = \frac{\pi}{2}, \frac{3\pi}{2} \\ \Rightarrow x = \frac{\pi}{2} \quad \text{where } x \in [0, 2\pi] \\ \because 2\pi \text{ is the period of } \sin x \\ \therefore \text{general values of } x \text{ are} \\ \frac{\pi}{2} + 2n\pi, n \in \mathbb{Z} \\ \text{For } \sin x = 0.3090 \\ \because \sin x \text{ is } +ve \text{ in I \& II quad.} \\ \text{with ref. angle} = \sin^{-1}(0.3090) \\ = 18^\circ = 18 \times \frac{\pi}{180} \text{ rad.} \\ = \frac{\pi}{10} \text{ radians} \\ \therefore x = \frac{\pi}{10} \quad \text{and } x = \pi - \frac{\pi}{10} = \frac{9\pi}{10} \text{ where} \\ x \in [0, 2\pi] \\ \because 2\pi \text{ is the period of } \sin x \\ \therefore \text{general values of } x \text{ are} \\ \frac{\pi}{10} + 2n\pi \text{ and } \frac{9\pi}{10} + 2n\pi, n \in \mathbb{Z} \end{aligned}$$

For  $\sin x = -0.8090$

$$\begin{aligned} \because \sin x \text{ is } -ve \text{ in III and IV quad.} \\ \text{with ref. angle} &= \sin^{-1}(-0.8090) \\ &= 144^\circ = 144 \times \frac{\pi}{180} \text{ rad.} \\ &= \frac{3\pi}{5} \text{ radians} \end{aligned}$$

$$\therefore x = \pi + \frac{3\pi}{10} = \frac{13\pi}{10} \quad \& x = 2\pi - \frac{3\pi}{10} = \frac{17\pi}{10}$$

where  $x \in [0, 2\pi]$

$\therefore 2\pi$  is the period of  $\sin x$

$\therefore$  general values of  $x$  are

$$\begin{aligned} \frac{13\pi}{10} + 2n\pi \text{ and } \frac{17\pi}{10} + 2n\pi, n \in \mathbb{Z} \\ \therefore \text{S.S.} = \left\{ \frac{\pi}{2} + 2n\pi \right\} \cup \left\{ \frac{\pi}{10} + 2n\pi \right\} \cup \left\{ \frac{9\pi}{10} + 2n\pi \right\} \\ \cup \left\{ \frac{13\pi}{10} + 2n\pi \right\} \cup \left\{ \frac{17\pi}{10} + 2n\pi \right\}, n \in \mathbb{Z} \end{aligned}$$



$$\begin{aligned}
 & (15) \sin x + \cos 3x = \cos 5x \\
 & \Rightarrow \sin x = \cos 5x - \cos 3x \\
 & \Rightarrow \sin x = -2 \sin\left(\frac{5x+3x}{2}\right) \cos\left(\frac{5x-3x}{2}\right) \\
 & \Rightarrow \sin x = -2 \sin 4x \cos x \\
 & \Rightarrow \sin x + 2 \sin 4x \cos x = 0 \\
 & \Rightarrow \sin x [1 + 2 \sin 4x] = 0 \\
 & \Rightarrow \sin x = 0 \quad | 1 + 2 \sin 4x = 0 \\
 & \Rightarrow x = 0 \text{ or } x = \pi \\
 & \text{where } x \in [0, 2\pi] \Rightarrow \sin 4x = -\frac{1}{2} \\
 & \because 2\pi \text{ is the period of } \sin 4x \text{ it is } -ve \text{ in } II \text{ quad. with ref. angle } \frac{\pi}{3} \\
 & \therefore x = 0 + 2n\pi = 2n\pi \\
 & 2x = \pi + 2n\pi \\
 & n \in \mathbb{Z} \\
 & \therefore 4x = \pi + \frac{\pi}{6} = \frac{7\pi}{6} \\
 & 4x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6} \\
 & \because 2\pi \text{ is the period of } \sin 4x \\
 & \therefore 4x = \frac{7\pi}{6} + 2n\pi \\
 & 4x = \frac{11\pi}{6} + 2n\pi \\
 & \Rightarrow x = \frac{7\pi}{24} + \frac{n\pi}{2} \\
 & x = \frac{11\pi}{24} + \frac{n\pi}{2}, n \in \mathbb{Z}
 \end{aligned}$$

$$\therefore S.S. = \{2n\pi\} \cup \{\pi + 2n\pi\} \cup \left\{ \frac{7\pi}{24} + \frac{n\pi}{2} \right\} \\
 \cup \left\{ \frac{11\pi}{24} + \frac{n\pi}{2} \right\}, n \in \mathbb{Z}.$$

$$\text{or } S.S. = \{n\pi\} \cup \left\{ \frac{7\pi}{24} + \frac{n\pi}{2} \right\} \cup \left\{ \frac{11\pi}{24} + \frac{n\pi}{2} \right\}, n \in \mathbb{Z}$$

$$\begin{aligned}
 & (16) \sin 3x + \sin 2x + \sin x = 0 \\
 & \Rightarrow (\sin 3x + \sin x) + \sin 2x = 0 \\
 & \Rightarrow 2 \sin\left(\frac{3x+x}{2}\right) \cos\left(\frac{3x-x}{2}\right) + \sin 2x = 0 \\
 & \Rightarrow 2 \sin 2x \cos x + \sin 2x = 0 \\
 & \Rightarrow \sin 2x (2 \cos x + 1) = 0 \\
 & \Rightarrow \sin 2x = 0 \\
 & \Rightarrow 2x = 0 \text{ or } 2x = \pi \\
 & \because 2\pi \text{ is the period of } \sin 2x \\
 & \therefore 2x = 0 + 2n\pi \\
 & 2x = \pi + 2n\pi \\
 & \Rightarrow x = n\pi \\
 & x = \frac{\pi}{2} + n\pi \\
 & n \in \mathbb{Z}
 \end{aligned}$$

$$\therefore S.S. = \{n\pi\} \cup \left\{ \frac{\pi}{2} + n\pi \right\} \cup \left\{ \frac{2\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{4\pi}{3} + 2n\pi \right\}, n \in \mathbb{Z}$$

$$\begin{aligned}
 & (17) \sin 7x - \sin x = \sin 3x \\
 & \Rightarrow 2 \cos\left(\frac{7x+x}{2}\right) \sin\left(\frac{7x-x}{2}\right) = \sin 3x \\
 & \Rightarrow 2 \cos 4x \sin 3x = \sin 3x \\
 & \Rightarrow 2 \cos 4x \sin 3x - \sin 3x = 0 \\
 & \Rightarrow \sin 3x (2 \cos 4x - 1) = 0 \\
 & \Rightarrow \sin 3x = 0 \quad | 2 \cos 4x - 1 = 0 \\
 & \Rightarrow 3x = 0 \text{ or } \\
 & \quad 3x = \pi \\
 & \quad \because 2\pi \text{ is the period of } \sin 3x \\
 & \quad \therefore 3x = 0 + 2n\pi \\
 & \quad 3x = \pi + 2n\pi \\
 & \quad \therefore 3x = \pi + 2n\pi \\
 & \quad \therefore 4x = \frac{\pi}{3} + 2n\pi \\
 & \quad 4x = \frac{5\pi}{3} + 2n\pi \\
 & \quad \Rightarrow x = \frac{\pi}{12} + \frac{n\pi}{3} \\
 & \quad x = \frac{5\pi}{12} + \frac{n\pi}{2}, n \in \mathbb{Z} \\
 & \therefore S.S. = \left\{ \frac{2n\pi}{3} \right\} \cup \left\{ \frac{\pi}{3} + \frac{2n\pi}{3} \right\} \cup \left\{ \frac{\pi}{12} + \frac{n\pi}{3} \right\} \\
 & \quad \cup \left\{ \frac{5\pi}{12} + \frac{n\pi}{2} \right\}, n \in \mathbb{Z}
 \end{aligned}$$

$$\begin{aligned}
 & (18) \sin x + \sin 3x + \sin 5x = 0 \\
 & \Rightarrow (\sin 5x + \sin x) + \sin 3x = 0 \\
 & \Rightarrow 2 \sin\left(\frac{5x+x}{2}\right) \cos\left(\frac{5x-x}{2}\right) + \sin 3x = 0 \\
 & \Rightarrow 2 \sin 3x \cos 2x + \sin 3x = 0 \\
 & \Rightarrow \sin 3x (2 \cos 2x + 1) = 0 \\
 & \Rightarrow \sin 3x = 0 \quad | 2 \cos 2x + 1 = 0 \\
 & \Rightarrow 3x = 0 \text{ or} \\
 & \quad 3x = \pi \\
 & \quad \because 2\pi \text{ is the period of } \sin 3x \\
 & \quad \therefore 3x = 0 + 2n\pi \\
 & \quad 3x = \pi + 2n\pi \\
 & \quad \Rightarrow x = \frac{2n\pi}{3} \\
 & \quad x = \frac{\pi}{3} + \frac{2n\pi}{3} \\
 & \quad n \in \mathbb{Z} \\
 & \quad \therefore 2x = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \text{ and} \\
 & \quad 2x = \pi + \frac{\pi}{3} = \frac{4\pi}{3} \\
 & \quad \because 2\pi \text{ is the period of } \cos 2x \\
 & \quad \therefore 2x = \frac{2\pi}{3} + 2n\pi \\
 & \quad 2x = \frac{4\pi}{3} + 2n\pi \\
 & \quad \Rightarrow x = \frac{\pi}{3} + n\pi \\
 & \quad x = \frac{2\pi}{3} + n\pi, n \in \mathbb{Z} \\
 & \therefore S.S. = \left\{ \frac{2n\pi}{3} \right\} \cup \left\{ \frac{\pi}{3} + \frac{2n\pi}{3} \right\} \cup \left\{ \frac{\pi}{3} + n\pi \right\} \\
 & \quad \cup \left\{ \frac{2\pi}{3} + n\pi \right\}, n \in \mathbb{Z}
 \end{aligned}$$

$$\begin{aligned}
 & (19) \sin 0 + \sin 30 + \sin 50 + \sin 70 = 0 \\
 & \text{general values of } x \text{ are } \Rightarrow (\sin 70 + \sin 0) + (\sin 50 + \sin 30) = 0 \\
 & \Rightarrow 2 \sin\left(\frac{70+0}{2}\right) \cos\left(\frac{70-0}{2}\right) + 2 \sin\left(\frac{50+30}{2}\right) \cos\left(\frac{50-30}{2}\right) = 0
 \end{aligned}$$

$$\Rightarrow 2 \sin 40 \cos 30 + 2 \sin 40 \cos 50 = 0$$

$$\Rightarrow 2 \sin 40 (\cos 30 + \cos 50) = 0$$

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$$\begin{aligned}
 & \Rightarrow 2 \sin \theta [2 \cos\left(\frac{\theta+0}{2}\right) \cos\left(\frac{\theta-0}{2}\right)] = 0 \\
 & \Rightarrow 4 \sin \theta \cos 2\theta \cos 0 = 0 \\
 & \Rightarrow \sin \theta \cos 2\theta \cos 0 = 0 \quad \because 4 \neq 0 \\
 & \Rightarrow \sin \theta = 0 \quad \cos 2\theta = 0 \quad \cos 0 = 0 \\
 & \Rightarrow \theta = n\pi \quad \theta = \frac{\pi}{2} + n\pi \quad \theta = \frac{3\pi}{2} + n\pi \\
 & \because 2\pi \text{ is the period} \quad \theta = \frac{3\pi}{2} + n\pi \quad \theta = \frac{3\pi}{2} + n\pi \\
 & \text{of } \sin \theta \quad \theta = \frac{\pi}{2} + n\pi \quad \because 2\pi \text{ is the} \\
 & \therefore \theta = 0 + 2n\pi = 2n\pi \quad \text{period of } \cos \theta \\
 & \theta = \pi + 2n\pi \quad \theta = \frac{\pi}{2} + 2n\pi \quad \theta = \frac{3\pi}{2} + 2n\pi \\
 & \Rightarrow \theta = \frac{n\pi}{2} + \pi \quad \theta = \frac{3\pi}{2} + 2n\pi \quad \theta = \frac{3\pi}{2} + 2n\pi \\
 & \theta = \frac{\pi}{4} + \frac{n\pi}{2} \quad \theta = \frac{\pi}{4} + n\pi \quad \theta = \frac{3\pi}{4} + n\pi \\
 & \quad n \in \mathbb{Z} \quad \theta = \frac{3\pi}{4} + n\pi \quad n \in \mathbb{Z} \\
 & \therefore S.S. = \left\{ \frac{n\pi}{2} \right\} \cup \left\{ \frac{\pi}{4} + \frac{n\pi}{2} \right\} \cup \left\{ \frac{\pi}{4} + n\pi \right\} \cup \left\{ \frac{3\pi}{4} + n\pi \right\} \\
 & \quad \cup \left\{ \frac{3\pi}{4} + 2n\pi \right\} \cup \left\{ \frac{3\pi}{2} + 2n\pi \right\}, n \in \mathbb{Z}
 \end{aligned}$$

$$\begin{aligned}
 & \textcircled{(2)} \cos 0 + \cos 30 + \cos 50 + \cos 70 = 0 \\
 & \Rightarrow (\cos 70 + \cos 0) + (\cos 50 + \cos 30) = 0 \\
 & \Rightarrow 2 \cos\left(\frac{70+0}{2}\right) \cos\left(\frac{70-0}{2}\right) + 2 \cos\left(\frac{50+30}{2}\right) \cos\left(\frac{50-30}{2}\right) = 0 \\
 & \Rightarrow 2 \cos 40 \cos 30 + 2 \cos 40 \cos 0 = 0 \\
 & \Rightarrow 2 \cos 40 [\cos 30 + \cos 0] = 0 \\
 & \Rightarrow 2 \cos 40 [2 \cos\left(\frac{30+0}{2}\right) \cos\left(\frac{30-0}{2}\right)] = 0 \\
 & \Rightarrow 2 \cos 40 [2 \cos 2\theta \cos 0] = 0 \\
 & \Rightarrow 4 \cos 40 \cos 2\theta \cos 0 = 0 \\
 & \Rightarrow \cos 40 \cos 2\theta \cos 0 = 0 \\
 & \Rightarrow \cos 40 = 0 \quad \cos 2\theta = 0 \quad \cos 0 = 0 \\
 & \Rightarrow \theta = \frac{\pi}{2} + n\pi \quad \theta = \frac{\pi}{2} + n\pi \quad \theta = \frac{3\pi}{2} + n\pi \\
 & \because 2\pi \text{ is period} \quad \theta = \frac{3\pi}{2} + n\pi \quad \theta = \frac{3\pi}{2} + n\pi \\
 & \text{of } \cos \theta \quad \theta = \frac{\pi}{2} + n\pi \quad \because 2\pi \text{ is the} \\
 & \therefore \theta = \frac{\pi}{2} + 2n\pi \quad \theta = \frac{\pi}{2} + 2n\pi \quad \text{period of } \cos \theta \\
 & \theta = \frac{3\pi}{2} + 2n\pi \quad \theta = \frac{3\pi}{2} + 2n\pi \quad \theta = \frac{3\pi}{2} + 2n\pi \\
 & \Rightarrow \theta = \frac{\pi}{8} + \frac{n\pi}{2} + \pi \quad \theta = \frac{3\pi}{4} + n\pi \quad \theta = \frac{3\pi}{2} + 2n\pi \\
 & \theta = \frac{3\pi}{8} + \frac{n\pi}{2} \quad \theta = \frac{3\pi}{4} + n\pi \quad n \in \mathbb{Z} \\
 & \quad , \quad n \in \mathbb{Z} \quad , \quad n \in \mathbb{Z}
 \end{aligned}$$

$$\begin{aligned}
 & \therefore S.S. = \left\{ \frac{\pi}{8} + \frac{n\pi}{2} \right\} \cup \left\{ \frac{3\pi}{8} + \frac{n\pi}{2} \right\} \cup \left\{ \frac{\pi}{4} + n\pi \right\} \\
 & \quad \cup \left\{ \frac{3\pi}{4} + n\pi \right\} \cup \left\{ \frac{\pi}{2} + 2n\pi \right\} \cup \left\{ \frac{3\pi}{2} + 2n\pi \right\} \\
 & \quad , \quad n \in \mathbb{Z}
 \end{aligned}$$