

**Question # 1**

Without using the tables, find the value of :

- |                         |                         |                          |
|-------------------------|-------------------------|--------------------------|
| (i) $\sin(-780^\circ)$  | (ii) $\cot(-855^\circ)$ | (iii) $\csc(2040^\circ)$ |
| (iv) $\csc(2040^\circ)$ | (v) $\tan(1110^\circ)$  | (vi) $\sec(-960^\circ)$  |

**Solution**

$$(i) \sin(-780^\circ) = -\sin 780^\circ = -\sin(8(90) + 60)$$

$$= -\sin(60) = -\frac{\sqrt{3}}{2} \quad \because 780^\circ \text{ is in the Ist quad.}$$

$$(ii) \cot(-855^\circ) = -\cot 855^\circ = -\cot(9(90) + 45)$$

$$= -(-\tan 45^\circ) = \tan 45^\circ = 1 \quad \because 855^\circ \text{ is in the IIInd quad.}$$

$$(iii) \csc(2040^\circ) = \csc(22(90) + 60) = -\csc(60)$$

$$= -\frac{1}{\sin(60)} = -\frac{1}{\sqrt{3}/2} = -\frac{2}{\sqrt{3}}$$

$\therefore 2040^\circ$  is in the Ist quad.

$$(iv) \sec(-960^\circ) = \sec(960^\circ) = \sec(10(90) + 60) = -\sec 60^\circ$$

$$= -\frac{1}{\cos 60^\circ} = -\frac{1}{1/2} = -2$$

$\therefore 960^\circ$  is in the IIIrd quad.

$$(v) \tan(1110^\circ) = \tan(12(90) + 30) = \tan(30) = \frac{1}{\sqrt{3}}$$

$\therefore 1110^\circ$  is in the Ist quad

$$(vi) \sin(-300^\circ) = -\sin(300^\circ) = -\sin(3(90) + 30)$$

$$= -(-\cos 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$\therefore 300^\circ$  is in the IIIrd quad.

**Question # 2**

Express each of the following as a trigonometric function of an angle of positive degree measure of less than  $45^\circ$ .

- |                      |                       |                          |                           |
|----------------------|-----------------------|--------------------------|---------------------------|
| (i) $\sin 196^\circ$ | (ii) $\cos 147^\circ$ | (iii) $\sin 319^\circ$   | (iv) $\cos 254^\circ$     |
| (v) $\tan 294^\circ$ | (vi) $\cos 728^\circ$ | (vii) $\sin(-625^\circ)$ | (viii) $\cos(-435^\circ)$ |

**Solution**

$$(i) \sin 196^\circ = \sin(180 + 16) = \sin 180^\circ \cos 16^\circ + \cos 180^\circ \sin 16^\circ$$

$$= (0) \cos 16^\circ + (-1) \sin 16^\circ = -\sin 16^\circ$$

$$(ii) \cos 147^\circ = \cos(180 - 33) = \cos 180^\circ \cos 33^\circ + \sin 180^\circ \sin 33^\circ$$

$$= (-1) \cos 33^\circ + (0) \sin 33^\circ = -\cos 33^\circ$$

(iii)  $\sin 319^\circ = \sin(360 - 41) = \sin 360^\circ \cos 41^\circ - \cos 360^\circ \sin 41^\circ$  **Now Do yourself**

(iv)  $\cos 254^\circ = \cos(270 - 16)$  **Do yourself**

$$\begin{aligned} \text{(v)} \quad \tan 294^\circ &= \frac{\sin 294^\circ}{\cos 294^\circ} = \frac{\sin(270 + 24)}{\cos(270 + 24)} \\ &= \frac{\sin 270^\circ \cos 24^\circ + \cos 270^\circ \sin 24^\circ}{\cos 270^\circ \cos 24^\circ - \sin 270^\circ \sin 24^\circ} = \frac{(-1)\cos 24^\circ + (0)\sin 24^\circ}{(0)\cos 24^\circ - (-1)\sin 24^\circ} \\ &= \frac{-\cos 24^\circ + 0}{0 + \sin 24^\circ} = \frac{-\cos 24^\circ}{\sin 24^\circ} = -\cot 24^\circ \end{aligned}$$

**Alternative Method:**

$$\begin{aligned} \tan 294^\circ &= \tan(270 + 24) = \frac{\tan 270^\circ + \tan 24^\circ}{1 - \tan 270^\circ \tan 24^\circ} \\ &= \frac{\tan 270^\circ \left(1 + \frac{\tan 24^\circ}{\tan 270^\circ}\right)}{\tan 270^\circ \left(\frac{1}{\tan 270^\circ} - \tan 24^\circ\right)} = \frac{\left(1 + \frac{\tan 24^\circ}{\infty}\right)}{\left(\frac{1}{\infty} - \tan 24^\circ\right)} \\ &= \frac{(1+0)}{(0-\tan 24^\circ)} = -\frac{1}{\tan 24^\circ} = -\cot 24^\circ \quad \square \end{aligned}$$

(vi)  $\cos 728^\circ = \cos(720 + 8)$  **Now Do yourself**

$$\begin{aligned} \text{(vii)} \quad \sin(-625^\circ) &= -\sin 625^\circ = -\sin(630 - 5) \\ &= -(\sin 630^\circ \cos 5^\circ - \cos 630^\circ \sin 5^\circ) = -((-1)\cos 5^\circ - (0)\sin 5^\circ) \\ &= -(-\cos 5^\circ - 0) = \cos 5^\circ \end{aligned}$$

$$\begin{aligned} \text{(viii)} \quad \cos(-435^\circ) &= \cos 435^\circ \\ &= \cos(450 - 15) \quad \text{Now Do yourself} \end{aligned}$$


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### Question # 3

Prove the following:

- (i)  $\sin(180 + \alpha)\sin(90 - \alpha) = -\sin \alpha \cos \alpha$
- (ii)  $\sin 780^\circ \sin 480^\circ + \cos 120^\circ \sin 30^\circ = \frac{1}{2}$
- (iii)  $\sin 306^\circ + \cos 234^\circ + \cos 162^\circ + \cos 18^\circ = 0$
- (iv)  $\cos 330^\circ \sin 600^\circ + \cos 120^\circ \sin 150^\circ = -1$

**Solution**

$$\begin{aligned} \text{(i)} \quad \text{L.H.S} &= \sin(180 + \alpha)\sin(90 - \alpha) \\ &= (\sin 180^\circ \cos \alpha + \cos 180^\circ \sin \alpha)(\sin 90^\circ \cos \alpha - \cos 90^\circ \sin \alpha) \\ &= ((0)\cos \alpha + (-1)\sin \alpha)((1)\cos \alpha - (0)\sin \alpha) \end{aligned}$$

$$= (0 - \sin \alpha)(\cos \alpha - 0) = -\sin \alpha \cos \alpha = \text{R.H.S} \quad \square$$

(ii) First we calculate

$$\sin 780^\circ = \sin(720 + 60) = \sin(2 \times 360 + 60) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\begin{aligned}\sin 480^\circ &= \sin(450 + 30) = \sin 450^\circ \cos 30^\circ + \cos 450^\circ \sin 30^\circ \\ &= (1) \cos 30 + (0) \sin 30 = \cos 30 + 0 = \frac{\sqrt{3}}{2}\end{aligned}$$

$$\cos 120^\circ = -\frac{1}{2} \quad \text{and} \quad \sin 30^\circ = \frac{1}{2}.$$

$$\text{So L.H.S} = \sin 780^\circ \sin 480^\circ + \cos 120^\circ \sin 30^\circ$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2} = \text{R.H.S} \quad \square$$

(iii) First we calculate

$$\begin{aligned}\cos 306^\circ &= \cos(270 + 36) = \cos 270^\circ \cos 36^\circ - \sin 270^\circ \sin 36^\circ \\ &= (0) \cos 36^\circ - (-1) \sin 36^\circ = 0 + \sin 36^\circ = \sin 36^\circ\end{aligned}$$

$$\begin{aligned}\cos 234^\circ &= \cos(270 - 36) = \cos 270 \cos 36 + \sin 270 \cos 36 \\ &= (0) \cos 36^\circ + (-1) \sin 36^\circ = 0 - \sin 36^\circ = -\sin 36^\circ\end{aligned}$$

$$\begin{aligned}\cos 162^\circ &= \cos(180 - 18) = \cos 180^\circ \cos 18^\circ + \sin 180^\circ \sin 18^\circ \\ &= (-1) \cos 18 + (0) \sin 18 = -\cos 18 + 0 = -\cos 18\end{aligned}$$

$$\begin{aligned}\text{So L.H.S} &= \sin 306^\circ + \cos 234^\circ + \cos 162^\circ + \cos 18^\circ \\ &= \sin 36^\circ - \sin 36^\circ - \cos 18^\circ + \cos 18^\circ = 0 = \text{R.H.S} \quad \square\end{aligned}$$

(iv) First we calculate (*Alternative Method*)

$$\cos 330^\circ = \cos(360 - 30) = \cos(-30^\circ) = \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin 600^\circ = \sin(6 \times 90 + 60) = -\sin 60 = -\frac{\sqrt{3}}{2} \quad \because 600^\circ \text{ is in the IIIrd quad}$$

$$\cos 120^\circ = \cos(90 + 30) = -\sin 30 = -\frac{1}{2} \quad \because 120^\circ \text{ is in the IIInd quad}$$

$$\sin 150^\circ = \sin(90 + 60) = \cos 60 = \frac{1}{2} \quad \because 150^\circ \text{ is in the IIInd quad}$$

$$\begin{aligned}\text{So L.H.S} &= \cos 330^\circ \sin 600^\circ + \cos 120^\circ \sin 150^\circ \\ &= \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) = -\frac{3}{4} - \frac{1}{4} = -\frac{4}{4} = -1 = \text{R.H.S}\end{aligned}$$


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**Question # 4**

Prove that;

$$(i) \frac{\sin^2(\pi + \theta) \tan\left(\frac{3\pi}{2} + \theta\right)}{\cot^2\left(\frac{3\pi}{2} - \theta\right) \cos^2(\pi - \theta) \csc(2\pi - \theta)} = \cos \theta$$

$$(ii) \frac{\cos(90^\circ + \theta) \sec(-\theta) \tan(180^\circ - \theta)}{\sec(360^\circ - \theta) \sin(180^\circ + \theta) \cot(90^\circ - \theta)} = -1$$

**Solution**

(i) First we calculate

$$\begin{aligned}\sin(\pi + \theta) &= \sin \pi \cos \theta + \cos \pi \sin \theta = (0)\cos \theta + (-1)\sin \theta \\ &= 0 - \sin \theta = -\sin \theta\end{aligned}$$

$$\tan\left(\frac{3\pi}{2} + \theta\right) = \tan\left(3 \cdot \frac{\pi}{2} + \theta\right) = -\cot \theta \quad \because \frac{3\pi}{2} + \theta \text{ is in the IVth quad}$$

$$\cot\left(\frac{3\pi}{2} - \theta\right) = \cot\left(3 \cdot \frac{\pi}{2} - \theta\right) = \tan \theta \quad \because \frac{3\pi}{2} - \theta \text{ is in the IIIrd quad}$$

$$\begin{aligned}\cos(\pi - \theta) &= \cos \pi \cos \theta + \sin \pi \sin \theta = (-1)\cos \theta + (0)\sin \theta \\ &= -\cos \theta + 0 = -\cos \theta\end{aligned}$$

$$\csc(2\pi - \theta) = \csc(-\theta) = -\csc \theta$$

Now

$$\begin{aligned}\text{L.H.S} &= \frac{\sin^2(\pi + \theta) \tan\left(\frac{3\pi}{2} + \theta\right)}{\cot\left(\frac{3\pi}{2} - \theta\right) \cos^2(\pi - \theta) \csc(2\pi - \theta)} \\ &= \frac{(-\sin \theta)^2 (-\cot \theta)}{(\tan \theta)^2 (-\cos \theta)^2 (-\csc \theta)} = \frac{\sin^2 \theta (-\cot \theta)}{\tan^2 \theta \cos^2 \theta (-\csc \theta)} \\ &= \frac{\sin^2 \theta \frac{\cos \theta}{\sin \theta}}{\frac{\sin^2 \theta}{\cos^2 \theta} \cos^2 \theta \frac{1}{\sin \theta}} = \frac{\sin \theta \cos \theta}{\sin \theta} = \cos \theta = \text{R.H.S}\end{aligned}$$

(ii) First we calculate

$$\cos(90 + \theta) = -\sin \theta \quad \because 90 + \theta \text{ is in the IIInd quad.}$$

$$\sec(-\theta) = \sec \theta$$

$$\tan(180 - \theta) = \tan(2(90) - \theta) = -\tan \theta \quad \because 180 - \theta \text{ is in the IIInd quad.}$$

$$\sec(360 - \theta) = \sec(-\theta) = \sec \theta$$

$$\sin(180 + \theta) = \sin(2(90) + \theta) = -\sin \theta \quad \because 180 + \theta \text{ is in the IIIrd quad.}$$

$$\cot(90 - \theta) = \tan \theta \quad \because 90 - \theta \text{ is in the Ist quad.}$$

Now

$$\begin{aligned} \text{L.H.S} &= \frac{\cos(90+\theta) \sec(-\theta) \tan(180-\theta)}{\sec(360-\theta) \sin(180+\theta) \cot(90-\theta)} \\ &= \frac{(-\sin\theta)\sec\theta (-\tan\theta)}{\sec\theta (-\sin\theta) (-\tan\theta)} = 1 = \text{R.H.S} \end{aligned}$$


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**Question # 5**

If  $\alpha, \beta, \gamma$  are the angles of a triangle  $ABC$ , then prove that;

- |  |   |
|--|---|
| (i) $\sin(\alpha + \beta) = \sin \gamma$   | (ii) $\cos\left(\frac{\alpha + \beta}{2}\right) = \sin\frac{\gamma}{2}$ |
| (iii) $\cos(\alpha + \beta) = \cos \gamma$ | (iv) $\tan(\alpha + \beta) + \tan \gamma = 0$                           |

**Solution**

- (i) Since  $\alpha, \beta$  and  $\gamma$  are angles of triangle therefore

$$\alpha + \beta + \gamma = 180 \Rightarrow \alpha + \beta = 180 - \gamma$$

$$\begin{aligned} \text{Now L.H.S} &= \sin(\alpha + \beta) = \sin(180 - \gamma) \\ &= \sin 180 \cos \gamma - \cos 180 \sin \gamma \\ &= (0) \cos \gamma - (-1) \sin \gamma = 0 + \sin \gamma = \sin \gamma = \text{R.H.S} \end{aligned}$$

- (ii) Since  $\alpha, \beta$  and  $\gamma$  are angles of triangle therefore

$$\alpha + \beta + \gamma = 180$$

$$\Rightarrow \alpha + \beta = 180 - \gamma \Rightarrow \frac{\alpha + \beta}{2} = \frac{180 - \gamma}{2}$$

$$\begin{aligned} \text{Now L.H.S} &= \cos\left(\frac{\alpha + \beta}{2}\right) = \cos\left(\frac{180 - \gamma}{2}\right) = \cos\left(\frac{180}{2} - \frac{\gamma}{2}\right) \\ &= \cos\left(90 - \frac{\gamma}{2}\right) = \cos 90 \cos \frac{\gamma}{2} + \sin 90 \sin \frac{\gamma}{2} \\ &= (0) \cos \frac{\gamma}{2} + (1) \sin \frac{\gamma}{2} = 0 + \sin \frac{\gamma}{2} = \sin \frac{\gamma}{2} = \text{R.H.S} \quad \square \end{aligned}$$

- (iii) Since  $\alpha, \beta$  and  $\gamma$  are angles of triangle therefore

$$\alpha + \beta + \gamma = 180 \Rightarrow \alpha + \beta = 180 - \gamma$$

$$\begin{aligned} \text{Now L.H.S} &= \cos(\alpha + \beta) = \cos(180 - \gamma) \\ &= \cos 180 \cos \gamma + \sin 180 \sin \gamma \\ &= (-1) \cos \gamma + (0) \sin \gamma = -\cos \gamma + 0 = -\cos \gamma = \text{R.H.S} \end{aligned}$$

- (iv) Since  $\alpha, \beta$  and  $\gamma$  are angles of triangle therefore

$$\alpha + \beta + \gamma = 180 \Rightarrow \alpha + \beta = 180 - \gamma$$

$$\begin{aligned} \text{Now L.H.S} &= \tan(\alpha + \beta) + \tan \gamma = \tan(180 - \gamma) + \tan \gamma \\ &= \frac{\tan 180 - \tan \gamma}{1 + \tan 180 \tan \gamma} + \tan \gamma \end{aligned}$$

$$\begin{aligned}
 &= \frac{(0) - \tan \gamma}{1 + (0) \tan \gamma} + \tan \gamma = \frac{-\tan \gamma}{1 + 0} + \tan \gamma \\
 &= -\tan \gamma + \tan \gamma = 0 = \text{R.H.S. } \square
 \end{aligned}$$


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**Remember:**

- $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$
- $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$
- $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$
- $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$
- $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$
- $\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$

**Three Steps to solve**  $\sin\left(n \cdot \frac{\pi}{2} \pm \theta\right)$

**Step I:** First check that  $n$  is even or odd

**Step II:** If  $n$  is even then the answer will be in  $\sin$  and if the  $n$  is odd then  $\sin$  will be converted to  $\cos$  and vice versa (i.e.  $\cos$  will be converted to  $\sin$ ).

**Step III:** Now check in which quadrant  $n \cdot \frac{\pi}{2} \pm \theta$  is lying if it is in *Ist* or *IIInd* quadrant the answer will be positive as  $\sin$  is positive in these quadrant and if it is in the *IIIrd* or *IVth* quadrant the answer will be negative.

e.g.  $\sin 667^\circ = \sin(7(90) + 37)$

Since  $n = 7$  is odd so answer will be in  $\cos$  and  $667$  is in *IVth* quadrant and  $\sin$  is negative in *IVth* quadrant therefore answer will be in negative. i.e.  $\sin 667^\circ = -\cos 37$ . Similar technique is used for other trigonometric ratios. i.e.  $\tan \rightleftharpoons \cot$  and  $\sec \rightleftharpoons \csc$ .

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