

Chapter 13

Exercise 13.2

Question # 1

Prove that:

$$(1) \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{7}{25} = \cos^{-1} \frac{253}{325}$$

Solution

$$\text{L.H.S} = \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{7}{25}$$

$$\because \sin^{-1} A + \sin^{-1} B = \sin^{-1} \left(A\sqrt{1-B^2} + B\sqrt{1-A^2} \right)$$

$$= \sin^{-1} \left(\frac{5}{13} \sqrt{1 - \left(\frac{7}{25} \right)^2} + \frac{7}{25} \sqrt{1 - \left(\frac{5}{13} \right)^2} \right)$$

$$= \sin^{-1} \left(\frac{5}{13} \sqrt{1 - \frac{49}{625}} + \frac{7}{25} \sqrt{1 - \frac{25}{169}} \right)$$

$$= \sin^{-1} \left(\frac{5}{13} \sqrt{\frac{576}{625}} + \frac{7}{25} \sqrt{\frac{144}{169}} \right) = \sin^{-1} \left(\frac{5}{13} \left(\frac{24}{25} \right) + \frac{7}{25} \left(\frac{12}{13} \right) \right)$$

$$= \sin^{-1} \left(\frac{120}{325} + \frac{84}{325} \right) = \sin^{-1} \left(\frac{204}{325} \right)$$

$$= \frac{\pi}{2} - \cos^{-1} \left(\frac{204}{325} \right) \quad \because \sin^{-1} \theta = \frac{\pi}{2} - \cos^{-1} \theta$$

$$= \cos^{-1} (0) - \cos \left(\frac{204}{325} \right) \quad \because \frac{\pi}{2} = \cos^{-1} (0)$$

$$\therefore \cos^{-1} A - \cos^{-1} B = \cos^{-1} \left(AB + \sqrt{(1-A^2)(1-B^2)} \right)$$

$$= \cos^{-1} \left((0) \left(\frac{204}{325} \right) + \sqrt{\left(1 - (0)^2 \right) \left(1 - \left(\frac{204}{325} \right)^2 \right)} \right)$$

$$= \cos^{-1} \left(0 + \sqrt{\left(1 - 0 \right) \left(1 - \frac{41616}{105625} \right)} \right) = \cos^{-1} \left(\sqrt{\left(1 \right) \left(\frac{64009}{105625} \right)} \right)$$

$$= \cos^{-1} \left(\sqrt{\frac{64009}{105625}} \right) = \cos^{-1} \frac{253}{325} = \text{R.H.S}$$

Question # 2

$$\text{Prove that: } \tan^{-1} \frac{1}{4} + \tan^{-1} \frac{1}{5} = \tan^{-1} \left(\frac{9}{19} \right)$$

Solution

$$\text{L.H.S} = \tan^{-1} \frac{1}{4} + \tan^{-1} \frac{1}{5}$$

$$\begin{aligned}
 &= \tan^{-1} \left(\frac{\frac{1}{4} + \frac{1}{5}}{1 - \left(\frac{1}{4} \right) \left(\frac{1}{5} \right)} \right) \quad \because \tan^{-1} A + \tan^{-1} B = \tan^{-1} \left(\frac{A+B}{1-AB} \right) \\
 &= \tan^{-1} \left(\frac{\frac{9}{20}}{1 - \frac{1}{20}} \right) = \tan^{-1} \left(\frac{\frac{9}{20}}{\frac{19}{20}} \right) \\
 &= \tan^{-1} \left(\frac{9}{19} \right) = \text{R.H.S}
 \end{aligned}$$

Question # 3

Prove that:

$$2 \tan^{-1} \frac{2}{3} = \sin^{-1} \frac{12}{13}$$

Solution Suppose

$$\alpha = \sin^{-1} \frac{12}{13} \quad \dots \dots \dots \text{(i)}$$

$$\Rightarrow \sin \alpha = \frac{12}{13}$$

$$\text{Now } \cos \alpha = \sqrt{1 - \sin^2 \alpha} = \sqrt{1 - \left(\frac{12}{13} \right)^2} = \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

$$\text{Now } \tan \frac{\alpha}{2} = \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} = \sqrt{\frac{1 - \frac{5}{13}}{1 + \frac{5}{13}}} = \sqrt{\frac{\frac{8}{13}}{\frac{18}{13}}} = \sqrt{\frac{8}{18}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$$

$$\Rightarrow \frac{\alpha}{2} = \tan^{-1} \left(\frac{2}{3} \right) \Rightarrow \alpha = 2 \tan^{-1} \frac{2}{3} \quad \dots \dots \dots \text{(ii)}$$

from (i) and (ii)

$$2 \tan^{-1} \frac{2}{3} = \sin^{-1} \frac{12}{13} \quad \text{proved.}$$

Question # 4

Prove that:

$$\tan^{-1} \frac{120}{119} = 2 \cos^{-1} \frac{12}{13}$$

Solution Suppose

$$\alpha = \tan^{-1} \frac{120}{119} \quad \dots \dots \dots \text{(i)}$$

$$\Rightarrow \tan \alpha = \frac{120}{119}$$

$$\text{Now } \sec \alpha = \sqrt{1 + \tan^2 \alpha}$$

$$\begin{aligned}
 &= \sqrt{1 + \left(\frac{120}{119}\right)^2} = \sqrt{1 + \frac{14400}{14161}} \\
 &= \sqrt{\frac{28561}{14161}} = \frac{169}{119} \\
 \text{So } \cos \alpha &= \frac{1}{\sec \alpha} = \frac{1}{\cancel{169}/119} = \frac{119}{169} \\
 \text{Now } \cos \frac{\alpha}{2} &= \sqrt{\frac{1 + \cos \alpha}{2}} = \sqrt{\frac{1 + 119/169}{2}} = \sqrt{\frac{288/169}{2}} = \sqrt{\frac{288}{2 \times 169}} = \sqrt{\frac{144}{169}} = \frac{12}{13} \\
 \Rightarrow \frac{\alpha}{2} &= \cos^{-1} \frac{12}{13} \quad \Rightarrow \alpha = 2 \cos^{-1} \frac{12}{13} \quad \dots \dots \dots \text{(ii)}
 \end{aligned}$$

From (i) and (ii)

$$\tan^{-1} \frac{120}{119} = 2 \cos^{-1} \frac{12}{13} \quad \text{proved.}$$

Question # 5

Prove that:

$$\sin^{-1} \frac{1}{\sqrt{5}} + \cot^{-1} 3 = \frac{\pi}{4}$$

Solution Suppose

$$\alpha = \sin^{-1} \frac{1}{\sqrt{5}} \quad \dots \dots \dots \text{(i)}$$

$$\Rightarrow \sin \alpha = \frac{1}{\sqrt{5}}$$

$$\text{Now } \cos \alpha = \sqrt{1 - \sin^2 \alpha} = \sqrt{1 - \left(\frac{1}{\sqrt{5}}\right)^2} = \sqrt{1 - \frac{1}{5}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

$$\begin{aligned}
 \text{So } \tan \alpha &= \frac{\sin \alpha}{\cos \alpha} = \frac{\frac{1}{\sqrt{5}}}{\frac{2}{\sqrt{5}}} = \frac{1}{2} \\
 \Rightarrow \alpha &= \tan^{-1} \frac{1}{2} \quad \dots \dots \dots \text{(ii)}
 \end{aligned}$$

From (i) and (ii)

$$\sin^{-1} \frac{1}{\sqrt{5}} = \tan^{-1} \frac{1}{2}$$

$$\text{Now } \cot^{-1} 3 = \tan^{-1} \frac{1}{3} \qquad \qquad \because \cot^{-1} x = \tan^{-1} \frac{1}{x}$$

$$\text{And } \text{L.H.S} = \sin^{-1} \frac{1}{\sqrt{5}} + \cot^{-1} 3$$

$$\begin{aligned}
 &= \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} \\
 &= \tan^{-1} \left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)} \right) = \tan^{-1} \left(\frac{\frac{5}{6}}{1 - \frac{1}{6}} \right) \\
 &= \tan^{-1} \left(\frac{\frac{5}{6}}{\frac{5}{6}} \right) = \tan^{-1}(1) = \frac{\pi}{4} = \text{R.H.S} \quad \text{proved.}
 \end{aligned}$$

Question # 6

Prove that:

$$\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{8}{17} = \sin^{-1} \left(\frac{77}{85} \right)$$

$$\begin{aligned}
 \text{Solution} \quad \text{L.H.S} &= \sin^{-1} \frac{3}{5} + \sin^{-1} \frac{8}{17} \\
 &= \sin^{-1} \left(\frac{3}{5} \sqrt{1 - \left(\frac{8}{17}\right)^2} + \frac{8}{17} \sqrt{1 - \left(\frac{3}{5}\right)^2} \right) \\
 &= \sin^{-1} \left(\frac{3}{5} \sqrt{1 - \frac{64}{289}} + \frac{8}{17} \sqrt{1 - \frac{9}{25}} \right) = \sin^{-1} \left(\frac{3}{5} \sqrt{\frac{225}{289}} + \frac{8}{17} \sqrt{\frac{16}{25}} \right) \\
 &= \sin^{-1} \left(\frac{3}{5} \left(\frac{15}{17} \right) + \frac{8}{17} \left(\frac{4}{5} \right) \right) = \sin^{-1} \left(\frac{45}{85} + \frac{32}{85} \right) \\
 &= \sin^{-1} \left(\frac{77}{85} \right) = \text{R.H.S} \quad \text{proved.}
 \end{aligned}$$

Question # 7

Prove that:

$$\sin^{-1} \frac{77}{85} - \sin^{-1} \frac{3}{5} = \cos^{-1} \left(\frac{15}{17} \right)$$

$$\begin{aligned}
 \text{Solution} \quad \text{L.H.S} &= \sin^{-1} \frac{77}{85} - \sin^{-1} \frac{3}{5} \\
 &= \left(\frac{\pi}{2} - \cos^{-1} \frac{77}{85} \right) - \left(\frac{\pi}{2} - \cos^{-1} \frac{3}{5} \right) \quad \because \sin^{-1} x = \frac{\pi}{2} - \cos^{-1} x \\
 &= \frac{\pi}{2} - \cos^{-1} \frac{77}{85} - \frac{\pi}{2} + \cos^{-1} \frac{3}{5} = \cos^{-1} \frac{3}{5} - \cos^{-1} \frac{77}{85} \\
 &= \cos^{-1} \left(\left(\frac{3}{5} \right) \left(\frac{77}{85} \right) + \sqrt{\left(1 - \left(\frac{3}{5}\right)^2\right) \left(1 - \left(\frac{77}{85}\right)^2\right)} \right) \\
 &= \cos^{-1} \left(\frac{231}{425} + \sqrt{\left(1 - \frac{9}{25}\right) \left(1 - \frac{5929}{7225}\right)} \right)
 \end{aligned}$$

$$\begin{aligned}
 &= \cos^{-1} \left(\frac{231}{425} + \sqrt{\left(\frac{16}{25}\right)\left(\frac{1296}{7225}\right)} \right) \\
 &= \cos^{-1} \left(\frac{231}{425} + \sqrt{\frac{20736}{180625}} \right) = \cos^{-1} \left(\frac{231}{425} + \frac{144}{425} \right) \\
 &= \cos^{-1} \left(\frac{375}{425} \right) = \cos^{-1} \left(\frac{15}{17} \right) = \text{L.H.S}
 \end{aligned}$$

Question # 8

Prove that:

$$\cos^{-1} \frac{63}{65} + 2 \tan^{-1} \frac{1}{5} = \sin^{-1} \left(\frac{3}{5} \right)$$

Solution Suppose $\alpha = \cos^{-1} \frac{63}{65}$ (i)

$$\Rightarrow \cos \alpha = \frac{63}{65}$$

$$\begin{aligned}
 \text{Now } \sin \alpha &= \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \left(\frac{63}{65}\right)^2} = \sqrt{1 - \frac{3969}{4225}} = \sqrt{\frac{256}{4225}} = \frac{16}{65} \\
 \Rightarrow \alpha &= \sin^{-1} \left(\frac{16}{65} \right) \text{ (ii)}
 \end{aligned}$$

So from equation (i) and (ii)

$$\cos^{-1} \left(\frac{63}{65} \right) = \sin^{-1} \left(\frac{16}{65} \right)$$

Now suppose $\beta = \tan^{-1} \frac{1}{5}$ (iii)

$$\Rightarrow \tan \beta = \frac{1}{5}$$

$$\text{So } \sec \alpha = \sqrt{1 + \tan^2 \alpha} = \sqrt{1 + \left(\frac{1}{5}\right)^2} = \sqrt{1 + \frac{1}{25}} = \sqrt{\frac{26}{25}} = \frac{\sqrt{26}}{5}$$

$$\text{So } \cos \beta = \frac{1}{\sec \beta} = \frac{1}{\sqrt{26}/5} = \frac{5}{\sqrt{26}}$$

$$\text{As } \frac{\sin \beta}{\cos \beta} = \tan \beta \Rightarrow \sin \beta = \tan \beta \cdot \cos \beta$$

$$\Rightarrow \sin \beta = \left(\frac{1}{5}\right) \left(\frac{5}{\sqrt{26}}\right) = \frac{1}{\sqrt{26}}$$

$$\Rightarrow \beta = \sin^{-1} \frac{1}{\sqrt{26}} \text{ (iv)}$$

From (iii) and (iv)

$$\tan^{-1} \frac{1}{5} = \sin^{-1} \frac{1}{\sqrt{26}}$$

$$\begin{aligned}
 \text{Now L.H.S} &= \cos^{-1} \frac{63}{65} + 2 \tan^{-1} \frac{1}{5} \\
 &= \sin^{-1} \frac{16}{65} + 2 \sin^{-1} \frac{1}{\sqrt{26}} = \sin^{-1} \frac{16}{65} + \left(\sin^{-1} \frac{1}{\sqrt{26}} + \sin^{-1} \frac{1}{\sqrt{26}} \right) \\
 &= \sin^{-1} \frac{16}{65} + \sin^{-1} \left(\frac{1}{\sqrt{26}} \sqrt{1 - \left(\frac{1}{\sqrt{26}} \right)^2} + \frac{1}{\sqrt{26}} \sqrt{1 - \left(\frac{1}{\sqrt{26}} \right)^2} \right) \\
 &= \sin^{-1} \frac{16}{65} + \sin^{-1} \left(\frac{1}{\sqrt{26}} \sqrt{1 - \frac{1}{26}} + \frac{1}{\sqrt{26}} \sqrt{1 - \frac{1}{26}} \right) \\
 &= \sin^{-1} \frac{16}{65} + \sin^{-1} \left(\frac{1}{\sqrt{26}} \sqrt{\frac{25}{26}} + \frac{1}{\sqrt{26}} \sqrt{\frac{25}{26}} \right) \\
 &= \sin^{-1} \frac{16}{65} + \sin^{-1} \left(\frac{1}{\sqrt{26}} \cdot \frac{5}{\sqrt{26}} + \frac{1}{\sqrt{26}} \cdot \frac{5}{\sqrt{26}} \right) \\
 &= \sin^{-1} \frac{16}{65} + \sin^{-1} \left(\frac{5}{26} + \frac{5}{26} \right) = \sin^{-1} \frac{16}{65} + \sin^{-1} \left(\frac{5}{13} \right) \\
 &= \sin^{-1} \left(\frac{16}{65} \sqrt{1 - \left(\frac{5}{13} \right)^2} + \frac{5}{13} \sqrt{1 - \left(\frac{16}{65} \right)^2} \right) \\
 &= \sin^{-1} \left(\frac{16}{65} \sqrt{1 - \frac{25}{169}} + \frac{5}{13} \sqrt{1 - \frac{256}{4225}} \right) \\
 &= \sin^{-1} \left(\frac{16}{65} \sqrt{1 - \frac{25}{169}} + \frac{5}{13} \sqrt{1 - \frac{256}{4225}} \right) = \sin^{-1} \left(\frac{16}{65} \sqrt{\frac{144}{169}} + \frac{5}{13} \sqrt{\frac{3969}{4225}} \right) \\
 &= \sin^{-1} \left(\frac{16}{65} \sqrt{\frac{144}{169}} + \frac{5}{13} \sqrt{\frac{3969}{4225}} \right) = \sin^{-1} \left(\frac{16}{65} \left(\frac{12}{13} \right) + \frac{5}{13} \left(\frac{63}{65} \right) \right) \\
 &= \sin^{-1} \left(\frac{192}{845} + \frac{315}{845} \right) = \sin^{-1} \left(\frac{3}{5} \right) = \text{R.H.S}
 \end{aligned}$$

Question # 9

Prove that:

$$\tan^{-1} \frac{3}{4} + \tan^{-1} \frac{3}{5} - \tan^{-1} \frac{8}{19} = \frac{\pi}{4}$$

$$\text{Solution} \quad \text{L.H.S} = \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{3}{5} - \tan^{-1} \frac{8}{19}$$

$$\begin{aligned}
 &= \tan^{-1} \left(\frac{\frac{3}{4} + \frac{3}{5}}{1 - \left(\frac{3}{4} \right) \left(\frac{3}{5} \right)} \right) - \tan^{-1} \frac{8}{19} \\
 &= \tan^{-1} \left(\frac{\frac{27}{20}}{1 - \frac{9}{20}} \right) - \tan^{-1} \frac{8}{19} = \tan^{-1} \left(\frac{\frac{27}{20}}{\frac{11}{20}} \right) - \tan^{-1} \frac{8}{19} \\
 &= \tan^{-1} \left(\frac{27}{11} \right) - \tan^{-1} \left(\frac{8}{19} \right) = \tan^{-1} \left(\frac{\frac{27}{11} - \frac{8}{19}}{1 + \left(\frac{27}{11} \right) \left(\frac{8}{19} \right)} \right) \\
 &= \tan^{-1} \left(\frac{\frac{425}{209}}{1 + \frac{216}{209}} \right) = \tan^{-1} \left(\frac{\frac{425}{209}}{1 + \frac{216}{209}} \right) \\
 &= \tan^{-1} \left(\frac{\frac{425}{209}}{\frac{425}{209}} \right) = \tan^{-1}(1) = \frac{\pi}{4} = \text{R.H.S} \quad \text{proved.}
 \end{aligned}$$

Question # 10

Do Yourself

Question # 11

Prove that:

$$\tan^{-1} \frac{1}{11} + \tan^{-1} \frac{5}{6} = \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2}$$

$$\begin{aligned}
 \text{Solution} \quad \text{L.H.S} &= \tan^{-1} \frac{1}{11} + \tan^{-1} \frac{5}{6} \\
 &= \text{Solve this} \dots \text{(i)}
 \end{aligned}$$

$$\begin{aligned}
 \text{R.H.S} &= \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2} \\
 &= \text{Solve this} \dots \text{(ii)}
 \end{aligned}$$

From (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

Question # 12

Prove that:

$$2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} = \frac{\pi}{4}$$

$$\begin{aligned}
 \text{Solution} \quad \text{L.H.S} &= 2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} \\
 &= \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7}
 \end{aligned}$$

$$\begin{aligned}
 &= \tan^{-1} \left(\frac{\frac{1}{3} + \frac{1}{3}}{1 - \frac{1}{3} \cdot \frac{1}{3}} \right) + \tan^{-1} \frac{1}{7} \\
 &= \tan^{-1} \left(\frac{\frac{2}{3}}{1 - \frac{1}{9}} \right) + \tan^{-1} \frac{1}{7} \\
 &= \tan^{-1} \left(\frac{\frac{2}{3}}{\frac{9-1}{9}} \right) + \tan^{-1} \frac{1}{7} \\
 &= \tan^{-1} \left(\frac{\frac{2}{3}}{\frac{8}{9}} \right) + \tan^{-1} \frac{1}{7} \\
 &= \tan^{-1} \left(\frac{2 \times 9}{3 \times 8} \right) + \tan^{-1} \frac{1}{7} \\
 &= \tan^{-1} \left(\frac{3}{4} \right) + \tan^{-1} \frac{1}{7} \Rightarrow \tan^{-1} \left(\frac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{4} \times \frac{1}{7}} \right) \\
 &= \tan^{-1} \left(\frac{\frac{21+4}{28}}{1 - \frac{3}{28}} \right) \Rightarrow \tan^{-1} \left(\frac{\frac{25}{28}}{\frac{28-3}{28}} \right) \\
 &= \tan^{-1} \left(\frac{\frac{25}{28}}{\frac{25}{28}} \right) \Rightarrow \tan^{-1}(1) \\
 &= \frac{\pi}{4} = R.H.S.
 \end{aligned}$$

Question # 13

Show that:

$$\cos(\sin^{-1} x) = \sqrt{1-x^2}$$

Solution Suppose $y = \sin^{-1} x$

$$\Rightarrow \sin y = x$$

$$\text{Since } \cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$$

$$\Rightarrow \cos(\sin^{-1} x) = \sqrt{1-x^2} \quad \text{Proved}$$

Question # 14

Show that:

$$\sin(2\cos^{-1} x) = 2x\sqrt{1-x^2}$$

Solution Suppose $y = \cos^{-1} x$
 Then $\cos y = x$
 Also $\sin y = \sqrt{1-\cos^2 y} = \sqrt{1-x^2}$
 Now $\begin{aligned} \sin(2\cos^{-1} x) &= \sin(2y) \\ &= 2\sin y \cdot \cos y \\ &= 2\sqrt{1-x^2} \cdot x \\ &= 2x\sqrt{1-x^2} \end{aligned}$

Question # 15

Show that:

$$\cos(2\sin^{-1} x) = 1-2x^2$$

Solution Suppose $y = \sin^{-1} x \Rightarrow \sin y = x$
 & $\cos y = \sqrt{1-\sin^2 y} = \sqrt{1-x^2}$
 Now $\begin{aligned} \cos(2\sin^{-1} x) &= \cos 2y \\ &= \cos^2 y - \sin^2 y \\ &= (\sqrt{1-x^2})^2 - x^2 = 1-x^2 - x^2 \\ &= 1-2x^2 \quad \text{Proved} \end{aligned}$

Question # 16

Show that:

$$\tan^{-1}(-x) = -\tan^{-1} x$$

Solution Suppose $y = \tan^{-1}(-x)$ (i)
 $\Rightarrow \tan y = -x \Rightarrow -\tan y = x$
 $\Rightarrow \tan(-y) = x \quad \because -\tan \theta = \tan(-\theta)$
 $\Rightarrow -y = \tan^{-1} x$
 $\Rightarrow y = -\tan^{-1} x$ (ii)

From equation (i) and (ii)

$$\tan^{-1}(-x) = -\tan^{-1} x \quad \text{Proved}$$

Question # 17*Do yourself as above***Question # 18**

Show that:

$$\cos^{-1}(-x) = \pi - \cos^{-1} x$$

Solution Suppose $y = \pi - \cos^{-1} x$ (i)

$$\Rightarrow \pi - y = \cos^{-1} x \Rightarrow \cos(\pi - y) = x$$

$$\Rightarrow \cos\pi \cos y + \sin\pi \sin y = x \Rightarrow (-1)\cos y + (0)\sin y = x$$

$$\Rightarrow -\cos y + 0 = x \Rightarrow -\cos y = x$$

$$\Rightarrow \cos y = -x \Rightarrow y = \cos^{-1}(-x) \text{ (ii)}$$

From (i) and (ii)

$$\cos^{-1}(-x) = \pi - \cos^{-1} x \quad \text{Proved}$$

Question # 19

Show that:

$$\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}$$

Solution Suppose $y = \sin^{-1} x \Rightarrow \sin y = x$

$$\text{Now } \cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$$

$$\& \tan y = \frac{\sin y}{\cos y} = \frac{x}{\sqrt{1-x^2}}$$

$$\text{Now } \tan(\sin^{-1} x) = \tan y = \frac{x}{\sqrt{1-x^2}} \quad \text{proved}$$

Question # 20

Given that $x = \sin^{-1} \frac{1}{2}$, find the values of the following trigonometric functions:

$\sin x, \cos x, \tan x, \cot x, \sec x$ and $\csc x$.

Solution Since $x = \sin^{-1} \frac{1}{2} \Rightarrow \sin x = \frac{1}{2}$

$$\text{Now } \cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}, \quad \cot x = \frac{1}{\tan x} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}, \quad \cosec x = \frac{1}{\sin x} = \frac{1}{\frac{1}{2}} = 2$$

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