

Exercise 2.3

Q# 1

- Commutative property of Union

$$A \cup B = B \cup A$$

- Commutative property of Intersection

$$A \cap B = B \cap A$$

See Example 1 at page 43

Q#2:

- i) Associativity of union.

$$A \cup (B \cup C) = (A \cup B) \cup C$$

- ii) Associativity of intersection

$$A \cap (B \cap C) = (A \cap B) \cap C$$

- iii) Distributivity of Union over intersection

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

- iv) Distributivity of intersection over union

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Do yourself

Q#3:

De Morgan's Law

$$\bullet (A \cup B)' = A' \cap B'$$

$$\bullet (A \cap B)' = A' \cup B'$$

Q#4

Same as Q#3

Note:-

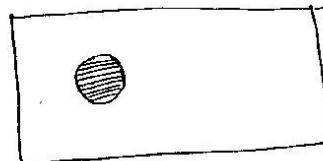
Question # 1 to 4 are not important question but you must know all these properties and De Morgan's law. Also you must know analytic proof of these properties and De Morgan's Law given at page 42. (Also by Venn Diagram)

Q#5:

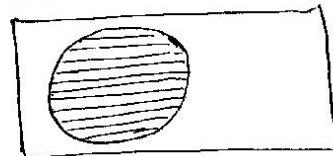
$$a) A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$b) A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

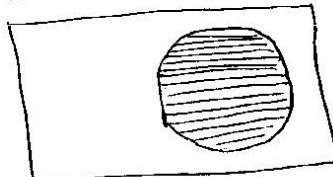
i)



B:



C:



Exercise 2.3

Proof of a)

$$\begin{aligned}
 B \cap C &: \quad \text{Diagram showing two overlapping circles shaded in the intersection area.} \\
 A \cup (B \cap C) &: \quad \text{Diagram showing two overlapping circles where the intersection area is shaded.} \\
 \text{Now} & \qquad \longrightarrow \text{(i)} \\
 A \cup B &: \quad \text{Diagram showing one large circle containing the other circle, both shaded.} \\
 A \cup C &: \quad \text{Diagram showing two separate circles, one shaded and one unshaded.} \\
 (A \cup B) \cap (A \cup C) &: \quad \text{Diagram showing two overlapping circles, both shaded.} \\
 & \qquad \longrightarrow \text{(ii)}
 \end{aligned}$$

From (i) and (ii)

$$L.H.S = R.H.S$$

Proof of b)

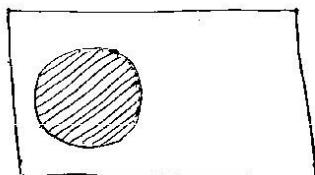
$$\begin{aligned}
 B \cup C &: \quad \text{Diagram showing two overlapping circles, both shaded.} \\
 A \cap (B \cup C) &: \quad \text{Diagram showing two overlapping circles where the intersection area is shaded.} \\
 A \cap B &: \quad \text{Diagram showing one large circle containing the other circle, only the inner circle is shaded.} \\
 A \cap C &: \quad \text{Diagram showing two separate circles, only the left circle is shaded.} \\
 (A \cap B) \cup (A \cap C) &: \quad \text{Diagram showing two overlapping circles, only the intersection area is shaded.} \\
 & \qquad \longrightarrow \text{(iv)}
 \end{aligned}$$

From (iii) and (iv)

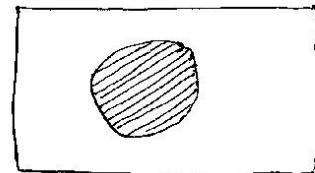
$$L.H.S = R.H.S$$

ii)

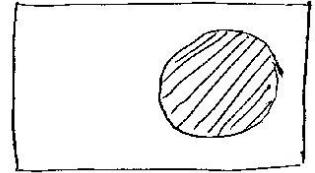
A:



B:

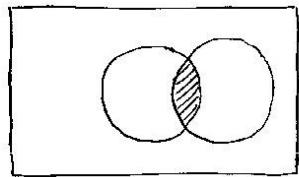


C:

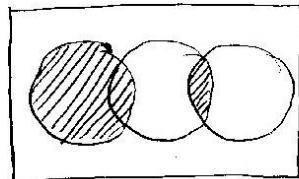


Proof of a)

$B \cap C$:

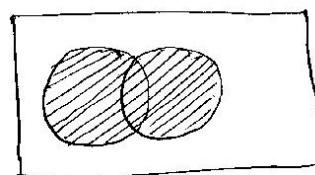


$A \cup (B \cap C)$:

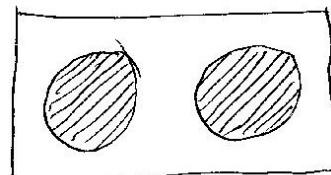


Now

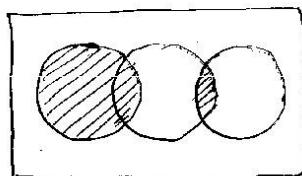
$A \cup B$:



$A \cup C$:



$(A \cup B) \cap (A \cup C)$:



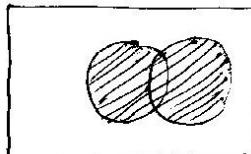
→ (vi)

From (v) and (vi)

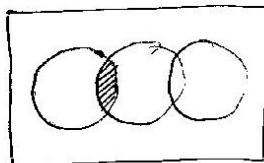
$$L.H.S = R.H.S$$

Proof of b)

$B \cup C$:



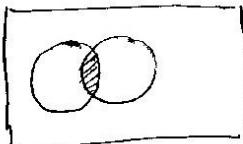
$A \cap (B \cup C)$:



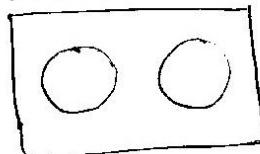
→ (vii)

Now

$A \cap B$:

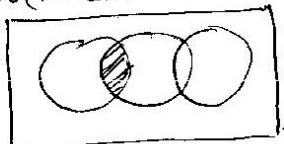


$A \cap C$:



$= \emptyset$

$(A \cap B) \cup (A \cap C)$:



→ (viii)

From (vii) & (viii)

$$L.H.S = R.H.S$$

Exercise 2.3

Q # 6 and 7:

Do yourself [Very low standard]

Q # 8:

i)

$$L.H.S = A \cap (A \cup B)$$

$$= (A \cap A) \cup (A \cap B)$$

by distributivity of intersection over union

$$= A \cup (A \cap B) \quad \because A \cap A = A$$

$$= R.H.S$$

ii) $L.H.S = A \cup (A \cap B)$

$$= (A \cup A) \cap (A \cup B)$$

by distributivity of union over intersection

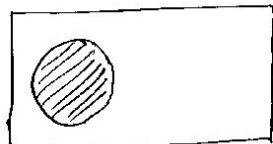
$$= A \cap (A \cup B) \quad \because A \cup A = A$$

$$= R.H.S$$

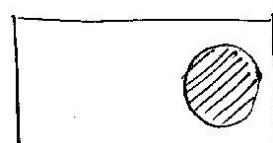
Q # 9

i) $A \cap B' = A$ if $A \cap B = \emptyset$
* Correction.

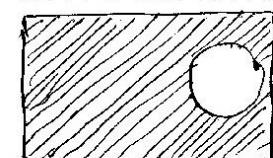
A :



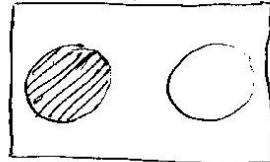
B :



B' :



$A \cap B'$:



$$= A$$

Q # 9 (ii), (iii), (iv)

Condition on A and B are not given in this question, so this is incomplete question or you have to discuss all cases

There are four cases:

a) $A \subseteq B$

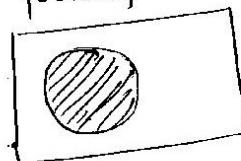
b) $B \subseteq A$

c) A and B are overlapping

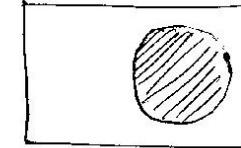
d) A and B are disjoint.
i.e. $A \cap B = \emptyset$

Here I am going to solve only case c), you can solve other cases yourself.

A :

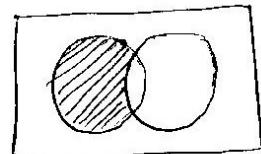


B :



iii) ii)

$$A - B =$$



$$(A-B) \cup B : \quad \boxed{\text{Two overlapping circles}} \\ = A \cup B \quad \text{proved}$$

iii)

$$A-B : \quad \boxed{\text{Two overlapping circles}}$$

$$(A-B) \cap B : \quad \boxed{\text{Two overlapping circles}}$$

$$= \emptyset \quad \text{proved}$$

iv) *Correct

$$A \cup B : \quad \boxed{\text{Two overlapping circles}}$$

$$A' : \quad \boxed{\text{One circle inside a shaded rectangle}}$$

$$A' \cap B : \quad \boxed{\text{Two overlapping circles}}$$

$$A \cup (A' \cap B) : \quad \boxed{\text{Two overlapping circles}}$$

$$= A \cup B \quad \text{proved}$$

iv) with case 2).

$$A : \quad \boxed{\text{One circle}}$$

$$B : \quad \boxed{\text{One circle}}$$

$$A \cup B : \quad \boxed{\text{One circle}}$$

$$A' : \quad \boxed{\text{One circle inside a shaded rectangle}}$$

$$A' \cap B : \quad \boxed{\text{One circle}}$$

$$A \cup (A' \cap B) : \quad \boxed{\text{One circle}}$$

$$= A \cup B \quad \text{proved}$$

—: END : —