

Exercise 2.4

Question # 1

Write the converse, inverse and contrapositive of the following conditions:

$$(i) \sim p \rightarrow q \quad (ii) q \rightarrow p \quad (iii) \sim p \rightarrow \sim q$$

Solution

(i)	Conditional:	$\sim p \rightarrow q$
	Converse:	$q \rightarrow \sim p$
	Inverse:	$p \rightarrow \sim q$
	Contrapositive:	$\sim q \rightarrow p$
(ii)	Conditional:	$q \rightarrow p$
	Converse:	$p \rightarrow q$
	Inverse:	$\sim q \rightarrow \sim p$
	Contrapositive:	$\sim p \rightarrow \sim q$
(iii)	Conditional:	$\sim p \rightarrow \sim q$
	Converse:	$\sim q \rightarrow \sim p$
	Inverse:	$p \rightarrow q$
	Contrapositive:	$q \rightarrow p$
(iv)	<i>Do yourself as above</i>	

Question # 2

Construct truth tables for the following statements:

$$(i) (p \rightarrow \sim p) \vee (p \rightarrow q) \quad (ii) (p \wedge \sim p) \rightarrow q \quad (iii) \sim (p \rightarrow q) \leftrightarrow (p \wedge \sim q)$$

Solution

$$(i) \quad \text{Statement: } (p \rightarrow \sim p) \vee (p \rightarrow q)$$

p	Q	$\sim p$	$p \rightarrow \sim p$	$p \rightarrow q$	$(p \rightarrow \sim p) \vee (p \rightarrow q)$
T	T	F	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	T	T

$$(ii) \quad \text{Statement: } (p \wedge \sim p) \rightarrow q$$

p	Q	$\sim p$	$p \wedge \sim p$	$(p \wedge \sim p) \rightarrow q$
T	T	F	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(iii)

Statement: $\sim(p \rightarrow q) \leftrightarrow (p \wedge \sim q)$

p	Q	$\sim q$	$p \rightarrow q$	$\sim(p \rightarrow q)$	$p \wedge \sim q$	$(p \wedge \sim q) \leftrightarrow \sim(p \rightarrow q)$
T	T	F	T	F	F	T
T	F	T	F	T	T	T
F	T	F	T	F	F	T
F	F	T	T	F	F	T

Tautology:

The statement, which is true for all possible values of the variables in it, is called *tautology*.

Contingency:

The statement, which is true or false depending upon the truth values of the variables involved in it, is called a *contingency*.

Absurdity or Contradiction:

The statement, which is false for all the possible values of the variables involved in it, is called an *absurdity* or *contradiction*.

Question # 3

Show that each of the following statements is a tautology:

- | | |
|---|---|
| (i) $(p \wedge q) \rightarrow p$ | (ii) $p \rightarrow (p \vee q)$ |
| (iii) $\sim(p \rightarrow q) \rightarrow p$ | (iv) $\sim q \wedge (p \rightarrow q) \rightarrow \sim p$ |

Solution

(i)

Statement: $(p \wedge q) \rightarrow p$

P	q	$p \wedge q$	$p \wedge q \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

The last column of the above table shows that the statement is true for all values of p and q thus given statement is tautology.

(ii)

Statement: $p \rightarrow (p \vee q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

The last column of the above table shows that the statement is true for all values of p and q thus given statement is tautology

(iii) Statement: $\sim(p \rightarrow q) \rightarrow p$

p	Q	$p \rightarrow q$	$\sim(p \rightarrow q)$	$\sim(p \rightarrow q) \rightarrow p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

The last column of the above table shows that the statement is true for all values of p and q thus given statement is tautology.

(iv) Statement: $\sim q \wedge(p \rightarrow q) \rightarrow \sim p$

p	Q	$\sim p$	$\sim q$	$p \rightarrow q$	$\sim q \wedge(p \rightarrow q)$	$\sim q \wedge(p \rightarrow q) \rightarrow \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

The last column of the above table shows that the statement is true for all values of p and q thus given statement is tautology.

Question # 4

Determined whether each of the following is a tautology, a contingency or an absurdity:

(i) $p \wedge \sim p$ (ii) $p \rightarrow(q \rightarrow p)$ (iii) $q \vee(\sim q \vee p)$ **Solution**

(i)

Statement: $p \wedge \sim p$

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

The last column of the above table shows that the statement is false for all values of p and q thus given statement is absurdity.

(ii)

Statement: $p \rightarrow(q \rightarrow p)$

p	q	$q \rightarrow p$	$p \rightarrow(q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

The last column of the above table shows that the statement is true for all values of p and q thus given statement is tautology.

(iii)

Statement: $q \vee(\sim q \vee p)$

P	q	$\sim q$	$\sim q \vee p$	$q \vee (\sim q \vee p)$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	T

The last column of the above table shows that the statement is true for all values of p and q thus given statement is tautology.

Question # 5

Prove that

$$p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$$

Solution Consider the truth table

P	Q	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \wedge \sim q$	$p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$	$p \vee (\sim p \wedge \sim q)$
T	T	F	F	T	F	T	T
T	F	F	T	F	F	T	T
F	T	T	F	F	F	F	F
F	F	T	T	F	T	T	T

The last two column of the above table are identical this shows that the statement $p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$ and $p \vee (\sim p \wedge \sim q)$ are equal

$$\text{i.e. } p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$$

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Book:

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