

Exercise 2.6

1. For $A = \{1, 2, 3, 4\}$ find the following relations in A. State the domain and range of each relation. Also draw the graph of each.

$$(i) R_1 = \{(x, y) \mid y = x\}$$

$$R_1 = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$$

$$\text{Dom } R_1 = \{1, 2, 3, 4\} = A$$

$$\text{Range } R_1 = \{1, 2, 3, 4\} = A$$

$$(ii) R_2 = \{(x, y) \mid y + x = 5\}$$

$$R_2 = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$$

$$\text{Dom } R_2 = \{1, 2, 3, 4\} = A$$

$$\text{Range } R_2 = \{1, 2, 3, 4\} = A$$

$$(iii) R_3 = \{(x, y) \mid x + y < 5\}$$

$$R_3 = \{(1, 2), (1, 3), (2, 1), (3, 1), (2, 2), (1, 1)\}$$

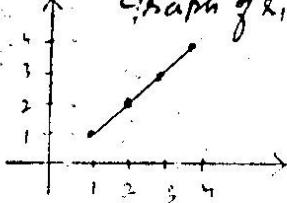
$$\text{Dom } R_3 = \{1, 2, 3\}, \text{ Range } R_3 = \{1, 2, 3\}$$

$$(iv) R_4 = \{(x, y) \mid x + y \geq 5\}$$

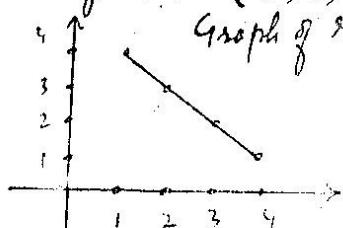
$$R_4 = \{(2, 4), (3, 3), (4, 3), (3, 4), (4, 2), (4, 4)\}$$

$$\text{Dom } R_4 = \{2, 3, 4\}, \text{ Range } R_4 = \{2, 3, 4\}$$

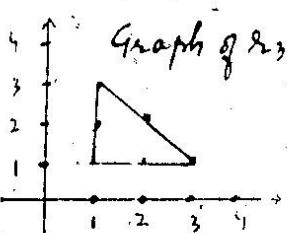
Graph of R_1



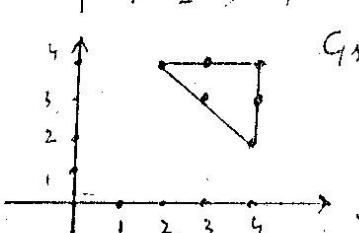
Graph of R_2



Graph of R_3



Graph of R_4



2. Repeat Q.1 when $A = \mathbb{R}$ Set of real Numbers which of the real lines are functions.

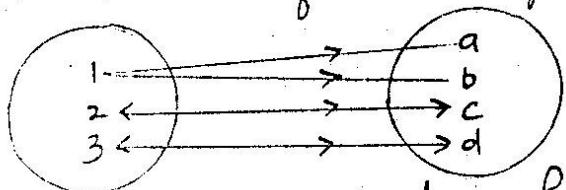
$R_1 = \{(x, y) \mid y = x\}$ is a function

$R_2 = \{(x, y) \mid x + y = 5\}$ is a function

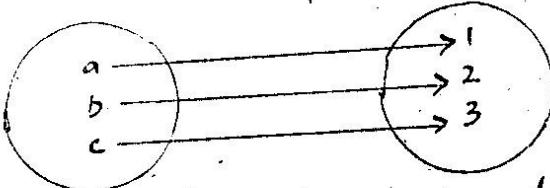
$R_3 = \{(x, y) \mid x + y < 5\}$ is not a function

because Domain is repeated

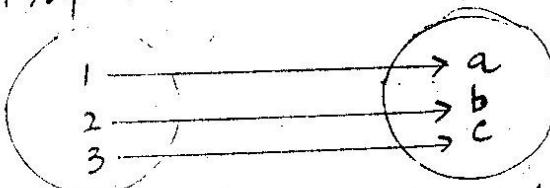
$R_4 = \{(x, y) \mid x+y > 5\}$ is not a function
because Domain is repeated.
3. which of the following diagrams represent functions and of which type.



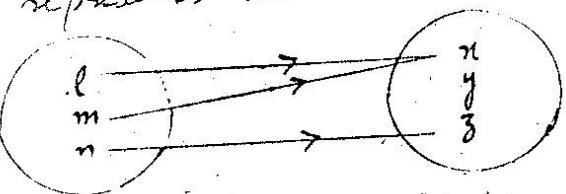
It does not represent a function.



It represents one-to-one and onto function.



It represents one-to-one and onto function.



It represents an injective (Into) function.

4. Find the inverse of each of the following relations. Tell whether each relation and its inverse is a function or not.

$$(i) R = \{(2, 1), (3, 2), (4, 3), (5, 4), (6, 5)\}$$

R is a function $\text{Dom}(R) = \{2, 3, 4, 5, 6\}$

$$R^{-1} = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$$

R^{-1} is also a function with $\text{Dom}(R^{-1}) = \{1, 2, 3, 4, 5\}$

$$(ii) R = \{(1, 3), (2, 5), (3, 7), (4, 9), (5, 11)\}$$

R is a function $\text{Dom}(R) = \{1, 2, 3, 4, 5\}$

$$R^{-1} = \{(3, 1), (5, 2), (7, 3), (9, 4), (11, 5)\}$$

R^{-1} is also a function

As domain is not repeated.

$$(iii) R = \{(x, y) | y = 2x + 3, x \in R\}$$

put $x = 0, 1, 2, \dots$

$$R = \{(0, 3), (1, 5), (2, 7), \dots\}$$

R is a function

$$R^{-1} = \{(x, y) | y = \frac{x-3}{2}, x \in R\}$$

$$R^{-1} = \{(-3, 0), (-1, 1), \dots\}$$

R^{-1} is a function

$$(iv) R = \{(x, y) | y^2 = 4ax, x \geq 0\}$$

put $x = 0, 1, 2, 3, \dots$

$$R = \{(0, 0), (1, 2\sqrt{a}), (2, \sqrt{8a}), \dots\}$$

R is a function

$$R^{-1} = \{(x, y) | y = \pm \frac{2}{a}x, x \geq 0\}$$

R^{-1} is a function

$$(v) R = \{(x, y) | x^2 + y^2 = 9, |x, y| \leq 3\}$$

$$R^{-1} = \{(x, y) | y^2 + x^2 = 9, |x, y| \leq 3\}$$

R , and R^{-1} are not functions

put $x = 0, \pm 1, \pm 2, \dots$

$$R = \{(0, 0)\}$$

As Domain is repeated.

