

## Chapter 4

## Exercise 4.1

### Quadratic Equation

An equation containing one or more terms in which the variable is raised to maximum positive power two. In general;

$ax^2 + bx + c = 0$  where  $a \neq 0$   
is called Quadratic Equation in variable  $x$ .

### 3 Methods.

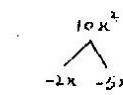
To solve Quadratic Equation there are three different methods named as;

1. Factorization method.
2. Completing Square method.
3. Quadratic Formula method.

### Example 1

Solve by Factorization  $x^2 - 7x + 10 = 0$

$$\begin{aligned} x^2 - 2x - 5x + 10 &= 0 \\ x(x-2) - 5(x-2) &= 0 \\ (x-2)(x-5) &= 0 \\ x-2 &= 0, \quad x-5 = 0 \\ \Rightarrow x = 2 &\Rightarrow x = 5 \end{aligned}$$



$$\{2, 5\}$$

### Example 2

Solve  $x^2 + 4x - 437 = 0$  by Completing Sq.

$$x^2 + 4x = 437$$

Adding  $(\frac{4}{2})^2 = (2)^2$  on both sides.

$$\begin{aligned} x^2 + 4x + (2)^2 &= 437 + (2)^2 \\ (x+2)^2 &= 437 + 4 \\ (x+2)^2 &= 441 \\ x+2 &= \pm 21 \quad \because \sqrt{441} = 21 \\ x+2 = 21 &, \quad x+2 = -21 \\ x = 21-2 &, \quad x = -21-2 \\ x = 19 &, \quad x = -23 \\ \{19, -23\} & \end{aligned}$$

### Example 3

Solve  $6x^2 + x - 15 = 0$  by Q. Formula

Comparing  $6x^2 + x - 15 = 0$   
with  $ax^2 + bx + c = 0$   
we have  $a = 6, b = 1, c = -15$

By using Quadratic Formula.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-1 \pm \sqrt{(1)^2 - 4(6)(-15)}}{2(6)} \\ x &= \frac{-1 \pm \sqrt{1+360}}{12} \Rightarrow x = \frac{-1 \pm \sqrt{361}}{12} \\ x &= \frac{-1 \pm 19}{12} \quad \because \sqrt{361} = 19 \\ x &= \frac{-1+19}{12}, \quad x = \frac{-1-19}{12} \\ x &= \frac{18}{12}, \quad x = \frac{-20}{12} \\ x &= \frac{3}{2}, \quad x = -\frac{5}{3} \quad \left\{ \frac{3}{2}, -\frac{5}{3} \right\} \end{aligned}$$

### Example 4

Solve  $8x^2 - 14x - 15 = 0$  by Quadratic formula

Comparing  $8x^2 - 14x - 15 = 0$   
with  $ax^2 + bx + c = 0$

We have  $a = 8, b = -14, c = -15$

$$\begin{aligned} \text{By using } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-(-14) \pm \sqrt{(-14)^2 - 4(8)(-15)}}{2(8)} \\ x &= \frac{14 \pm \sqrt{196+480}}{16} \Rightarrow x = \frac{14 \pm \sqrt{676}}{16} \\ x &= \frac{14 \pm 26}{16} \quad \because \sqrt{676} = 26 \\ x &= \frac{14+26}{16}, \quad x = \frac{14-26}{16} \\ x &= \frac{40}{16}, \quad x = \frac{-12}{16} \\ x &= \frac{5}{2}, \quad x = -\frac{3}{4} \quad \left\{ \frac{5}{2}, -\frac{3}{4} \right\} \end{aligned}$$

**Q.1 Derive the Quadratic Formula.**

Quadratic Equation in standard form

$$\text{is } ax^2 + bx + c = 0$$

Dividing by  $a$

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Now

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Adding  $(\frac{1}{2} \cdot \frac{b}{a})^2$  on both sides

$$x^2 + \frac{b}{a}x + (\frac{1}{2} \cdot \frac{b}{a})^2 = (\frac{1}{2} \cdot \frac{b}{a})^2 - \frac{c}{a}$$

$$x^2 + x \cdot \frac{b}{a} + (\frac{b}{2a})^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$(x+2)(x)(\frac{b}{2a}) + (\frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$$

As  $y^2 + 2y\beta + \beta^2 = (y+\beta)^2$

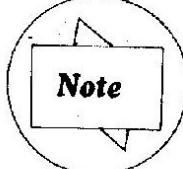
$$(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is called Quadratic Formula.



Note

In Quadratic Equation  $ax^2 + bx + c = 0$

1.  $a, b$  and  $c$  are real numbers.
2. The answer of Quadratic Equation are called its Roots.
3. Another name of Quadratic Equation is Second Degree Polynomial.

5

## EXERCISE.4.1

Solve by FACTORIZATION.

**Q.1**  $3x^2 + 4x + 1 = 0$

$$3x^2 + 3x + x + 1 = 0$$

$$3x(x+1) + 1(x+1) = 0$$

$$(x+1)(3x+1) = 0$$

$$x+1 = 0, 3x+1 = 0$$

$$x = -1, 3x = -1$$



$$\left\{ -1, -\frac{1}{3} \right\}$$

**Q.2**

$$x^2 + 7x + 12 = 0$$

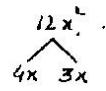
$$x^2 + 4x + 3x + 12 = 0$$

$$x(x+4) + 3(x+4) = 0$$

$$(x+3)(x+4) = 0$$

$$x+3 = 0, x+4 = 0$$

$$x = -3, x = -4$$



$$\left\{ -3, -4 \right\}$$

**Q.3**  $9x^2 - 12x - 5 = 0$

$$9x^2 + 3x - 15x - 5 = 0$$

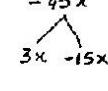
$$3x(3x+1) - 5(3x+1) = 0$$

$$(3x+1)(3x-5) = 0$$

$$3x+1 = 0, 3x-5 = 0$$

$$3x = -1, 3x = 5$$

$$x = -\frac{1}{3}, x = \frac{5}{3}$$



$$\left\{ -\frac{1}{3}, \frac{5}{3} \right\}$$

**Q.4**  $x^2 - x - 2 = 0$

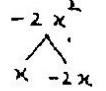
$$x^2 - x - 2 = 0$$

$$x^2 + x - 2x - 2 = 0$$

$$x(x+1) - 2(x+1) = 0$$

$$(x-2)(x+1) = 0$$

$$x-2 = 0, x+1 = 0$$



$$\left\{ 2, -1 \right\}$$

**Q.5**  $x(x+7) = (2x-1)(x+4)$

$$x^2 + 7x = 2x^2 + 8x - x - 4$$

$$x^2 + 7x = 2x^2 + 7x - 4$$

$$2x^2 - x^2 + 7x - 7x - 4 = 0$$

$$x^2 - 4 = 0$$

$$x^2 - (2)^2 = 0$$

$$(x+2)(x-2) = 0$$

$$x+2 = 0, x-2 = 0$$

$$x = -2, x = 2$$

**Q6**  $\frac{x}{x+1} + \frac{x+1}{x} = \frac{5}{2}$

Multiplying by  $2x(x+1)$

$$2x(x+1) \cdot \frac{x}{x+1} + 2x(x+1) \cdot \frac{x+1}{x} = 2x(x+1) \cdot \frac{5}{2}$$

$$2x^2 + 2(x+1)(x+1) = 5x(x+1)$$

$$2x^2 + 2(x^2 + 2x + 1) = 5x^2 + 5x$$

$$2x^2 + 2x^2 + 4x + 2 = 5x^2 + 5x$$

$$4x^2 + 4x + 2 = 5x^2 + 5x$$

$$5x^2 - 4x^2 + 5x - 4x - 2 = 0$$

$$\begin{array}{rcl} x^2 + x - 2 & = 0 \\ x - x + 2x - 2 & = 0 \\ x(x-1) + 2(x-1) & = 0 \\ (x-1)(x+2) & = 0 \end{array}$$

$$\begin{array}{l} x-1=0, \quad x+2=0 \\ \Rightarrow x=1, \quad x=-2 \quad \{1, -2\} \end{array}$$

**Q7**  $\frac{1}{x+1} + \frac{2}{x+2} = \frac{7}{x+5}$

Multiplying by  $(x+1)(x+2)(x+5)$

$$(x+1)(x+2)(x+5) \cdot \frac{1}{x+1} + (x+1)(x+2)(x+5) \cdot \frac{2}{x+2}$$

$$= (x+1)(x+2)(x+5) \cdot \frac{7}{x+5}$$

$$(x+2)(x+5) + 2(x+1)(x+5) = 7(x+1)(x+2)$$

$$x^2 + 5x + 2x + 10 + 2(x^2 + 5x + x + 5) = 7(x^2 + 2x + x + 2)$$

$$x^2 + 7x + 10 + 2x^2 + 12x + 10 = 7x^2 + 21x + 14$$

$$3x^2 + 19x + 20 = 7x^2 + 21x + 14$$

$$7x^2 - 3x^2 + 21x - 19x + 14 - 20 = 0$$

$$4x^2 + 2x - 6 = 0$$

$$2x^2 + x - 3 = 0$$

$$2x^2 - 2x + 3x - 3 = 0$$

$$2x(x-1) + 3(x-1) = 0$$

$$(x-1)(2x+3) = 0$$

$$x-1=0, \quad 2x+3=0$$

$$x=1, \quad 2x=-3$$

$$x = -\frac{3}{2} \quad \{1, -\frac{3}{2}\}$$

**Q8**  $\frac{a}{ax-1} + \frac{b}{bx-1} = a+b$

$$\begin{array}{l} \frac{a}{ax-1} - b + \frac{b}{bx-1} - a = 0 \\ \frac{a-b(ax-1)}{ax-1} + \frac{b-a(bx-1)}{bx-1} = 0 \end{array}$$

$$\begin{array}{l} \frac{a-abx+b}{ax-1} + \frac{b-abx+a}{bx-1} = 0 \\ \frac{a+b-abx}{ax-1} + \frac{a+b-abx}{bx-1} = 0 \end{array}$$

$$(a+b-abx) \left\{ \frac{1}{ax-1} + \frac{1}{bx-1} \right\} = 0$$

$$(a+b-abx) \left\{ \frac{bx-1+ax-1}{(ax-1)(bx-1)} \right\} = 0$$

$$(a+b-abx)(ax+bx-2) = 0(ax-1)(bx-1)$$

$$(a+b-abx)(ax+bx-2) = 0$$

Either  $a+b-abx = 0$  or  $ax+bx-2 = 0$

$$\Rightarrow abx = a+b, \quad (a+b)x = 2$$

$$\Rightarrow x = \frac{a+b}{ab}, \quad x = \frac{2}{a+b}$$

$$\left\{ \frac{a+b}{ab}, \frac{2}{a+b} \right\}$$

\* Solve By Completing Square.

**Q9**  $x^2 - 2x - 899 = 0$

$$x^2 - 2x = 899$$

Adding  $(\frac{-2}{2})^2 = (-1)^2$  on both sides

$$x^2 - 2x + (-1)^2 = 899 + (-1)^2$$

$$(x-1)^2 = 899 + 1$$

$$(x-1)^2 = 900$$

$$\Rightarrow x-1 = \pm 30$$

$$x-1 = 30, \quad x-1 = -30$$

$$x = 30+1, \quad x = -30+1$$

$$x = 31, \quad x = -29 \quad \{31, -29\}$$

**Q10**  $x^2 + 4x - 1085 = 0$

$$x^2 + 4x = 1085$$

Adding  $(\frac{4}{2})^2 = (2)^2$  on both sides

$$x^2 + 4x + (2)^2 = 1085 + (2)^2$$

$$\begin{aligned} (x+2)^2 &= 1085 + 4 \\ (x+2)^2 &= 1089 \\ \Rightarrow x+2 &= \pm 33, \\ x+2 &= 33, \quad x+2 = -33 \\ x &= 33-2, \quad x = -33-2 \\ x &= 31, \quad x = -35 \\ &\{ 31, -35 \} \end{aligned}$$

**Q.11**  $x^2 + 6x - 567 = 0$

$$\begin{aligned} x^2 + 6x &= 567 \\ \text{Adding } (\frac{6}{2})^2 = (3)^2 \text{ on both sides} \\ x^2 + 6x + (3)^2 &= 567 + (3)^2 \\ (x+3)^2 &= 567 + 9 \\ (x+3)^2 &= 576 \\ x+3 &= \pm 24 \\ x+3 &= 24, \quad x+3 = -24 \\ x &= 24-3, \quad x = -24-3 \\ x &= 21, \quad x = -27 \\ &\{ 21, -27 \} \end{aligned}$$

**Q.12**  $x^2 - 3x - 648 = 0$

$$\begin{aligned} x^2 - 3x &= 648 \\ \text{Adding } (\frac{3}{2})^2 \text{ on both sides} \\ x^2 - 3x + (\frac{3}{2})^2 &= 648 + (\frac{3}{2})^2 \\ (x - \frac{3}{2})^2 &= 648 + \frac{9}{4} \\ (x - \frac{3}{2})^2 &= \frac{2592+9}{4} \\ (x - \frac{3}{2})^2 &= \frac{2601}{4} \\ \Rightarrow x - \frac{3}{2} &= \pm \frac{51}{2} \\ x - \frac{3}{2} &= \frac{51}{2}, \quad x - \frac{3}{2} = -\frac{51}{2} \\ x &= \frac{51}{2} + \frac{3}{2}, \quad x = -\frac{51}{2} + \frac{3}{2} \\ x &= \frac{51+3}{2}, \quad x = -\frac{51+3}{2} \\ x &= \frac{54}{2}, \quad x = -\frac{48}{2} \\ x &= 27, \quad x = -24 \quad \{ 27, -24 \} \end{aligned}$$

**Q.13**  $x^2 - x - 1806 = 0$

$$\begin{aligned} x^2 - x &= 1806 \\ \text{Adding } (\frac{1}{2})^2 \text{ on both sides} \\ x^2 - x + (\frac{1}{2})^2 &= 1806 + (\frac{1}{2})^2 \\ (x - \frac{1}{2})^2 &= 1806 + \frac{1}{4} \\ (x - \frac{1}{2})^2 &= \frac{7225+1}{4} \\ (x - \frac{1}{2})^2 &= \frac{7226}{4} \\ \Rightarrow x - \frac{1}{2} &= \pm \frac{85}{2} \\ x - \frac{1}{2} &= \frac{85}{2}, \quad x - \frac{1}{2} = -\frac{85}{2} \\ x &= \frac{85}{2} + \frac{1}{2}, \quad x = -\frac{85}{2} + \frac{1}{2} \\ x &= \frac{85+1}{2}, \quad x = -\frac{85+1}{2} \\ x &= \frac{86}{2}, \quad x = -\frac{84}{2} \\ x &= 43, \quad x = -42 \quad \{ 43, -42 \} \end{aligned}$$

**Q.14**  $2x^2 + 12x - 110 = 0$

$$\begin{aligned} \text{Dividing by 2.} \quad x^2 + 6x - 55 &= 0 \\ x^2 + 6x &= 55 \\ \text{Adding } (\frac{6}{2})^2 = (3)^2 \text{ on both sides} \\ x^2 + 6x + (3)^2 &= 55 + (3)^2 \\ (x+3)^2 &= 55 + 9 \\ (x+3)^2 &= 64 \\ x+3 &= \pm 8 \\ x+3 &= 8, \quad x+3 = -8 \\ x &= 8-3, \quad x = -8-3 \\ x &= 5, \quad x = -11 \quad \{ 5, -11 \} \end{aligned}$$

\* Find roots by using Q. Formula.

**Q.15**  $5x^2 - 13x + 6 = 0$

$$\begin{aligned} \text{Comparing } ax^2 + bx + c &= 0 \\ \text{We have } a &= 5, b = -13, c = 6 \\ \text{Using } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-(-13) \pm \sqrt{(-13)^2 - 4(5)(6)}}{2(5)} \\ x &= \frac{13 \pm \sqrt{169 - 120}}{10} \end{aligned}$$

$$x = \frac{13 \pm \sqrt{49}}{10} \Rightarrow x = \frac{13 \pm 7}{10}$$

$$x = \frac{13+7}{10}, \quad x = \frac{13-7}{10}$$

$$x = \frac{20}{10}, \quad x = \frac{6}{10}$$

$$x = 2, \quad x = \frac{3}{5} \quad \{2, \frac{3}{5}\}$$

**Q.16**  $4x^2 + 7x - 1 = 0$

Comparing  $ax^2 + bx + c = 0$

We get  $a=4, b=7, c=-1$

$$\text{Using } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(4)(-1)}}{2(4)}$$

$$x = \frac{-7 \pm \sqrt{49 + 16}}{8}$$

$$x = \frac{-7 \pm \sqrt{65}}{8}, \quad \left\{ \frac{-7 \pm \sqrt{65}}{8} \right\}$$

**Q.17**  $15x^2 + 2ax - a^2 = 0$

Comparing  $ax^2 + bx + c = 0$

$$a=15, b=2a, c=-a^2$$

$$\text{Using } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2a \pm \sqrt{(2a)^2 - 4(15)(-a^2)}}{2(15)}$$

$$x = \frac{-2a \pm \sqrt{4a^2 + 60a^2}}{30}$$

$$x = \frac{-2a \pm \sqrt{64a^2}}{30} \Rightarrow x = \frac{-2a \pm 8a}{30}$$

$$x = \frac{-2a + 8a}{30}, \quad x = \frac{-2a - 8a}{30}$$

$$x = \frac{6a}{30}, \quad x = \frac{-10a}{30}$$

$$x = \frac{a}{5}, \quad x = -\frac{a}{3} \quad \left\{ \frac{a}{5}, -\frac{a}{3} \right\}$$

**Q.18**  $16x^2 + 8x + 1 = 0$

Comparing  $ax^2 + bx + c = 0$

We get  $a=16, b=8, c=1$

$$\text{Using, } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

8

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(16)(1)}}{2(16)}$$

$$x = \frac{-8 \pm \sqrt{64 - 64}}{32} \Rightarrow x = \frac{-8 \pm \sqrt{0}}{32}$$

$$x = \frac{-8}{32} \Rightarrow x = -\frac{1}{4} \quad \left\{ -\frac{1}{4} \right\}$$

**Q.19**

$$(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0$$

Simplyfying

$$x^2 - bx - ax + ab + x^2 - cx - bx + bc + x^2 - ax - cx + ac = 0$$

$$3x^2 - 2ax - 2bx - 2cx + ab + bc + ac = 0$$

$$3x^2 - 2(a+b+c)x + ab + bc + ac = 0$$

$$\text{Using } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-[-2(a+b+c)] \pm \sqrt{[-2(a+b+c)]^2 - 4(3)(ab+bc+ac)}}{2(3)}$$

$$x = \frac{2(a+b+c) \pm 2\sqrt{(a+b+c)^2 - 3(ab+bc+ac)}}{6}$$

$$x = \frac{(a+b+c) \pm \sqrt{a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - 3ab - 3bc}}{3}$$

$$x = \frac{(a+b+c) \pm \sqrt{a^2 + b^2 + c^2 - ab - bc - ca}}{3}$$

**Q.20**  $(a+b)x^2 + (a+2b+c)x + b+c = 0$

$$\text{Using } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(a+2b+c) \pm \sqrt{(a+2b+c)^2 - 4(a+b)(b+c)}}{2(a+b)}$$

$$x = \frac{-(a+2b+c) \pm \sqrt{(a+2b+c)^2 - 4(ab+ac+b^2+bc)}}{2(a+b)}$$

$$x = \frac{-(a+2b+c) \pm \sqrt{a^2 + 4b^2 + c^2 + 4ab + 4bc + 2ac - 4ab - 4ac - 4b^2 - 4bc}}{2(a+b)}$$

$$x = \frac{-(a+2b+c) \pm \sqrt{a^2 + c^2 - 2ac}}{2(a+b)}$$

$$\begin{aligned}
 & 9) \quad x = \frac{-(a+2b+c) \pm \sqrt{(a-c)^2}}{2(a+b)} \\
 & x = \frac{-(a+2b+c) \pm (a-c)}{2(a+b)} \\
 & x = \frac{-(a+2b+c)+a-c}{2(a+b)}, \quad x = \frac{-(a+2b+c)-a+c}{2(a+b)} \\
 & x = \frac{-a-2b-c+a-c}{2(a+b)}, \quad x = \frac{-a-2b-c-a+c}{2(a+b)} \\
 & x = \frac{-2b-2c}{2(a+b)}, \quad x = \frac{-2b-2a}{2(a+b)} \\
 & x = \frac{-2(b+c)}{2(a+b)}, \quad x = \frac{-2(a+b)}{2(a+b)} \\
 & x = -\frac{(b+c)}{a+b}, \quad x = -1 \\
 & \left\{ -\frac{(b+c)}{a+b}, -1 \right\}
 \end{aligned}$$

Put  $x^{\frac{1}{4}} = y$  then  
 $y^2 - y - 6 = 0$   
 Factorizing,  
 $y^2 + 2y - 3y - 6 = 0$   
 $y(y+2) - 3(y+2) = 0$   
 $(y+2)(y-3) = 0$   
 $y+2 = 0, \quad y-3 = 0$   
 $y = -2, \quad y = 3$   
 If  $y = -2$ , If  $y = 3$   
 Then  $x^{\frac{1}{4}} = -2$  Then  $x^{\frac{1}{4}} = 3$   
 $(x^{\frac{1}{4}})^4 = (-2)^4, \quad (x^{\frac{1}{4}})^4 = (3)^4$   
 $x = 16, \quad x = 81$   
 $\{ 16, 81 \}$   
 Tumma No 2