

Chapter 4

Exercise 4.3

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Radical Another word used for root, the sign $\sqrt{}$ is called radical sign. A number placed to the left of the sign shows the type of root e.g. $\sqrt{ } \cdot \sqrt[3]{ } \cdot \sqrt[4]{ } \cdot \dots \cdot \sqrt[n]{ }$ denote square root, Cube root, Fourth root, ..., n th roots respectively. Here numbers 2, 3, 4 and n are called index. If there is no number to the left of the sign $\sqrt{}$, then root is called square root.

Radical Equations The equations involving radical expressions of the variable are called radical equations.

For example; $\sqrt{2x+8} + \sqrt{x+5} = 7$

$$3x^2 + 2x - \sqrt{3x^2 + 2x - 1} - 7 = 0$$

$$\sqrt{x^2 + 2x - 3} + \sqrt{x^2 + 7x - 8} = \sqrt{5x^2 + 15x - 20}$$

are radical equations.

Extraneous Root A number obtained in the process of solving an equation, which is actually not a root of the given equation. In other words, we may say that, a root that does not satisfy given equation is called an extraneous root.

* There are some types of Radical Equations

Type No. 1

i) The equation of the form

$$l(ax^2 + bx) + m\sqrt{ax^2 + bx + c} = 0$$

Questions of this type are

Example 1, Q. No. 1, 2, 10

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Example.1

$$\begin{aligned} 3x^2 + 15x - 2\sqrt{x^2 + 5x + 1} &= 2 \\ 3(x^2 + 5x) - 2\sqrt{x^2 + 5x + 1} &= 2 \\ \text{put } \sqrt{x^2 + 5x + 1} &= y \\ \Rightarrow x^2 + 5x + 1 &= y^2 - 1 \end{aligned}$$

then given equation becomes

$$\begin{aligned} 3(y^2 - 1) - 2y &= 2 \\ 3y^2 - 3 - 2y - 2 &= 0 \\ 3y^2 - 2y - 5 &= 0 \\ 3y^2 + 3y - 5y - 5 &= 0 \\ 3y(y+1) - 5(y+1) &= 0 \\ (y+1)(3y-5) &= 0 \\ y+1 = 0, 3y-5 &= 0 \\ y = -1, y = 5/3 & \end{aligned}$$

If $y = -1$ then, If $y = 5/3$ then

$$\sqrt{x^2 + 5x + 1} = -1, \sqrt{x^2 + 5x + 1} = 5/3$$

Squaring both sides

$$\begin{aligned} x^2 + 5x + 1 &= 1, x^2 + 5x + 1 = \frac{25}{9} & (\sqrt{x^2 + 11x + 24})^2 &= (5x+1)^2 \\ x^2 + 5x + 1 - 1 &= 0, 9x^2 + 45x + 9 = 25 & x^2 + 11x + 24 &= 25x^2 + 10x + 1 \\ x^2 + 5x &= 0, 9x^2 + 45x + 9 - 25 = 0 & 25x^2 - x^2 + 10x - 11x + 1 - 24 &= 0 \\ x(x+5) &= 0 & 24x^2 - x - 23 &= 0 \\ x = 0, x+5 &= 0 & 24x^2 - 24x + 23x - 23 &= 0 \\ x = -5 & & 24x(x-1) + 23(x-1) &= 0 \\ 3x(3x-1) + 16(3x-1) &= 0 & (x-1)(24x+23) &= 0 \\ (3x-1)(3x+16) &= 0 & x-1 = 0, 24x+23 &= 0 \\ 3x-1 &= 0, 3x+16 & x = 1 & \Rightarrow x = -23/24 \\ x = 1/3 & \Rightarrow x = -16/3 & \end{aligned}$$

On checking we found that 0 and -5 not satisfy given equation, so 0 and -5 are extraneous roots which can not be written in solution set.

So solution set is $\{1/3, -16/3\}$

Type No.2

When given equation is of the form

$$\sqrt{x+a} + \sqrt{x+b} = \sqrt{x+c}$$

such as. Example. 2, Q. No. 3, 4, 5

Example.2 $\sqrt{x+8} + \sqrt{x+3} = \sqrt{12x+13}$

Squaring both sides

$$\begin{aligned} (\sqrt{x+8} + \sqrt{x+3})^2 &= (\sqrt{12x+13})^2 \\ (\sqrt{x+8})^2 + (\sqrt{x+3})^2 + 2\sqrt{x+8}\sqrt{x+3} &= 12x+13 \\ x+8 + x+3 + 2\sqrt{(x+8)(x+3)} &= 12x+13 \\ 2x+11 + 2\sqrt{x^2 + 11x + 24} &= 12x+13 \\ 2\sqrt{x^2 + 11x + 24} &= 12x-2x+13-11 \\ 2\sqrt{x^2 + 11x + 24} &= 10x+2 \\ 2\sqrt{x^2 + 11x + 24} &= 2(5x+1) \\ \sqrt{x^2 + 11x + 24} &= 5x+1 \end{aligned}$$

Squaring again

$$\begin{aligned} (\sqrt{x^2 + 11x + 24})^2 &= (5x+1)^2 \\ x^2 + 11x + 24 &= 25x^2 + 10x + 1 \\ 25x^2 - x^2 + 10x - 11x + 1 - 24 &= 0 \\ 24x^2 - x - 23 &= 0 \\ 24x^2 - 24x + 23x - 23 &= 0 \\ 24x(x-1) + 23(x-1) &= 0 \\ (x-1)(24x+23) &= 0 \\ x-1 = 0, 24x+23 &= 0 \\ x = 1 & \Rightarrow x = -23/24 \end{aligned}$$

On checking we found that $-23/24$ is an extraneous root.

Hence solution set is $\{1\}$.
OR. S.S. $\{1\}$.

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Type No. 3

When given equation is of the form

$$\sqrt{ax^2+bx+c} + \sqrt{px^2+qx+r} = \sqrt{mx^2+nx+n}$$

such as. Example 3, Q. No. 7, 8, 9

Example.3

$$\sqrt{x^2+4x-21} + \sqrt{x^2-x-6} = \sqrt{6x^2-5x-39}$$

Factorizing the expression under radical sign.

$$\sqrt{x^2+7x-21} + \sqrt{x^2+2x-3x-6} = \sqrt{6x^2-18x+13x-39}$$

On checking $-5/3$ is to be found as an extraneous root.

$$\sqrt{(x+7)(x-3)} + \sqrt{x(x+2)-3(x+2)} = \sqrt{6(x-3)+13(x+3)}$$

so S.S is $\{3, 2\}$.

$$\sqrt{(x-3)(x+7)} + \sqrt{(x-3)(x+2)} - \sqrt{(x-3)(6x+13)} = 0$$

$$\sqrt{x-3} \left\{ \sqrt{x+7} + \sqrt{x+2} - \sqrt{6x+13} \right\} = 0$$

$$\sqrt{x-3} = 0, \quad \sqrt{x+7} + \sqrt{x+2} - \sqrt{6x+13} = 0$$

$$x-3 = 0, \quad \sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$$

$$x=3 \quad \text{Squaring on both sides}$$

$$(\sqrt{x+7} + \sqrt{x+2})^2 = (\sqrt{6x+13})^2$$

$$(\sqrt{x+7})^2 + (\sqrt{x+2})^2 + 2\sqrt{x+7}\sqrt{x+2} = 6x+13$$

$$x+7 + x+2 + 2\sqrt{(x+7)(x+2)} = 6x+13$$

$$2x+9 + 2\sqrt{x^2+2x+7x+14} = 6x+13$$

$$2\sqrt{x^2+9x+14} = 6x-2x+13-9$$

$$2\sqrt{x^2+9x+14} = 4x+4$$

$$\sqrt{x^2+9x+14} = 2x+2$$

Squaring again

$$(\sqrt{x^2+9x+14})^2 = (2x+2)^2$$

$$x^2+9x+14 = 4x^2+8x+4$$

$$4x^2 - x^2 + 8x - 9x + 4 - 14 = 0$$

$$3x^2 - x - 10 = 0$$

$$3x^2 - 6x + 5x - 10 = 0$$

$$3x(x-2) + 5(x-2) = 0$$

$$(x-2)(3x+5) = 0$$

$$x-2 = 0, \quad 3x+5 = 0$$

$$x = 2, \quad x = -5/3$$

Type No.4

When given equations are of the form

$$\sqrt{ax^2+bx+c} + \sqrt{px^2+qx+r} = mx+n$$

such as Example 4, Q. 6, 11, 12

Example.4

$$\sqrt{3x^2-7x-30} - \sqrt{2x^2-7x-5} = x-5$$

$$\text{put } \sqrt{3x^2-7x-30} = a, \quad \sqrt{2x^2-7x-5} = b$$

Then given equation becomes

$$a - b = x - 5 \rightarrow ①$$

To find the values of a and b we find $a^2 - b^2$ as;

$$a^2 - b^2 = (\sqrt{3x^2-7x-30})^2 - (\sqrt{2x^2-7x-5})^2$$

$$a^2 - b^2 = 3x^2 - 7x - 30 - (2x^2 - 7x - 5)$$

$$a^2 - b^2 = 3x^2 - 7x - 30 - 2x^2 + 7x + 5$$

$$a^2 - b^2 = x^2 - 25$$

$$a^2 - b^2 = (x)^2 - (5)^2$$

$$\therefore a^2 - b^2 = (a-b)(a+b)$$

$$(a-b)(a+b) = (x-5)(x+5) \rightarrow ②$$

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Putting value $a - b = x - 5$ from eq ①
in eq ②

$$(x-5)(a+b) = (x-5)(x+5)$$

$$\rightarrow a+b = x+5 \rightarrow ③$$

Adding ① and ③

$$\begin{array}{r} a-b = x-5 \\ a+b = x+5 \\ \hline 2a = 2x \end{array}$$

$$\rightarrow a = x$$

Putting value of a in equation ①

$$x - b = x - 5$$

$$\rightarrow b = 5$$

If $a = x$ then

$$\sqrt{3x^2 - 7x - 30} = x$$

Squaring

$$3x^2 - 7x - 30 = x^2$$

$$3x^2 - x^2 - 7x - 30 = 0$$

$$2x^2 - 7x - 30 = 0$$

Solving any one

$$2x^2 - 7x - 30 = 0$$

$$2x^2 - 12x + 5x - 30 = 0$$

$$2x(x-6) + 5(x-6) = 0$$

$$(x-6)(2x+5) = 0$$

$$x-6=0, 2x+5=0$$

$$x=6, x=-\frac{5}{3}$$

If $b = 5$ then

$$\sqrt{2x^2 - 7x - 5} = 5$$

Squaring

$$2x^2 - 7x - 5 = 25$$

$$2x^2 - 7x - 5 - 25 = 0$$

$$2x^2 - 7x - 30 = 0$$

EXERCISE 4.3

$$Q.1 \quad 3x^2 + 2x - \sqrt{3x^2 + 2x - 1} = 7$$

$$\text{put } \sqrt{3x^2 + 2x - 1} = y$$

$$\rightarrow 3x^2 + 2x - 1 = y^2$$

$$3x^2 + 2x = y^2 + 1$$

Then given equation takes form

$$y^2 + 1 - y = 7$$

$$y^2 - y + 1 - 7 = 0$$

$$y^2 - y - 6 = 0$$

$$y^2 + 2y - 3y - 6 = 0$$

$$y(y+2) - 3(y+2) = 0$$

$$(y+2)(y-3) = 0$$

$$y+2=0, y-3=0$$

$$y=-2, y=3$$

If $y = -2$ Then, If $y = 3$ Then

$$\sqrt{3x^2 + 2x - 1} = -2 \quad \sqrt{3x^2 + 2x - 1} = 3$$

$$3x^2 + 2x - 1 = 4 \quad 3x^2 + 2x - 1 = 9$$

$$3x^2 + 2x - 4 = 0 \quad 3x^2 + 2x - 9 = 0$$

$$3x^2 + 2x - 5 = 0 \quad 3x^2 + 2x - 10 = 0$$

$$3x^2 - 3x + 5x - 5 = 0 \quad 3x^2 + 2x - 10 = 0$$

$$3x(x-1) + 5(x-1) = 0 \quad \text{Using } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$(x-1)(3x+5) = 0 \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4(3)(-1)}}{2(3)}$$

$$x-1=0, 3x+5=0 \quad x-1=0, 3x+5=0$$

$$x=1, x=-\frac{5}{3} \quad x=1, x=-\frac{5}{3}$$

$$x = \frac{-2 \pm \sqrt{4+120}}{6}$$

$$x = \frac{-2 \pm \sqrt{124}}{6}$$

$$x = \frac{-2 \pm \sqrt{4x31}}{6} \Rightarrow x = \frac{-2 \pm 2\sqrt{31}}{6}$$

$$x = \frac{2(-1 \pm \sqrt{31})}{6} \Rightarrow x = \frac{-1 \pm \sqrt{31}}{3}$$

On checking we found that 1 and $-\frac{5}{3}$ are extraneous roots. Hence

$$S.S \text{ is } \left\{ \frac{-1 \pm \sqrt{31}}{3} \right\}$$

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Q.2 $x - \frac{x}{2} - 7 = x - 3\sqrt{2x^2 - 3x + 2}$
Multiplying by 2

$$2x^2 - x - 14 = 2x - 6\sqrt{2x^2 - 3x + 2}$$

$$2x^2 - x - 2x - 14 + 6\sqrt{2x^2 - 3x + 2} = 0$$

$$2x^2 - 3x - 14 + 6\sqrt{2x^2 - 3x + 2} = 0$$

put $\sqrt{2x^2 - 3x + 2} = y$

$$\Rightarrow 2x^2 - 3x + 2 = y^2$$

$$2x^2 - 3x = y^2 - 2$$

Given equation takes form

$$y^2 - 2 - 14 + 6y = 0$$

$$y^2 + 6y - 16 = 0$$

$$y^2 - 2y + 8y - 16 = 0$$

$$y(y-2) + 8(y-2) = 0$$

$$(y-2)(y+8) = 0$$

$$y-2 = 0, \quad y+8 = 0$$

$$y = 2, \quad y = -8$$

If $y = 2$ then

$$\sqrt{2x^2 - 3x + 2} = 2$$

Squaring

$$2x^2 - 3x + 2 = 4$$

$$2x^2 - 3x + 2 - 4 = 0$$

$$2x^2 - 3x - 2 = 0$$

$$2x^2 + x - 4x - 2 = 0$$

$$x(2x+1) - 2(2x+1) = 0$$

$$(x-2)(2x+1) = 0$$

$$x-2=0, \quad 2x+1=0$$

$$x=2, \quad x=-\frac{1}{2}$$

$$x = \frac{3 \pm \sqrt{505}}{4}$$

On checking we found $\frac{3 \pm \sqrt{505}}{4}$ is an extraneous root.

Hence S.S. $\{2, -\frac{1}{2}\}$

Q.3 $\sqrt{2x+8} + \sqrt{x+5} = 7$
Squaring both sides

$$(\sqrt{2x+8} + \sqrt{x+5})^2 = 7^2$$

$$(\sqrt{2x+8})^2 + (\sqrt{x+5})^2 + 2\sqrt{2x+8}\sqrt{x+5} = 49$$

$$2x+8 + x+5 + 2\sqrt{(2x+8)(x+5)} = 49$$

$$3x+13 + 2\sqrt{2x^2 + 18x + 40} = 49$$

$$2\sqrt{2x^2 + 18x + 40} = 49 - 13 - 3x$$

$$2\sqrt{2x^2 + 18x + 40} = 36 - 3x$$

Squaring both sides again

$$4(2x^2 + 18x + 40) = (36 - 3x)^2$$

$$8x^2 + 72x + 160 = 1296 + 9x^2 - 216x$$

$$9x^2 - 8x^2 - 216x - 72x + 1296 - 160 = 0$$

$$x^2 - 288x + 1136 = 0$$

$$x^2 - 4x - 284x + 1136 = 0$$

$$x(x-4) - 284(x-4) = 0$$

$$(x-4)(x-284) = 0$$

$$x-4=0, \quad x-284=0$$

$$x=4, \quad x=284$$

On checking we found 284 is an extraneous root. So

S.S. is $\{4\}$

Q.4 $\sqrt{3x+4} = 2 + \sqrt{2x-4}$

$$\sqrt{3x+4} - \sqrt{2x-4} = 2$$

Squaring both sides

$$(\sqrt{3x+4} - \sqrt{2x-4})^2 = 2^2$$

$$(\sqrt{3x+4})^2 + (\sqrt{2x-4})^2 - 2\sqrt{3x+4}\sqrt{2x-4} = 4$$

$$3x+4 + 2x-4 - 2\sqrt{(3x+4)(2x-4)} = 4$$

$$5x - 2\sqrt{6x^2 - 12x + 8x - 16} = 4$$

$$\begin{aligned} -2\sqrt{6x^2-4x-16} &= 4-5x \\ \text{Squaring again} \\ 4(6x^2-4x-16) &= (4-5x)^2 \\ 24x^2-16x-64 &= 16+25x^2-40x \\ 25x^2-24x^2+40x+16x+16+64 &= 0 \\ x^2-24x+80 &= 0 \\ x^2-4x-20x+80 &= 0 \\ x(x-4)-20(x-4) &= 0 \\ (x-4)(x-20) &= 0 \\ x-4=0, & x-20=0 \\ x=4, & x=20 \end{aligned}$$

On checking we found that no root is an extraneous root.

Hence S.S. {4, 20}

Q.5 $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$

Squaring on both sides.

$$\begin{aligned} (\sqrt{x+7} + \sqrt{x+2})^2 &= (\sqrt{6x+13})^2 \\ (\sqrt{x+7})^2 + (\sqrt{x+2})^2 + 2\sqrt{(x+7)(x+2)} &= 6x+13 \\ x+7+x+2+2\sqrt{(x+7)(x+2)} &= 6x+13 \\ 2x+9+2\sqrt{x^2+2x+7x+14} &= 6x+13 \end{aligned}$$

$$2\sqrt{x^2+9x+14} = 6x-2x+13-9$$

$$2\sqrt{x^2+9x+14} = 4x+4$$

$$\sqrt{x^2+9x+14} = 2x+2$$

Squaring again

$$\begin{aligned} (\sqrt{x^2+9x+14})^2 &= (2x+2)^2 \\ x^2+9x+14 &= 4x^2+8x+4 \\ 4x^2-x^2+8x-9x+4-14 &= 0 \\ 3x^2-x-10 &= 0 \\ 3x^2-6x+5x-10 &= 0 \\ 3x(x-2)+5(x-2) &= 0 \\ (x-2)(3x+5) &= 0 \end{aligned}$$

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$$\begin{aligned} x-2 &= 0, \quad 3x+5=0 \\ x=2 &, \quad x=-5/3 \end{aligned}$$

On checking we found that $-5/3$ is an extraneous root. Hence
S.S. { }.

Q6 $\sqrt{x^2+x+1} - \sqrt{x^2+x-1} = 1$

put $\sqrt{x^2+x+1} = a, \sqrt{x^2+x-1} = b$

Given equation takes form

$$a-b=1 \rightarrow ①$$

To find 'a' and 'b' we find a^2-b^2 as

$$a^2-b^2 = (\sqrt{x^2+x+1})^2 - (\sqrt{x^2+x-1})^2$$

$$a^2-b^2 = x^2+x+1 - (x^2+x-1)$$

$$a^2-b^2 = x^2+x+1 - x^2-x+1$$

$$a^2-b^2 = 2 \quad \because a^2-b^2=(a-b)(a+b)$$

$$(a-b)(a+b)=2 \rightarrow ②$$

Putting value of $a-b$ from ① in ②

$$1(a+b) = 2 \Rightarrow a+b=2 \rightarrow ③$$

Adding ① and ③

$$a-b=1$$

$$a+b=2$$

$$2a=3 \Rightarrow a=\frac{3}{2}$$

Putting values of 'a' in ①

$$\frac{3}{2}-1=b \Rightarrow b=\frac{3-2}{2}=\frac{1}{2}$$

If $a=\frac{3}{2}$ then If $b=\frac{1}{2}$ then

$$\sqrt{x^2+x+1} = \frac{3}{2} \quad \sqrt{x^2+x-1} = \frac{1}{2}$$

$$2\sqrt{x^2+x+1} = 3 \quad 2\sqrt{x^2+x-1} = 1$$

Squaring

$$4(x^2+x+1) = 9 \quad 4(x^2+x-1) = 1$$

$$4x^2+4x+4-9=0 \quad 4x^2+4x-4-1=0$$

$$4x^2+4x-5=0 \quad 4x^2+4x-5=0$$

Solving any one

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$$\begin{aligned} & 4x^2 + 4x - 5 = 0 \\ \text{Using } & x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ & x = \frac{-4 \pm \sqrt{(4)^2 - 4(4)(-5)}}{2(4)} \\ & x = \frac{-4 \pm \sqrt{16+80}}{8} \Rightarrow x = \frac{-4 \pm \sqrt{96}}{8} \\ & x = \frac{-4 \pm \sqrt{16 \times 6}}{8} \Rightarrow x = \frac{-4 \pm 4\sqrt{6}}{8} \\ & x = \frac{4(-1 \pm \sqrt{6})}{8} \Rightarrow x = \frac{-1 \pm \sqrt{6}}{2} \end{aligned}$$

On checking we found that no root is an extraneous root. So

$$\text{S.S. } \left\{ \frac{-1 \pm \sqrt{6}}{2} \right\}$$

Q.7

$$\sqrt{x^2+2x-3} + \sqrt{x^2+7x-8} = \sqrt{5(x^2+3x-4)}$$

Factorizing the expressions under the radical sign.

$$\sqrt{x^2-x+3x-3} + \sqrt{x^2-x+8x-8} = \sqrt{5(x-1)(x+4)}$$

$$\sqrt{x(x-1)+3(x-1)} + \sqrt{x(x-1)+8(x-1)} = \sqrt{5[x(x-1)+4(x-1)]}$$

$$\sqrt{(x-1)(x+3)} + \sqrt{(x-1)(x+8)} = \sqrt{5(x-1)(x+4)}$$

$$\sqrt{(x-1)(x+3)} + \sqrt{(x-1)(x+8)} - \sqrt{(x-1)(x+4)} \cdot 5 = 0$$

$$\sqrt{x-1} \left\{ \sqrt{x+3} + \sqrt{x+8} - \sqrt{5(x+4)} \right\} = 0$$

$$\sqrt{x-1} = 0, \sqrt{x+3} + \sqrt{x+8} - \sqrt{5(x+4)} = 0$$

$$\sqrt{x-1} = 0, \sqrt{x+3} + \sqrt{x+8} = \sqrt{5(x+4)}$$

$$\sqrt{x-1} = 0, \text{ Squaring both sides}$$

$$(\sqrt{x+3} + \sqrt{x+8})^2 = (\sqrt{5(x+4)})^2$$

$$\sqrt{x+3}^2 + (\sqrt{x+8})^2 + 2\sqrt{x+3}\sqrt{x+8} = 5(x+4)$$

$$x+3 + x+8 + 2\sqrt{(x+3)(x+8)} = 5x+20$$

$$2x+11 + 2\sqrt{x^2+11x+24} = 5x+20$$

$$2\sqrt{x^2+11x+24} = 5x-2x+20-11$$

$$2\sqrt{x^2+11x+24} = 3x+9$$

Squaring again

$$4(x^2+11x+24) = (3x+9)^2$$

$$4x^2+44x+96 = 9x^2+54x+81$$

$$9x^2-4x^2+54x-44x+81-96 = 0$$

$$5x^2+10x-15 = 0$$

Dividing by 5

$$x^2+2x-3 = 0$$

$$x^2-x+3x-3 = 0$$

$$x(x-1)+3(x-1) = 0$$

$$(x-1)(x+3) = 0$$

$$x-1 = 0, x+3 = 0$$

$$x = 1, x = -3$$

On checking we found that no root is an extraneous root. So

$$\text{S.S. u } \left\{ 1, -3 \right\}$$

Q.8

$$\sqrt{2x^2-5x-3} + 3\sqrt{2x+1} = \sqrt{2x^2+25x+12}$$

Factorizing the expressions under the radical sign.

$$\sqrt{2x^2+x-6x-3} + 3\sqrt{2x+1} = \sqrt{2x^2+x+24x+12}$$

$$\sqrt{x(2x+1)-3(2x+1)} + 3\sqrt{2x+1} = \sqrt{x(2x+1)+12(2x+1)}$$

$$\sqrt{(2x+1)(x-3)} + 3\sqrt{2x+1} - \sqrt{(2x+1)(x+12)} = 0$$

$$\sqrt{2x+1} \left\{ \sqrt{x-3} + 3 - \sqrt{x+12} \right\} = 0$$

$$\sqrt{2x+1} = 0, \sqrt{x-3} + 3 - \sqrt{x+12} = 0$$

$$2x+1 = 0, \sqrt{x-3} + 3 = \sqrt{x+12}$$

$$x = -\frac{1}{2}, \text{ Squaring}$$

$$(\sqrt{x-3} + 3)^2 = (\sqrt{x+12})^2$$

$$(\sqrt{x-3})^2 + (3)^2 + 2(3)\sqrt{x-3} = x+12$$

$$x-3 + 9 + 6\sqrt{x-3} = x+12$$

$$x+6 + 6\sqrt{x-3} = x+12$$

$$6\sqrt{x-3} = x-x+12-6$$

$$6\sqrt{x-3} = 6 \Rightarrow \sqrt{x-3} = 1$$

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$x - 3 = 1 \Rightarrow x = 4$
On checking we found no root is an
extraneous root. Hence
S.S is $\{-\frac{1}{2}, 4\}$

Q.9

$$\sqrt{3x^2 - 5x + 2} + \sqrt{6x^2 - 11x + 5} = \sqrt{5x^2 - 9x + 4}$$

Factorizing the expressions under radical sign.

$$\sqrt{3x^2 - 3x - 2x + 2} + \sqrt{6x^2 - 6x - 5x + 5} = \sqrt{5x^2 - 5x - 4x + 4}$$

$$\sqrt{3x(x-1) - 2(x-1)} + \sqrt{6x(x-1) - 5(x-1)} = \sqrt{5x(x-1) - 4(x-1)}$$

$$\sqrt{(x-1)(3x-2)} + \sqrt{(x-1)(6x-5)} = \sqrt{(x-1)(5x-4)}$$

$$\sqrt{(x-1)(3x-2)} + \sqrt{(x-1)(6x-5)} - \sqrt{(x-1)(5x-4)} = 0$$

$$\sqrt{x-1} \{ \sqrt{3x-2} + \sqrt{6x-5} - \sqrt{5x-4} \} = 0$$

$$\sqrt{x-1} = 0, \quad \sqrt{3x-2} + \sqrt{6x-5} - \sqrt{5x-4} = 0$$

$$x-1 = 0, \quad \sqrt{3x-2} + \sqrt{6x-5} = \sqrt{5x-4}$$

$$x = 1, \quad \text{Squaring both sides}$$

$$(\sqrt{3x-2} + \sqrt{6x-5})^2 = (\sqrt{5x-4})^2$$

$$(\sqrt{3x-2})^2 + (\sqrt{6x-5})^2 + 2\sqrt{3x-2}\sqrt{6x-5} = 5x-4$$

$$3x-2 + 6x-5 + 2\sqrt{(3x-2)(6x-5)} = 5x-4$$

$$9x-7 + 2\sqrt{18x^2 - 15x - 12x + 10} = 5x-4$$

$$2\sqrt{18x^2 - 27x + 10} = 5x - 9x + 4 + 7$$

$$2\sqrt{18x^2 - 27x + 10} = 3 - 4x$$

Squaring again

$$4(18x^2 - 27x + 10) = (3 - 4x)^2$$

$$72x^2 - 108x + 40 = 9 + 16x^2 - 24x$$

$$72x^2 - 16x^2 - 108x + 24x + 40 - 9 = 0$$

$$56x^2 - 84x + 31 = 0$$

$$\text{Using } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-84) \pm \sqrt{(-84)^2 - 4(56)(31)}}{2(56)}$$

$$x = \frac{84 + \sqrt{7056 - 6944}}{112} \Rightarrow x = \frac{84 \pm \sqrt{112}}{112}$$

$$x = \frac{84 \pm \sqrt{112}}{112}$$

$$x = \frac{84 + 4\sqrt{7}}{112} \Rightarrow x = 4\left(\frac{21 \pm \sqrt{7}}{112}\right)$$

$$x = \frac{21 \pm \sqrt{7}}{28}$$

Q.10

On checking we found that no root is an extraneous root. So

$$\text{S.S is } \left\{ \frac{21 \pm \sqrt{7}}{28} \right\}$$

$$(x+4)(x+1) = \sqrt{x^2 + 2x - 15} + 3x + 31$$

$$x^2 + x + 4x + 4 = \sqrt{x^2 + 2x - 15} + 3x + 31$$

$$x^2 + 5x + 4 = \sqrt{x^2 + 2x - 15} + 3x + 31$$

$$x^2 + 5x - 3x + 4 - 31 - \sqrt{x^2 + 2x - 15} = 0$$

$$x^2 + 2x - 27 - \sqrt{x^2 + 2x - 15} = 0$$

$$\text{put } \sqrt{x^2 + 2x - 15} = y$$

$$\Rightarrow x^2 + 2x - 15 = y^2$$

$$x^2 + 2x = y^2 + 15$$

Given equation takes form

$$y^2 + 15 - 27 - y = 0$$

$$y^2 - y - 12 = 0$$

$$y^2 + 3y - 4y - 12 = 0$$

$$y(y+3) - 4(y+3) = 0$$

$$(y-4)(y+3) = 0$$

$$y-4=0, \quad y+3=0$$

$$y=4, \quad y=-3$$

If $y=4$ then

$$\sqrt{x^2 + 2x - 15} = 4$$

$$x^2 + 2x - 15 = 16$$

$$x^2 + 2x - 15 - 16 = 0$$

$$x^2 + 2x - 31 = 0$$

$$a=1, b=2, c=-31$$

$$\text{Using } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{4 - 4(-31)}}{2}$$

$$\sqrt{x^2 + 2x - 15} = -3$$

$$x^2 + 2x - 15 = 9$$

$$x^2 + 2x - 15 - 9 = 0$$

$$x^2 + 2x - 24 = 0$$

$$x - 4x + 6x - 24 = 0$$

$$x(x-4) + 6(x-4) = 0$$

$$\begin{aligned}
 x &= \frac{-2 \pm \sqrt{(12)^2 - 4(1)(-3)}}{2(1)} & (x-4)(x+6) = 0 \\
 x &= \frac{-2 \pm \sqrt{4+124}}{2} & x-4 = 0, x+6 = 0 \\
 x &= \frac{-2 \pm \sqrt{128}}{2} & x = 4, x = -6 \\
 x &= \frac{-2 \pm 8\sqrt{2}}{2} & \Rightarrow x = \frac{-2 \pm \sqrt{64 \times 2}}{2} \\
 x &= \frac{-2 \pm 8\sqrt{2}}{2} & \Rightarrow x = \frac{2(-1 \pm 4\sqrt{2})}{2}
 \end{aligned}$$

$$x = -1 \pm 4\sqrt{2}$$

On checking we found that 4 and -6 are extraneous roots. Hence

$$\text{S.S. is } \{-1 \pm 4\sqrt{2}\}$$

Q.11

$$\sqrt{3x^2 - 2x + 9} + \sqrt{3x^2 - 2x - 4} = 13$$

$$\text{put } \sqrt{3x^2 - 2x + 9} = a, \sqrt{3x^2 - 2x - 4} = b$$

Given equation takes form

$$a+b = 13 \rightarrow ①$$

To find a and b we find $a^2 - b^2$ as

$$a^2 - b^2 = (\sqrt{3x^2 - 2x + 9})^2 - (\sqrt{3x^2 - 2x - 4})^2$$

$$a^2 - b^2 = 3x^2 - 2x + 9 - (3x^2 - 2x - 4)$$

$$a^2 - b^2 = 3x^2 - 2x + 9 - 3x^2 + 2x + 4$$

$$a^2 - b^2 = 13 \quad \therefore a^2 - b^2 = (a+b)(a-b)$$

$$+b)(a-b) = 13 \rightarrow ②$$

Putting value of a+b from ① in ②

$$13(a-b) = 13 \rightarrow a-b = 1 \rightarrow ③$$

Adding ① and ③

$$a+b = 13$$

$$a-b = 1$$

$$\underline{2a = 14} \rightarrow a = 7$$

Putting value of a in ①

$$7+b = 13 \rightarrow b = 6$$

a = 7 then

If b = 6 then

$$x^2 - 2x + 9 = 7 \quad \sqrt{3x^2 - 2x - 4} = 6$$

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$$\begin{aligned}
 3x^2 - 2x + 9 &= 49 & 3x^2 - 2x - 4 &= 36 \\
 3x^2 - 2x + 9 - 49 &= 0 & 3x^2 - 2x - 4 - 36 &= 0 \\
 3x^2 - 2x - 40 &= 0 & 3x^2 - 2x - 40 &= 0
 \end{aligned}$$

Solving any one

$$3x^2 - 2x - 40 = 0$$

$$3x^2 - 12x + 10x - 40 = 0$$

$$3x(x-4) + 10(x-4) = 0$$

$$(x-4)(3x+10) = 0$$

$$x-4 = 0, \quad 3x+10 = 0$$

$$x = 4, \quad x = -10/3$$

On checking we found no root is extraneous root. So.

$$\text{S.S. is } \{4, -10/3\}$$

Q.12

$$\sqrt{5x^2 + 7x + 2} - \sqrt{4x^2 + 7x + 18} = x-4$$

$$\text{put } \sqrt{5x^2 + 7x + 2} = a, \sqrt{4x^2 + 7x + 18} = b$$

Given equation takes form

$$a - b = x - 4 \rightarrow ①$$

To find a and b we find $a^2 - b^2$ as

$$a^2 - b^2 = (\sqrt{5x^2 + 7x + 2})^2 - (\sqrt{4x^2 + 7x + 18})^2$$

$$a^2 - b^2 = 5x^2 + 7x + 2 - (4x^2 + 7x + 18)$$

$$a^2 - b^2 = 5x^2 + 7x + 2 - 4x^2 - 7x - 18$$

$$a^2 - b^2 = x^2 - 16$$

$$a^2 - b^2 = (x-4)^2 \quad \therefore a^2 - b^2 = (a-b)(a+b)$$

$$(a-b)(a+b) = (x-4)(x+4) \rightarrow ②$$

Putting value of a-b from ① in ②

$$(x-4)(a+b) = (x-4)(x+4)$$

$$\rightarrow a+b = x+4 \rightarrow ③$$

Adding eq ① and ③

$$a-b = x-4$$

$$a+b = x+4$$

$$\underline{2a = 2x} \rightarrow a = x$$

Putting value of a in ①

$$x-b = x-4 \rightarrow b = 4$$

$$\begin{array}{l|l}
 \text{If } a = x \text{ then} & \text{If } b = 4 \text{ then} \\
 \sqrt{5x^2 + 7x + 2} = x & \sqrt{4x^2 + 7x + 18} = 4 \\
 5x^2 + 7x + 2 = x^2 & 4x^2 + 7x + 18 = 16 \\
 5x^2 - x^2 + 7x + 2 = 0 & 4x^2 + 7x + 18 - 16 = 0 \\
 4x^2 + 7x + 2 = 0 & 4x^2 + 7x + 2 = 0
 \end{array}$$

Solving any one

$$4x^2 + 7x + 2 = 0$$

$$\text{Using } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(4)(2)}}{2(4)}$$

$$x = \frac{-7 \pm \sqrt{49 - 32}}{8} \Rightarrow x = \frac{-7 \pm \sqrt{17}}{8}$$

On checking we found that no root
is extraneous root. Hence (ii)

$$\text{s.s is } \left\{ \frac{-7 \pm \sqrt{17}}{8} \right\}$$