

## Chapter 4

## Exercise 4.8

$$\begin{aligned}x^2 + [-(2x+4)]^2 + 4x &= 1 \\x^2 + (2x+4)^2 + 4x - 1 &= 0 \\x^2 + 4x^2 + 16x + 16 + 4x - 1 &= 0 \\5x^2 + 20x + 15 &= 0\end{aligned}$$

Dividing by 5 we get.

$$\begin{aligned}x^2 + 4x + 3 &= 0 \\x^2 + x + 3x + 3 &= 0 \\x(x+1) + 3(x+1) &= 0 \\(x+1)(x+3) &= 0 \\x+1=0, x+3=0 \rightarrow x &= -1, x = -3\end{aligned}$$

If $x=-1$ then from eq. ③	If $x=-3$ then from eq. ③
$y = -(2(-1)+4)$	$y = -(2(-3)+4)$
$y = -(-2+4)$	$y = -(-6+4)$
$y = -2$	$y = 2$

Hence  $\{(-1, -2), (-3, 2)\}$

### EXERCISE 4.8

**Q.1**  $2x-y=4 \rightarrow ①$

$$2x^2 - 4xy - y^2 = 6 \rightarrow ②$$

From ①  $y = 2x-4 \rightarrow ③$

Putting value of  $y$  in equation ②

$$\begin{aligned}2x^2 - 4x(2x-4) - (2x-4)^2 &= 6 \\2x^2 - 8x^2 + 16x - (4x^2 - 16x + 16) - 6 &= 0 \\-6x^2 + 16x - 4x^2 + 16x - 16 - 6 &= 0 \\-10x^2 + 32x - 22 &= 0\end{aligned}$$

Dividing by -2 we get

$$\begin{aligned}5x^2 - 16x - 11 &= 0 \\5x^2 - 5x - 11x + 11 &= 0 \\5x(x-1) - 11(x-1) &= 0 \\(x-1)(5x-11) &= 0 \\x-1=0, 5x-11=0 \rightarrow x &= 1, x = 11/5\end{aligned}$$

If $x=1$ then from ③	If $x=11/5$ then from ③
$y = 2(1)-4$	$y = 2(11/5) - 4$
$y = 2-4 = -2$	$y = 22/5 - 4$
	$y = \frac{22-20}{5} = 2/5$

$$\{(1, -2), (11/5, 2/5)\}$$

**Q.2**  $x+y=5 \rightarrow ①$

$$x^2 + 2y^2 = 17 \rightarrow ②$$

From ①  $y = 5-x \rightarrow ③$

Putting value of  $y$  in equation ②

$$\begin{aligned}x^2 + 2(5-x)^2 &= 17 \\x^2 + 2(25 - 10x + x^2) - 17 &= 0 \\x^2 + 50 - 20x + 2x^2 - 17 &= 0 \\3x^2 - 20x + 33 &= 0 \\3x^2 - 9x - 11x + 33 &= 0 \\3x(x-3) - 11(x-3) &= 0 \\(x-3)(3x-11) &= 0 \\x-3=0, 3x-11 &= 0 \\x=3, x &= 11/3\end{aligned}$$

If $x=3$ then from ③	If $x=11/3$ then from ③
$y = 5-3$	$y = 5 - 11/3$
$y = 2$	$y = \frac{15-11}{3}$
	$y = 4/3$

$$\{(3, 2), (11/3, 4/3)\}$$

**Q.3**  $3x+2y=7 \rightarrow ①$

$$3x^2 = 25 + 2y^2$$

or  $3x^2 - 2y^2 = 25 \rightarrow ②$

From ①  $2y = 7 - 3x \rightarrow y = \frac{7-3x}{2}$

Putting value of  $y$  in eq. ②

$$\begin{aligned}3x^2 - 2\left(\frac{7-3x}{2}\right)^2 &= 25 \\3x^2 - 2\left(\frac{49-42x+9x^2}{4}\right) - 25 &= 0 \\3x^2 - \left(\frac{9x^2 - 42x + 49}{2}\right) - 25 &= 0 \\6x^2 - (9x^2 - 42x + 49) - 50 &= 0 \\6x^2 - 9x^2 + 42x - 49 - 50 &= 0 \\-3x^2 + 42x - 99 &= 0\end{aligned}$$

Dividing by -3 we get

$$x^2 - 14x + 33 = 0$$

$$x^2 - 3x - 11x + 33 = 0$$

$$x(x-3) - 11(x-3) = 0$$

$$(x-3)(x-11) = 0$$

$$x-3=0, x-11=0 \rightarrow x=3, x=11$$

If  $x=3$  then from ③

$$y = \frac{7-3(3)}{2}$$

$$y = \frac{7-9}{2} = \frac{-2}{2} = -1$$

$$\{(3, -1), (11, -13)\}$$

Q.4

$$x+y=5 \rightarrow ①$$

$$\frac{2}{x} + \frac{3}{y} = 2$$

Multiplying by  $xy$  we get

$$\text{or } 2y+3x = 2xy \rightarrow ②$$

$$\text{From } ① y = 5-x \rightarrow ③$$

Putting value of  $y$  in equation ②

$$2(5-x)+3x = 2x(5-x)$$

$$10-2x+3x = 10x-2x^2$$

$$10+x = 10x-2x^2$$

$$2x^2-10x+x+10 = 0$$

$$2x^2-9x+10 = 0$$

$$2x^2-4x-5x+10 = 0$$

$$2x(x-2)-5(x-2) = 0$$

$$(x-2)(2x-5) = 0$$

$$x-2=0, 2x-5=0$$

$$x=2, x=\frac{5}{2}$$

If  $x=2$  then from ③

$$y=5-2$$

$$y=3$$

$$y=\frac{5-5}{2}$$

$$y=\frac{10-5}{2}$$

$$y=\frac{5}{2}$$

If  $x=\frac{5}{2}$  then from ③

$$y=5-\frac{5}{2}$$

$$y=\frac{10-5}{2}$$

$$y=\frac{5}{2}$$

$$\{(2, 3), (\frac{5}{2}, \frac{5}{2})\}$$

Q.5

$$x+y = a+b \rightarrow ①$$

$$\frac{a}{x} + \frac{b}{y} = 2$$

$$\text{or } ay+bx = 2xy \rightarrow ②$$

$$\text{From } ① y = a+b-x \rightarrow ③$$

Putting value of  $y$  in eq. ②

$$a(a+b-x)+bx = 2x(a+b-x)$$

$$a^2+ab-ax+bx = 2ax+2bx-2x^2$$

$$2x^2-2ax-2bx-ax+bx+a^2+ab=0$$

$$2x^2-3ax-bx+a^2+ab=0$$

$$\text{Using } x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(3a+b) \pm \sqrt{(-(3a+b))^2 - 4(2)(a^2+ab)}}{2(2)}$$

$$x = \frac{(3a+b) \pm \sqrt{(3a+b)^2 - 8(a^2+ab)}}{4}$$

$$x = \frac{(3a+b) \pm \sqrt{9a^2+b^2+6ab-8a^2-8ab}}{4}$$

$$x = \frac{(3a+b) \pm \sqrt{a^2+b^2-2ab}}{4}$$

$$x = \frac{(3a+b) \pm \sqrt{(a-b)^2}}{4}$$

$$x = \frac{(3a+b) \pm (a-b)}{4}$$

$$x = \frac{3a+b+a-b}{4}, x = \frac{3a+b-(a-b)}{4}$$

$$x = \frac{4a}{4}, x = \frac{3a+b-a+b}{4}$$

$$x=a, x = \frac{2a+2b}{4} \rightarrow x = \frac{a+b}{2}$$

If  $x=a$  then from ③

$$\text{If } x=\frac{a+b}{2} \text{ then from ③}$$

$$y=a+b-a$$

$$y=b$$

$$y=\frac{2a+2b-a-b}{2}$$

$$y=\frac{a+b}{2}$$

$$\{(a, b), \left(\frac{a+b}{2}, \frac{a+b}{2}\right)\}$$

**Q.6**  $3x + 4y = 25 \rightarrow (1)$

$$\frac{3}{x} + \frac{4}{y} = 2$$

$$\text{or } 3y + 4x = 2xy \rightarrow (2)$$

$$\text{From (1) } 4y = 25 - 3x \Rightarrow y = \frac{25 - 3x}{4} \rightarrow (3)$$

Putting value of  $y$  in equation (2)

$$3\left(\frac{25 - 3x}{4}\right) + 4x = 2x\left(\frac{25 - 3x}{4}\right)$$

Multiplying by 4 we get

$$-3(25 - 3x) + 4x = 2x(25 - 3x)$$

$$75 - 9x + 4x = 50x - 6x^2$$

$$75 + 7x = 50x - 6x^2 \rightarrow (1)$$

$$(1) \quad 6x^2 - 50x + 7x + 75 = 0$$

$$6x^2 - 43x + 75 = 0$$

$$6x^2 - 18x - 25x + 75 = 0$$

$$6x(x-3) - 25(x-3) = 0$$

$$(x-3)(6x-25) = 0$$

$$x-3=0 \quad , \quad 6x-25=0$$

$$x=3 \quad , \quad x=\frac{25}{6}$$

If  $x=3$  then from (3) If  $x=\frac{25}{6}$  then from (3)

$$y = \frac{25 - 3(3)}{4} \quad y = \frac{25 - 3(\frac{25}{6})}{4}$$

$$y = \frac{25 - 9}{4} \quad y = \frac{1}{4}(25 - \frac{25}{2})$$

$$y = \frac{16}{4} = 4 \quad y = \frac{1}{4} \cdot \frac{25}{2} = \frac{25}{8}$$

$$\{(3, 4), (\frac{25}{6}, \frac{25}{8})\}$$

**Q.7**

$$(x-3)^2 + y^2 = 5$$

$$x^2 - 6x + 9 + y^2 = 5 \rightarrow (1)$$

$$\rightarrow x^2 + y^2 - 6x + 4 = 0 \rightarrow (1)$$

$$2x = y + 6 \rightarrow (2)$$

$$\text{From (2) } y = 2x - 6 \rightarrow (3)$$

Putting value of  $y$  in equation (1)

$$x^2 + (2x-6)^2 - 6x + 4 = 0$$

$$x^2 + 4x^2 - 24x + 36 - 6x + 4 = 0$$

$$5x^2 - 30x + 40 = 0$$

Dividing by 5 we get

$$x^2 - 6x + 8 = 0$$

$$x^2 - 2x - 4x + 8 = 0$$

$$x(x-2) - 4(x-2) = 0$$

$$(x-2)(x-4) = 0$$

$$x-2=0 \quad , \quad x-4=0$$

$$x=2 \quad , \quad x=4$$

If  $x=2$  then from (3) If  $x=4$  then from (3)

$$y = 2(2) - 6 = -2 \quad y = 2(4) - 6 = 2$$

$$y = 4 - 6 = -2 \quad y = 8 - 6 = 2$$

$$\{(2, -2), (4, 2)\}$$

**Q.8**

$$(x+3)^2 + (y+1)^2 = 5$$

$$\text{or } x^2 + 6x + 9 + y^2 + 2y + 1 - 5 = 0$$

$$x^2 + y^2 + 6x + 2y + 5 = 0 \rightarrow (1)$$

$$x^2 + y^2 + 2x = 9$$

$$\text{or } x^2 + y^2 + 2x - 9 = 0$$

Subtracting eq. (2) from eq. (1)

$$(1) \text{ or } y^2 + 6x + 2y + 5 = 0$$

$$x^2 + y^2 + 2x - 9 = 0$$

$$4x - 4y + 14 = 0$$

$$4x - 4y + 14 = 0$$

$$x - y + \frac{7}{2} = 0$$

$$\rightarrow y = x + \frac{7}{2} \rightarrow (3)$$

Putting value of  $y$  in equation (2)

$$x^2 + (2x+7)^2 + 2x - 9 = 0$$

$$\begin{aligned}x^2 + 4x^2 + 28x + 49 + 2x - 9 &= 0 \\5x^2 + 30x + 40 &= 0\end{aligned}$$

Dividing by 5 we get

$$\begin{aligned}x^2 + 6x + 8 &= 0 \\x^2 + 2x + 4x + 8 &= 0 \\x(x+2) + 4(x+2) &= 0 \\(x+2)(x+4) &= 0\end{aligned}$$

$$x+2 = 0 \quad , \quad x+4 = 0$$

$$x = -2, \quad x = -4$$

$$\text{If } x = -2 \text{ then from } ③ \quad \text{If } x = -4 \text{ then from } ③$$

$$y = 2(-2) + 7$$

$$y = -4 + 7$$

$$y = 3$$

$$\left\{ (-2, 3), (-4, 1) \right\}$$

$$\begin{aligned}\textbf{Q.9} \quad x^2 + (4+1)^2 &= 18 \\x^2 + y^2 + 2y + 1 - 18 &= 0 \\x^2 + y^2 + 2y - 17 &= 0 \rightarrow ①\end{aligned}$$

$$(x+2)^2 + y^2 = 21$$

$$x^2 + 4x + 4 + y^2 - 21 = 0$$

$$x^2 + y^2 + 4x - 17 = 0 \rightarrow ②$$

Subtracting eq ② from eq ①

$$x^2 + y^2 + 2y - 17 = 0$$

$$\underline{x^2 + y^2 + 4x - 17 = 0}$$

$$\underline{2y - 4x = 0}$$

$$2y - 4x = 0 \rightarrow 2x = 4x \rightarrow y = 2x$$

Putting value of y in equation ②

$$x^2 + (2x)^2 + 4x - 17 = 0$$

$$x^2 + 4x^2 + 4x - 17 = 0$$

$$5x^2 + 4x - 17 = 0$$

$$\text{Using } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(5)(-17)}}{2(5)}$$

$$x = \frac{-4 \pm \sqrt{16 + 340}}{10}$$

$$x = \frac{-4 \pm \sqrt{356}}{10} \rightarrow x = \frac{-4 \pm \sqrt{4 \times 89}}{10}$$

$$x = \frac{-4 \pm 2\sqrt{89}}{10} \rightarrow x = \frac{2(-2 \pm \sqrt{89})}{10}$$

$$x = \frac{-2 \pm \sqrt{89}}{5}$$

$$\text{If } x = \frac{-2 + \sqrt{89}}{5} \text{ then from } ③ \quad \text{If } x = \frac{-2 - \sqrt{89}}{5}$$

$$y = 2\left(\frac{-2 + \sqrt{89}}{5}\right)$$

$$y = \frac{-4 + 2\sqrt{89}}{5}$$

$$\left\{ \left( \frac{-2 + \sqrt{89}}{5}, \frac{-4 + 2\sqrt{89}}{5} \right), \left( \frac{-2 - \sqrt{89}}{5}, \frac{-4 - 2\sqrt{89}}{5} \right) \right\}$$

$$\textbf{Q.10} \quad x^2 + y^2 + 6x = 1 \rightarrow ①$$

$$x^2 + y^2 + 2(x+y) = 3$$

$$\text{or } x^2 + y^2 + 2x + 2y = 3 \rightarrow ②$$

Subtracting equation ② from ①

$$x^2 + y^2 + 6x = 1$$

$$\underline{x^2 + y^2 + 2x + 2y = 3}$$

$$4x - 2y = -2$$

Dividing by 2 we get

$$2x - y = -1$$

$$\rightarrow y = 2x + 1$$

Putting values of y in equation ①

$$x^2 + (2x+1)^2 + 6x = 1$$

$$x^2 + 4x^2 + 4x + 1 + 6x - 1 = 0$$

$$5x^2 + 10x = 0$$

$$5x(x+2) = 0$$

$$5x = 0 \rightarrow x+2=0$$

$$x = 0 \rightarrow x = -2$$

If  $x = 0$  then from ③ If  $x = -2$  then from ③

$$y = 2(0) + 1 \quad y = 2(-2) + 1$$

$$y = 1 \quad y = -4 + 1 = -3$$

$$\left\{ (0, 1), (-2, -3) \right\}$$