Question # 1

Write the first four terms of the following sequences, if

(i) $a_n = 2n-3$ (ii) $a_n = (-1)^n n^2$

(viii) $a_n = na_{n-1}$, $a_1 = 1$ (x) $a_n = \dfrac{1}{a+(n-1)d}$

Solution

(i) $a_n = 2n-3$

Put $n = 1$

$a_1 = 2(1)-3 \quad \Rightarrow \quad a_1 = 2-3 = -1$

Put $n = 2$

$a_2 = 2(2)-3 \quad \Rightarrow \quad a_2 = 4-3 = 1$

Put $n = 3$

$a_3 = 2(3)-3 \quad \Rightarrow \quad a_3 = 6-3 = 3$

Put $n = 4$

$a_4 = 2(4)-3 \quad \Rightarrow \quad a_4 = 8-3 = 5$

Hence -1, 1, 3 , 5 are the first four term of the sequence.

(ii) $a_n = (-1)^n n^2$

Put $n = 1$

$a_1 = (-1)^1 (1)^2 \quad \Rightarrow \quad a_1 = (-1)(1) = -1$

Put $n = 2$

$a_2 = (-1)^2 (2)^2 \quad \Rightarrow \quad a_2 = (1)(4) = 4$

Put $n = 3$

$a_3 = (-1)^3 (3)^2 \quad \Rightarrow \quad a_3 = (-1)(9) = -9$

Put $n = 4$

$a_4 = (-1)^4 (4)^2 \quad \Rightarrow \quad a_4 = (1)(16) = 16$

Hence -1, 4, -9, 16 are the first four terms of the sequence.

(iii), (iv), (v) and (vi) Do yourself as above.

(vii) $a_n - a_{n-1} = n+2$, $a_1 = 2$

Put $n = 2$

$a_2 - a_{2-1} = 2+2 \quad \Rightarrow \quad a_2 - a_1 = 4 \quad \Rightarrow \quad a_2 = 4 + a_1 = 4+2 = 6 \quad \because a_1 = 2$

Put $n = 3$

$a_3 - a_{3-1} = 3+2 \quad \Rightarrow \quad a_3 - a_2 = 5 \Rightarrow a_3 = 5 + a_2 = 5+6 = 11 \quad \because a_2 = 6$

Put $n = 4$

$a_4 - a_{4-1} = 4+2 \quad \Rightarrow \quad a_4 - a_3 = 6 \Rightarrow a_4 = 6 + a_3 = 6+11 = 17 \quad \because a_3 = 11$

Hence 2, 6, 11, 17 are the first four terms of the sequence.

(viii) $a_n = na_{n-1}$, $a_1 = 1$

Put $n = 2$

$a_2 = (2)a_{2-1} \quad \Rightarrow \quad a_2 = 2a_1 = 2(1) = 2 \quad \because a_1 = 1$

Put $n = 3$

$a_3 = (3)a_{3-1} \quad \Rightarrow \quad a_3 = 3a_2 = 3(2) = 6 \quad \because a_2 = 2$

Put $n = 4$

$a_4 = (4)a_{4-1} \quad \Rightarrow \quad a_4 = 4a_3 = 4(6) = 24 \quad \because a_3 = 6$

Hence 1, 2, 6, 24 are the first four terms of the sequence.

(ix) *Same as above*

(x) $a_n = \dfrac{1}{a+(n-1)d}$

Put $n = 1$

$a_1 = \dfrac{1}{a+(1-1)d} \Rightarrow a_1 = \dfrac{1}{a+(0)d} = \dfrac{1}{a+0} = \dfrac{1}{a}$

Put $n = 2$

$a_2 = \dfrac{1}{a+(2-1)d} \Rightarrow a_2 = \dfrac{1}{a+(1)d} = \dfrac{1}{a+d}$

Put $n = 3$

$a_3 = \dfrac{1}{a+(3-1)d} \Rightarrow a_3 = \dfrac{1}{a+(2)d} = \dfrac{1}{a+2d}$

Put $n = 4$

$a_4 = \dfrac{1}{a+(4-1)d} \Rightarrow a_4 = \dfrac{1}{a+(3)d} = \dfrac{1}{a+3d}$

Hence $\dfrac{1}{a}, \dfrac{1}{a+d}, \dfrac{1}{a+2d}, \dfrac{1}{a+3d}$ are the first four terms of the sequence.

Question # 2

Find the next two terms of the following sequences;

(i) $2, 6, 11, 17, ... a_7$ (ii) $1, 3, 12, 60, ... a_6$

(iii) $1, \dfrac{3}{2}, \dfrac{5}{4}, \dfrac{7}{8}, ... a_7$ (iv) $1, 1, -3, 5, -7, 9, ... a_8$

(v) $1, -3, 5, -7, 9, -11, ... a_8$

Solution

(i) $2, 6, 11, 17, ... a_7$

We see that the successive difference of the given terms are 4, 5, 6 and conclude that sequence of the differences is 4, 5, 6, 7, 8, 9,……….

So $a_5 = 17 + 7 = 24$, $a_6 = 24 + 8 = 32$ and $a_7 = 32 + 9 = 41$

Thus the required term is $a_7 = 41$

(ii) $1, 3, 12, 60, ... a_6$

We see that the successive multiplying factor are 3, 4, 5 and conclude that the sequence of multiplying factors is 3, 4, 5, 6, 7, 8, 9…………

So $a_5 = 60 \times 6 = 360$, $a_6 = 360 \times 7 = 2520$

Thus the required term is $a_6 = 2520$

(iii) $1, \dfrac{3}{2}, \dfrac{5}{4}, \dfrac{7}{8}, ... a_7$

The successive terms in numerator are 1, 3, 5, 7, ………….. , which are the consecutive odd numbers and next terms are 9, 11, 13.

And the successive terms in denominators are 1, 2, 4, 8, ………… with common ratio 2, so the next terms are 16, 32, 64.

Thus the required term is $a_7 = \dfrac{13}{64}$

(iv) $1, 1, -3, 5, -7, 9, ... a_8$ **Correction*

We see that the common difference of odd terms is -4, so $a_7 = -7 + (-4) = -11$

And the common difference of even terms is 4, so $a_8 = 9 + 4 = 13$

Thus the required term is $a_8 = 13$

(v) $1, -3, 5, -7, 9, -11, \ldots\ldots\ldots\ldots a_8$

We see that the common difference of odd terms is 4, so $a_7 = 9 + 4 = 13$.

And the common difference of the even terms is -4, so $a_8 = -11 + (-4) = -15$

Thus the required term is $a_8 = -15$

Question # 3

Find the next two terms of the following sequences;

(i) $7, 9, 12, 16, \ldots\ldots\ldots\ldots$ (ii) $1, 3, 7, 15, 31, \ldots\ldots\ldots\ldots$

(iii) $-1, 2, 12, 40, \ldots\ldots\ldots$ (iv) $1, -3, 5, -7, 9, -11, \ldots\ldots\ldots\ldots$

Solution

(i) $7, 9, 12, 16, \ldots\ldots\ldots\ldots$

We see that the sequence of the successive difference is 2, 3, 4, …….. so the next two differences are 5 and 6.

Thus the next two terms are 16 + 5 = 21 and 21 + 6 = 27.

(ii) $1, 3, 7, 15, 31, \ldots\ldots\ldots\ldots$

We see that the sequence of the successive difference is 2, 4, 8, 16, ……… so the next two differences are 32 and 64.

Thus the next two terms of the sequence are 31 + 32 = 63 and 63 + 64 = 127.

(iii) $-1, 2, 12, 40, \ldots\ldots\ldots$

The sequence of the above terms can be written as

$$-1\times1,\ 1\times2,\ 3\times4,\ 5\times8, \ldots\ldots\ldots\ldots$$

So the next two terms are $7\times16 = 112$ and $9\times32 = 288$.

(iv) $1, -3, 5, -7, 9, -11, \ldots\ldots\ldots\ldots$

We see that the common difference of odd terms is 4, so $a_7 = 9 + 4 = 13$.

And the common difference of the even terms is -4, so $a_8 = -11 + (-4) = -15$

Thus the next two terms are 13 and -15.