Question # 1

Find A.M. between

(i) $3\sqrt{5}$ and $5\sqrt{5}$ (ii) $x-3$ and $x+5$

(iii) $1-x+x^2$ and $1+x+x^2$

Solution

(i) $3\sqrt{5}$ and $5\sqrt{5}$

Here $a=3\sqrt{5}$ and $b=5\sqrt{5}$, so

$$\text{A.M.} = \frac{a+b}{2} = \frac{3\sqrt{5}+5\sqrt{5}}{2} = \frac{8\sqrt{5}}{2} = 4\sqrt{5}$$

(ii) $x-3$ and $x+5$

Here $a=x-3$ and $b=x+5$

$$\text{A.M.} = \frac{a+b}{2} = \frac{x-3+x+5}{2} = \frac{2x+2}{2} = x+1$$

(iii) $1-x+x^2$ and $1+x+x^2$

Here $a=1-x+x^2$ and $b=1+x+x^2$

$$\text{A.M.} = \frac{a+b}{2} = \frac{1-x+x^2+1+x+x^2}{2} = \frac{2+2x^2}{2} = 1+x^2$$

Question # 2

If $5,8$ are two A.Ms between a and b, find a and b.

Solution

Since 5, 8 are two A.Ms between a and b.

Therefor a, 5, 8, b are in A.P.

Here $a_1 = a$ and $d = 8-5 = 3$

Now $a_2 = a_1 + d \Rightarrow 5 = a+3 \Rightarrow 5-3 = a \Rightarrow \boxed{a=2}$

Also $a_4 = a_1 + 3d \Rightarrow b = 2+3(3) \Rightarrow \boxed{b=11}$

Question # 3

Find 6 A.Ms between 2 and 5 .

solution

Let A_1, A_2, A_3, A_4, A_5 and A_6 are six A.Ms between 2 and 5.

Then $2, A_1, A_2, A_3, A_4, A_5, A_6, 5$ are in A.P.

Here $a_1 = 2$ and $a_8 = 5$

$$\Rightarrow a_1 + 7d = 5 \Rightarrow 2+7d = 5$$

$$\Rightarrow 7d = 5-2 \Rightarrow 7d = 3 \Rightarrow d = 3/7$$

So $A_1 = a_2 = a_1 + d = 2 + 3/7 = 17/7$

$$A_2 = a_3 = a_1 + 2d = 2 + 2\left(\tfrac{3}{7}\right) = 2 + \tfrac{6}{7} = \tfrac{20}{7}$$
$$A_3 = a_4 = a_1 + 3d = 2 + 3\left(\tfrac{3}{7}\right) = 2 + \tfrac{9}{7} = \tfrac{23}{7}$$
$$A_4 = a_5 = a_1 + 4d = 2 + 4\left(\tfrac{3}{7}\right) = 2 + \tfrac{12}{7} = \tfrac{26}{7}$$
$$A_5 = a_6 = a_1 + 5d = 2 + 5\left(\tfrac{3}{7}\right) = 2 + \tfrac{15}{7} = \tfrac{29}{7}$$
$$A_6 = a_7 = a_1 + 6d = 2 + 6\left(\tfrac{3}{7}\right) = 2 + \tfrac{18}{7} = \tfrac{32}{7}$$

Hence $\frac{17}{7}, \frac{20}{7}, \frac{23}{7}, \frac{26}{7}, \frac{29}{7}, \frac{32}{7}$ are six A.Ms between 2 and 5.

Question # 4

Find 4 A.Ms between $\sqrt{2}$ and $\frac{12}{\sqrt{2}}$.

Solution

Suppose A_1, A_2, A_3 and A_4 are four A.Ms between $\sqrt{2}$ and $\frac{12}{\sqrt{2}}$.

Then $\sqrt{2}, A_1, A_2, A_3, A_4, \frac{12}{\sqrt{2}}$ are in A.P.

Here $a_1 = \sqrt{2}$ and $a_6 = \frac{12}{\sqrt{2}}$

$$\Rightarrow a_1 + 5d = \frac{12}{\sqrt{2}} \;\Rightarrow\; \sqrt{2} + 5d = \frac{12}{\sqrt{2}} \;\Rightarrow\; 5d = \frac{12}{\sqrt{2}} - \sqrt{2}$$
$$\Rightarrow 5d = \frac{12-2}{\sqrt{2}} \qquad \Rightarrow 5d = \frac{10}{\sqrt{2}}$$
$$\Rightarrow d = \frac{2}{\sqrt{2}} = \frac{\left(\sqrt{2}\right)^2}{\sqrt{2}} \Rightarrow d = \sqrt{2}$$

Now $A_1 = a_2 = a_1 + d = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$

$$A_2 = a_3 = a_1 + 2d = \sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$$
$$A_3 = a_4 = a_1 + 3d = \sqrt{2} + 3\sqrt{2} = 4\sqrt{2}$$
$$A_4 = a_5 = a_1 + 4d = \sqrt{2} + 4\sqrt{2} = 5\sqrt{2}$$

Hence $2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, 5\sqrt{2}$ are four A.Ms between $\sqrt{2}$ and $\frac{12}{\sqrt{2}}$.

Question # 5

Insert 7 A.Ms between 4 and 8.

Solution *Do yourself*

Question # 6

Find three A.Ms between 3 and 11

Solution *Do yourself*

Question # 7

Find n so that $\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}}$ may be the A.M. between a and b.

Solution

Since we know that A.M. (i)

But we have given A.M. $= \dfrac{a^n + b^n}{a^{n-1} + b^{n-1}} \cdots\cdots\cdots\cdots\cdots \text{(ii)}$

Comparing (i) and (ii)

$$\frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \frac{a+b}{2}$$

$$\Rightarrow \ 2(a^n + b^n) = (a+b)(a^{n-1} + b^{n-1}) \qquad \textit{Cross} \times \textit{ing}$$

$$\Rightarrow \ 2a^n + 2b^n = a^n + a^{n-1}b + ab^{n-1} + b^n$$

$$\Rightarrow \ 2a^n + 2b^n - a^n - b^n = a^{n-1}b + ab^{n-1}$$

$$\Rightarrow \ a^n + b^n = a^{n-1}b + ab^{n-1}$$

$$\Rightarrow \ a^n - a^{n-1}b = ab^{n-1} - b^n$$

$$\Rightarrow \ a^{n-1+1} - a^{n-1}b = ab^{n-1} - b^{n-1+1} \qquad \because \ n = n-1+1$$

$$\Rightarrow \ a^{n-1}(a-b) = b^{n-1}(a-b)$$

$$\Rightarrow \ a^{n-1} = b^{n-1}$$

$$\Rightarrow \ \frac{a^{n-1}}{b^{n-1}} = 1$$

$$\Rightarrow \ \left(\frac{a}{b}\right)^{n-1} = \left(\frac{a}{b}\right)^{0} \qquad \because \ \left(\frac{a}{b}\right)^{0} = 1$$

$$\Rightarrow \ n-1 = 0 \ \Rightarrow \ \boxed{n = 1}$$

Question # 8

Do Yourself