

QNo1 3, 6, 12, ...,  $\dots$ 

$$a_1 = 3, r = \frac{a_2}{a_1} = \frac{6}{3} = 2, n=5$$

$$\text{Since } a_n = a_1 r^{n-1}$$

$$\begin{aligned}\Rightarrow a_5 &= (3)(2)^{5-1} \\ &= (3)(2)^4 = (3)(16) \\ &= 48 \quad \text{Answe}\end{aligned}$$

QNo2 1+ $i$ , 2,  $\frac{4}{1+i}$ ,  $\dots$ 

$$a_1 = 1+i, r = \frac{a_2}{a_1} = \frac{2}{1+i}, n=11$$

$$\text{Since } a_n = a_1 r^{n-1}$$

$$\begin{aligned}\Rightarrow a_{11} &= (1+i)\left(\frac{2}{1+i}\right)^{11-1} \\ &= (1+i)\left(\frac{2}{1+i}\right)^{10} \\ &= (1+i)\left(\frac{2}{1+i} \cdot \frac{1-i}{1-i}\right)^{10} \\ &= (1+i)\left(\frac{2(1-i)}{(1+i)(1-i)}\right)^{10} \\ &= (1+i)\left(\frac{2(1-i)}{1+i}\right)^{10} \quad \because i^2 = -1 \\ &= (1+i)\left(\frac{2(1-i)}{2}\right)^{10} \\ &= (1+i)(1-i)^{10} \\ &= (1+i)[(1-i)^2]^5 \\ &= (1+i)[(1)^2 - 2(1)(i) + (i)^2]^5 \\ &= (1+i)[1 - 2i - 1]^5 \quad \because i^2 = -1 \\ &= (1+i)(-2i)^5 \\ &= (1+i)(-2i)^5(i^2)^2 \cdot 2 \\ &= (1+i)(-32)(i^4 - i^2) \\ &= (1+i)(-32)(1) \cdot 2 \\ &= -32i(1+i)$$

$$\begin{aligned}&= -32i - 32i^2 \\ &= -32i - 32(-1) = -32i + 32 \\ &= 32(-i + 1) = 32(1-i) \quad \text{Ans.}\end{aligned}$$

QNo3 1+i, 2i, ~~2-2+2i~~,  $\dots$ 

$$a_1 = 1+i, r = \frac{a_2}{a_1} = \frac{2i}{1+i}, n=12$$

$$\text{Since } a_n = a_1 r^{n-1}$$

$$\begin{aligned}\Rightarrow a_{12} &= (1+i)\left(\frac{2i}{1+i}\right)^{12-1} \\ &= (1+i)\left(\frac{2i}{1+i} \cdot \frac{1-i}{1-i}\right)^{11} \\ &= (1+i)\left(\frac{2i - 2i^2}{(1+i)(1-i)}\right)^{11} \\ &= (1+i)\left(\frac{2i + 2}{1+i}\right)^{11} \quad \because i^2 = -1 \\ &= (1+i)\left(\frac{2(i+1)}{2}\right)^{11} \\ &= (1+i)(i+1)^{11} = (1+i)^{12} \\ &= [(1+i)^2]^6 = (1+2i+i^2)^6 \\ &= (1+2i-1)^6 = (2i)^6 \\ &= 2^6 \cdot (i)^6 = 64(i^2)^3 \\ &= 64(-1)^3 = -64 \quad \text{Answe}\end{aligned}$$

QNo4 Do yourself as QNo.2 &amp; 3.

QNo.5 Here  $a_1 = 12000$ depreciation  $\approx 5\%$ 

$$\text{therefore } r = 1 - \frac{5}{100} = 1 - 0.05$$

$$= 0.95$$

$$n = 5$$

$$\text{Since } a_n = a_1 r^{n-1}$$

$$\Rightarrow a_5 = (12000)(0.95)^{5-1}$$

$$= (12000)(0.95)^4$$

$$= (12000)(0.8145)$$

$$= 9774.08$$

Thus value of automobile at the end of 4 year is 9774.08

<p>No. 6 <math>x^2 - y^2, x+y, \frac{x+y}{x-y}, \dots</math> is <math>\frac{x+y}{(x-y)^n}</math></p> <p>Here <math>a_1 = x^2 - y^2</math></p> $r = \frac{x+y}{x^2 - y^2} = \frac{x+y}{(x-y)(x+y)} = \frac{1}{x-y}$ $n = ? \rightarrow a_n = \frac{x+y}{(x-y)^n}$ <p>Since <math>a_n = a_1 r^{n-1}</math>,</p> $\Rightarrow \frac{x+y}{(x-y)^n} = (x^2 - y^2) \left(\frac{1}{x-y}\right)^{n-1}$ $\Rightarrow \frac{(x+y)}{(x-y)^n} = (x+y)(x-y) \frac{1}{(x-y)^{n-1}}$ $\Rightarrow \frac{1}{(x-y)^n} = \frac{1}{(x-y)^{n-1-1}}$ $\Rightarrow \frac{1}{(x-y)^n} = \frac{1}{(x-y)^{n-2}}$ $\Rightarrow \left(\frac{1}{x-y}\right)^n = \left(\frac{1}{x-y}\right)^{n-2}$ $\Rightarrow n = n-2 \Rightarrow n+2=n$ $\Rightarrow n = 11 \quad \text{Answer}$	$= \frac{b^2 - bc - bc + c^2}{(b-c)(a-b)} \quad \because ac = b^2 \\ \therefore ad = bc \\ bd = c^2$ $= \frac{b^2 - 2bc + c^2}{(b-c)(a-b)}$ $= \frac{(b-c)^2}{(b-c)(a-b)} = \frac{b-c}{a-b} = r \quad \text{--- (2)}$ <p>from (1) &amp; (2),  <math>r = r</math></p> <p>therefore <math>a-b, b-c, c-d</math> are in G.P.</p> <p>ii) To show <math>a^2-b^2, b^2-c^2, c^2-d^2</math> are in G.P.</p> <p>Let <math>r = \frac{b^2 - c^2}{a^2 - b^2} \quad \text{--- (1)}</math></p> <p>Also</p> $r = \frac{c^2 - d^2}{b^2 - c^2}$ $= \frac{c^2 - d^2}{b^2 - c^2} \cdot \frac{a^2 - b^2}{a^2 - b^2}$ $= \frac{a^2 c^2 - a^2 d^2 - b^2 c^2 + b^2 d^2}{(b^2 - c^2)(a^2 - b^2)}$ $= \frac{(ac)^2 - (ad)^2 - (bc)^2 + (bd)^2}{(b^2 - c^2)(a^2 - b^2)}$ $= \frac{(b^2 - c^2)(a^2 - b^2)}{(b^2 - c^2)(b^2 - c^2) \quad \text{from (i)}} \quad \text{iii) } \frac{\text{from (i)}}{\text{from (ii)}}$ $= \frac{b^4 - 2b^2c^2 + c^4}{(b^2 - c^2)(a^2 - b^2)}$ $= \frac{(b^2 - c^2)^2}{(b^2 - c^2)(b^2 - c^2)}$ $= \frac{b^2 - c^2}{a^2 - b^2} \quad \text{--- (2)}$ <p>From (1) and (2)</p> <p><math>r = r'</math></p> <p>Hence <math>a^2-b^2, b^2-c^2, c^2-d^2</math> are in G.P.</p> <p>iii) Do Yourself</p> <p>Hint: <math>r = \frac{b^2 + c^2}{a^2 + b^2} \quad \text{--- (1)}</math></p> <p>Also</p> $r = \frac{c^2 + d^2}{b^2 + c^2} = \frac{c^2 + d^2}{b^2 + c^2} \cdot \frac{a^2 + b^2}{a^2 + b^2}$ <p>(Same as ii)</p>
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No. 8  $a_1, a_1r, a_1r^2, a_1r^3, \dots$

The sequence of reciprocal of the term is

$$\frac{1}{a_1}, \frac{1}{a_1r}, \frac{1}{a_1r^2}, \dots$$

To show this is in G.P. let

$$r' = \frac{a_2}{a_1} = \frac{1/a_1 r^2}{1/a_1} = \frac{1}{a_1 r^2} = \frac{1}{a_1 r^2} \quad (i)$$

Also

$$\begin{aligned} r'_1 &= \frac{a_3}{a_2} = \frac{1/a_1 r^3}{1/a_1 r^2} = \frac{1}{a_1 r^2} \\ &= \frac{1}{r'} \quad (ii) \end{aligned}$$

From (i) & (ii)

$$r'_1 = r'$$

therefore the sequence of reciprocal of the term of G.P. is also in G.P.

When  $a_1 = \frac{2}{3}$  and  $r = \frac{2}{3}$

$$\begin{aligned} a_n &= \left(\frac{2}{3}\right) \left(\frac{2}{3}\right)^{n-1} \\ &= \left(\frac{2}{3}\right)^n \end{aligned}$$

When  $a_1 = -\frac{2}{3}$  and  $r = -\frac{2}{3}$

$$\begin{aligned} a_n &= \left(-\frac{2}{3}\right) \left(-\frac{2}{3}\right)^{n-1} \\ &= \left(-\frac{2}{3}\right)^n \text{ or } (-1)^n \left(\frac{2}{3}\right)^n \end{aligned}$$

No 10 Consider  $\frac{a_1}{r}, a_1, a_1r$

are three consecutive terms in G.P. by given condition

$$\frac{a_1}{r} + a_1 + a_1r = 26$$

$$\rightarrow a_1 \left( \frac{1}{r} + 1 + r \right) = 26$$

$$\rightarrow a_1 (1 + r + r^2) = 26r \quad \text{Xing by } r \quad (i)$$

Also we have given

$$\left(\frac{a_1}{r}\right)(a_1)(a_1r) = 216$$

$$\Rightarrow a_1^3 = 6^3 \Rightarrow a_1 = 6$$

putting in eq (i)

$$6(1+r+r^2) = 26r$$

$$\Rightarrow 6 + 6r + 6r^2 - 26r = 0$$

$$\Rightarrow 6r^2 - 20r + 6 = 0$$

$$\Rightarrow 3r^2 - 10r + 3 = 0$$

$$r = \frac{10 \pm \sqrt{100 - 4(3)(3)}}{2(3)}$$

$$= \frac{10 \pm \sqrt{64}}{6} = \frac{10 \pm 8}{6}$$

$$\begin{aligned} r &= \frac{10+8}{6} \quad \text{or} \quad r = \frac{10-8}{6} \\ &= \frac{18}{6} = 3, \quad = \frac{2}{6} = \frac{1}{3} \end{aligned}$$

When  $a_1 = 6, r = 3$

$$\frac{a_1}{r} = \frac{6}{3} = 2$$

$$a_1 = 6$$

$$a_1r = (6)(3) = 18$$

$$\text{Also } a_2 = \frac{4}{9}$$

$$\Rightarrow a_1r^{2-1} = \frac{4}{9} \Rightarrow a_1r = \frac{4}{9}$$

When  $r = \frac{2}{3}$

$$a_1 \left(\frac{2}{3}\right) = \frac{4}{9} \Rightarrow a_1 = \frac{4}{9} \cdot \frac{3}{2}$$

$$\Rightarrow a_1 = \frac{2}{3}$$

When  $r = -\frac{2}{3}$

$$a_1 \left(-\frac{2}{3}\right) = \frac{4}{9} \Rightarrow a_1 = \frac{4}{9} \left(-\frac{3}{2}\right)$$

$$\Rightarrow a_1 = -\frac{2}{3}$$

Since  $a_n = a_1r^{n-1}$

When  $a_1 = 6$ ,  $r = \frac{1}{3}$

$$\frac{a_1}{r} = \frac{6}{\frac{1}{3}} = 6 \times 3 = 18$$

$$a_1 = 6$$

$$a_1 r = 6 \times \frac{1}{3} = 2$$

Hence 2, 6, 18 OR 18, 6, 2  
are required number in G.P.

Ques 11 Let the four terms in  
G.P. be  $a_1, a_1 r, a_1 r^2, a_1 r^3$

By given condition,

$$a_1 + a_1 r + a_1 r^2 + a_1 r^3 = 80$$

$$\Rightarrow a_1(1+r+r^2+r^3) = 80$$

$$\Rightarrow a_1[1(1+r)+r^2(1+r)] = 80$$

$$\Rightarrow a_1(1+r)(1+r^2) = 80 \quad \text{(i)}$$

Also we have given

$$\frac{a_1 r + a_1 r^3}{2} = 30$$

$$\Rightarrow \frac{a_1 r(1+r^2)}{2} = 30$$

$$\Rightarrow a_1 r(1+r^2) = 60 \quad \text{(ii)}$$

From eq. (i)

$$1+r^2 = \frac{80}{a_1(1+r)} \quad \text{putting in (ii)}$$

$$\frac{a_1 r \cdot \frac{80}{a_1(1+r)}}{2} = 60$$

$$\Rightarrow \frac{80 r}{1+r} = 60 \Rightarrow 80 r = 60(1+r)$$

$$\Rightarrow 80r = 60 + 60r$$

$$\Rightarrow 80r - 60r = 60 \Rightarrow 20r = 60$$

$$\Rightarrow r = \frac{60}{20} \Rightarrow r = 3$$

Putting value of  $r$  in eq. (i)

$$a_1(1+3)(1+(3)^2) = 80$$

$$\Rightarrow a_1(4)(10) = 80 \Rightarrow 40a_1 = 80$$

$$\Rightarrow a_1 = \frac{80}{40} \Rightarrow a_1 = 2$$

So

$$a_1 r = (2)(3) = 6$$

$$a_1 r^2 = (2)(3)^2 = 2 \times 9 = 18$$

$$a_1 r^3 = (2)(3)^3 = 2 \times 27 = 54$$

Hence 2, 6, 18, 54 are required  
number

Ques 12 Since  $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$  are in G.P.

$$\therefore \text{Therefore } r = \frac{1/b}{1/a} = \frac{b}{a} \quad \text{(i)}$$

$$\text{Also } r = \frac{\frac{1}{c}}{\frac{1}{b}} = \frac{b}{c} \quad \text{(ii)}$$

from (i) & (ii)

$$r \cdot r = \frac{b}{a} \cdot \frac{b}{c} \Rightarrow r^2 = \frac{b^2}{ac}$$

$$\Rightarrow r = \pm \sqrt{\frac{b^2}{ac}} \quad \text{proved}$$

Ques 13 Let  $a-d, a, a+d$  are

three numbers in A.P.

By given condition:

$$a-d+a+a+d=21$$

$$\Rightarrow 3a = 21 \Rightarrow a = 7$$

Now  $a-d-1, a-4, a+d-3$  are  
in G.P. therefore

$$r = \frac{a_1-4}{a_1-d-1} = \frac{a_1+d-3}{a_1-4}$$

$$\Rightarrow (a_1-4)^2 = (a_1+d-3)(a_1-d-1)$$

$$\text{put } a_1 = 7$$

$$(7-4)^2 = (7+d-3)(7-d-1)$$

$$\Rightarrow (3)^2 = (d+4)(8-d)$$

$$\Rightarrow 9 = 6d + 24 - d^2 - 4d$$

$$\Rightarrow 9 - 6d - 24 + d^2 + 4d = 0$$

$$\Rightarrow d^2 - 2d - 15 = 0$$

$$\Rightarrow d^2 - 5d + 3d - 15 = 0$$

$$\Rightarrow d(d-5) + 3(d-5) = 0$$

$$\Rightarrow (d-5)(d+3) = 0$$

$$d-5 = 0, \quad d+3 = 0$$

$$d = 5, \quad d = -3$$

when  $a_1 = 7, d = 5$ ; when  $a_1 = 7, d = 3$

$$a_1-d = 7-5 = 2 \quad a_1-d = 7+3 = 10$$

$$a_1 = 7 \quad a_1 = 7$$

$$a_1+d = 7+5 = 12 \quad a_1+d = 7+3 = 10$$

hence 2, 7, 12 or 10, 7, 4 are

required number

Ques 14 Hint: Consider number

$a_1-d, a_1, a_1+d$ , then

$a_1-d+1, a_1+1, a_1+d+1$  are in G.P.

END