

Q.No 1 i) If -2 and 8 .

$$a = -2 \text{ and } b = 8$$

$$\text{Since } G.M = \pm \sqrt{ab}$$

$$= \pm \sqrt{(-2)(8)} \\ = \pm \sqrt{-16} = \pm 4^2$$

$$\text{i) } -2^2 \text{ and } 8^2$$

$$a = -2^2, b = 8^2$$

$$\text{Since } G.M = \pm \sqrt{ab}$$

$$= \pm \sqrt{(-2^2)(8^2)} \\ = \pm \sqrt{-16^2} \\ = \pm \sqrt{16} \quad \because 2^2 = -1 \\ = \pm 4$$

Q.No 2 i) Let G_1, G_2 are two G.Ms between 1 and 8 .

then $1, G_1, G_2, 8$ are in G.P.

$$\text{here } a_1 = 1, a_4 = 8$$

$$\Rightarrow a_1 r^3 = 8 \\ \Rightarrow (1) r^3 = 8 \\ \Rightarrow r^3 = (2)^3 \\ \Rightarrow r = 2$$

Now

$$G_1 = a_2 = a_1 r = (1)(2) = 2$$

$$G_2 = a_3 = a_1 r^2 = (1)(2)^2 = (1)(4) = 4$$

hence $2, 4$ are two

G.Ms between 1 and 8

ii) Do yourself as (i)

Q.No 3 i) Let G_1, G_2 and G_3 are three G.Ms between 1 and 16 .

then $1, G_1, G_2, G_3, 16$ are in G.P.

$$\text{here } a_1 = 1, a_5 = 16$$

$$\Rightarrow a_1 r^4 = 16 \\ \Rightarrow 1 \cdot r^4 = 16 \\ \Rightarrow r^4 = (2)^4 \\ \Rightarrow r = 2$$

$$\text{Now } G_1 = a_2 = a_1 r = (1)(2) = 2$$

$$G_2 = a_3 = a_1 r^2 = (1)(2)^2 = 4$$

$$G_3 = a_4 = a_1 r^3 = (1)(2)^3 = 8$$

hence $2, 4, 8$ are three G.Ms between 1 and 16

Q.No 3 ii) Do yourself

Q.No 4 Let G_1, G_2, G_3 and G_4

are four G.Ms between 3 and 96 .

then $3, G_1, G_2, G_3, G_4, 96$ are in G.P.

here $a_1 = 3, a_6 = 96$

$$\Rightarrow a_1 r^5 = 96$$

$$\Rightarrow 3 r^5 = 96$$

$$\Rightarrow r^5 = \frac{96}{3} = 32$$

$$\Rightarrow r^5 = (2)^5 \Rightarrow r = 2$$

Q.No 4 ii) Do yourself

$$G_1 = a_2 = a_1 r = (3)(2) = 6$$

$$G_2 = a_3 = a_1 r^2 = (3)(2)^2 = 12$$

$$G_3 = a_4 = a_1 r^3 = (3)(2)^3 = 24$$

$$G_4 = a_5 = a_1 r^4 = (3)(2)^4 = 48$$

hence $6, 12, 24, 48$ are four G.Ms between 3 and 96

Q.No 5

Suppose $A > G$

$$\text{then } \frac{x+y}{2} > \pm \sqrt{xy}$$

$$\Rightarrow x+y > \pm 2\sqrt{xy}$$

$$\Rightarrow x+y \mp 2\sqrt{xy} > 0$$

$$\Rightarrow (\sqrt{x} \mp \sqrt{y})^2 > 0$$

$$\Rightarrow (\sqrt{x} \mp \sqrt{y})^2 > 0$$

which is true as square is always +ve hence $A > G$

(Q) No 6 We know that

$$G.M = \sqrt{ab} \quad \text{(i)}$$

but we have given

$$G.M = \frac{a^n + b^n}{a^{n-1} + b^{n-1}} \quad \text{(ii)}$$

Comparing (i) and (ii)

$$\frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \sqrt{ab}$$

$$\Rightarrow a^n + b^n = a^{\frac{n}{2}} b^{\frac{n}{2}} (a^{n-1} + b^{n-1})$$

$$\Rightarrow a^n + b^n = a^{\frac{n-1}{2}} b^{\frac{n-1}{2}} + a^{\frac{n-1}{2}} b^{\frac{n-1}{2}}$$

$$\Rightarrow a^n - a^{\frac{n-1}{2}} b^{\frac{n-1}{2}} = a^{\frac{n-1}{2}} b^{\frac{n-1}{2}} - b^n$$

$$\Rightarrow a^{\frac{n-1}{2}} (a^{\frac{1}{2}} - b^{\frac{1}{2}}) = b^{\frac{n-1}{2}} (a^{\frac{1}{2}} - b^{\frac{1}{2}})$$

$$\Rightarrow a^{\frac{n-1}{2}} = b^{\frac{n-1}{2}}$$

$$\Rightarrow \frac{a^{\frac{n-1}{2}}}{b^{\frac{n-1}{2}}} = 1$$

$$\Rightarrow \left(\frac{a}{b}\right)^{\frac{n-1}{2}} = \left(\frac{a}{b}\right)^0 \because \left(\frac{a}{b}\right)^0 = 1$$

$$\Rightarrow n - \frac{1}{2} = 0 \Rightarrow n = \frac{1}{2}$$

(Q) No 7 Let a and b be two five integers then by given condition.

$$A.M = G.M + 2$$

$$\Rightarrow \frac{a+b}{2} = \sqrt{ab} + 2$$

$$\Rightarrow a+b = 2(\sqrt{ab} + 2) \quad \text{(i)}$$

Also we have given

$$a+b = 20 \quad \text{(ii)}$$

Comparing (i) & (ii)

$$2(\sqrt{ab} + 2) = 20 \Rightarrow 2\sqrt{ab} + 4 = 20$$

$$\Rightarrow 2\sqrt{ab} = 20 - 4$$

$$= 16$$

$$\Rightarrow \sqrt{ab} = \frac{16}{2} = 8$$

$\Rightarrow ab = 64$ by squaring

$$\Rightarrow b = \frac{64}{a} \quad \text{(iii)}$$

putting in (ii)

$$a + \frac{64}{a} = 20$$

$$\Rightarrow a^2 + 64 = 20a$$

$$\Rightarrow a^2 - 20a + 64 = 0$$

$$\Rightarrow a^2 - 16a - 4a + 64 = 0$$

$$\Rightarrow a(a-16) - 4(a-16) = 0$$

$$\Rightarrow (a-16)(a-4) = 0$$

$$\Rightarrow a-16 = 0 \Rightarrow a-4 = 0$$

$$\Rightarrow a = 16 \Rightarrow a = 4$$

putting in (iii)

$$b = \frac{64}{16} = 4, \quad b = \frac{64}{4} = 16$$

hence 16, 4 OR 4, 16 are required numbers

(Q) No 8 Set a , and b be two required numbers

$$\text{then } A.M = 5$$

$$\Rightarrow \frac{a+b}{2} = 5$$

$$\Rightarrow a+b = 10 \quad \text{(i)}$$

$$\text{Also } G.M = 4$$

$$\Rightarrow \sqrt{ab} = 4$$

$$\Rightarrow ab = 16 \quad \text{on squaring}$$

$$\Rightarrow b = \frac{16}{a} \quad \text{(ii)}$$

putting in (i)

$$a + \frac{16}{a} = 10$$

$$\Rightarrow a^2 + 16 = 10a$$

$$\Rightarrow a^2 - 10a + 16 = 0$$

$$\Rightarrow a^2 - 8a - 2a + 16 = 0$$

$$\Rightarrow a(a-8) - 2(a-8) = 0$$

$$\Rightarrow (a-8)(a-2) = 0$$

$$\Rightarrow a-8 = 0, \quad a-2 = 0$$

$$\Rightarrow a = 8, \quad a = 2$$

putting in (ii)

$$b = \frac{16}{8} = 2 \Rightarrow b = \frac{16}{2} = 8$$

hence 8, 2 OR 2, 8 are required numbers

END