Question # 1
Evaluate the following:
(i) ${}^{20}P_3$ (ii) ${}^{16}P_4$

Solution
(i)
$${}^{20}P_3=\frac{20!}{(20-3)!}=\frac{20!}{17!}=\frac{20\cdot 19\cdot 18\cdot 17!}{17!}=20\cdot 19\cdot 18=6840$$
(ii)
$${}^{16}P_4=\frac{16!}{(16-4)!}=\frac{16!}{12!}=\frac{16\cdot 15\cdot 14\cdot 13\cdot 12!}{12!}=16\cdot 15\cdot 14\cdot 13=43680$$
Others do yourself

Question # 2
Find the value of n when:
(i) ${}^{n}P_2=30$ (ii) ${}^{11}P_n=11\cdot 10\cdot 9$ (iii) ${}^{n}P_4:{}^{n-1}P_3=9:1$

Solution
(i)
$${}^{n}P_2=30 \quad\Rightarrow\quad \frac{n!}{(n-2)!}=30 \quad\Rightarrow\quad \frac{n(n-1)(n-2)!}{(n-2)!}=30$$
$$\Rightarrow\ n(n-1)=30 \quad\Rightarrow\ n(n-1)=6\cdot 5 \quad\Rightarrow\ \boxed{n=6}$$
(ii)
$${}^{11}P_n=11\cdot 10\cdot 9 \quad\Rightarrow\quad \frac{11\cdot 10\cdot 9\cdot 8!}{(11-n)!}=11\cdot 10\cdot 9$$
$$\Rightarrow\ \frac{8!}{(11-n)!}=1 \quad\Rightarrow\ 8!=(11-n)! \quad\Rightarrow\ 8=11-n \quad\Rightarrow\ n=11-8$$
$$\Rightarrow\ \boxed{n=3}$$
(iii)
$${}^{n}P_4:{}^{n-1}P_3=9:1 \quad\Rightarrow\quad \frac{{}^{n}P_4}{{}^{n-1}P_3}=\frac{9}{1} \quad\Rightarrow\quad {}^{n}P_4=9\ {}^{n-1}P_3$$
$$\Rightarrow\ \frac{n!}{(n-4)!}=9\,\frac{(n-1)!}{(n-1-3)!} \qquad\Rightarrow\ \frac{n(n-1)!}{(n-4)!}=9\,\frac{(n-1)!}{(n-4)!} \qquad\Rightarrow\ \boxed{n=9}$$

Question # 3
Prove from the first principle that:
(i) ${}^{n}P_r=n\cdot{}^{n-1}P_{r-1}$ (ii) ${}^{n}P_r={}^{n-1}P_r+r\cdot{}^{n-1}P_{r-1}$

Solution
(i)

$$\text{R.H.S } = n\cdot {}^{n-1}P_{r-1} = n\cdot\frac{(n-1)!}{\left(n-1-(r-1)\right)} = \frac{n(n-1)!}{\left(n-1-r+1\right)} = \frac{n!}{\left(n-r\right)} = {}^{n}P_{r} = \text{L.H.S}$$

(ii)

$$\text{R.H.S } = {}^{n-1}P_{r} + r\cdot {}^{n-1}P_{r-1} \quad = \frac{(n-1)!}{(n-1-r)!} + r\cdot\frac{(n-1)!}{(n-1-r+1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!} + r\cdot\frac{(n-1)!}{(n-r)!} \qquad = \frac{(n-1)!}{(n-r-1)!} + r\cdot\frac{(n-1)!}{(n-r)(n-r-1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!}\left(1 + r\cdot\frac{1}{(n-r)}\right) \qquad = \frac{(n-1)!}{(n-r-1)!}\left(\frac{n-r+r}{(n-r)}\right)$$

$$= \frac{(n-1)!}{(n-r-1)!}\left(\frac{n}{(n-r)}\right) \qquad = \frac{n(n-1)!}{(n-r)(n-r-1)!}$$

$$= \frac{n!}{(n-r)!} = {}^{n}P_{r} = \text{ L.H.S}$$

Question # 4

Do yourself as Question # 5

Question # 5

How many signals can be given by 6 flags of different colours, when any number of flags cab be used at a time?

Solution

Total number of flags = $n = 6$

Number of signal using one flag = ${}^{6}P_{1} = 6$

Number of signal using two flag = ${}^{6}P_{2} = 30$

Number of signal using three flag = ${}^{6}P_{3} = 120$

Number of signal using four flag = ${}^{6}P_{4} = 360$

Number of signal using five flag = ${}^{6}P_{5} = 720$

Number of signal using six flag = ${}^{6}P_{6} = 720$

Total number of signals = $6+30+120+360+720+720 = 1956$

Question # 6

How many words can be formed from the letters of the following words using all letters when no letter is to be repeated:

(i)PLSNE (ii)OBJECT (iii)FASTING

Solution

(i)

Since number of letters in PLANE = $n = 5$

Therefore total words form = ${}^{5}P_{5} = 120$

(ii)

Since number of letters in OBJECT = $n = 6$

Therefore total words forms = ${}^{6}P_{6} = 720$

(iii)

Since number of letters in FASTING = $n = 7$

Therefore total words forms = ${}^7P_7 = 5040$

Question # 7

How many $3-digit$ numbers can be formed by using each one of the digits $2,3,5,7,9$ only once?

Solution

Number of digits = $n = 5$

So numbers forms taken 3 digits at a time = ${}^5P_3 = 60$

Question # 8

Find the numbers greater than 23000 that can be formed from the digits $1,2,3,5,6$ without repeating any digit.

Solution

Number greater than 23000 can be formed as

Number of numbers of the form $23{*}{*}{*} = {}^3P_3 = 6$

Number of numbers of the form $25{*}{*}{*} = {}^3P_3 = 6$

Number of numbers of the form $26{*}{*}{*} = {}^3P_3 = 6$

Number of numbers of the form $3{*}{*}{*}{*} = {}^4P_4 = 24$

Number of numbers of the form $5{*}{*}{*}{*} = {}^4P_4 = 24$

Number of numbers of the form $6{*}{*}{*}{*} = {}^4P_4 = 24$

Thus the total number formed = 6 + 6 + 6 + 24 + 24 + 24 = 90

***Alternative* solution:**

Permutation of 5 digits numbers = ${}^5P_5 = 120$

Numbers less than 23000 are of the form $1{*}{*}{*}{*}$

Then permutations $= {}^4P_4 = 24$

If number less than 23000 are of the form $21{*}{*}{*}$

Then permutations $= {}^3P_3 = 6$

Thus number greater than 23000 formed = $120 - 24 - 6 = 90$

Question # 9

Find the number of 5 digits numbers that can be formed from the digits 1, 2, 4, 6, 8 (when no digit is repeated), but

(i) the digits 2 and 8 are next to each other.

(ii) the digits 2 and 8 are not next to each other.

Solution

Total number of digits = 5

(i) If we take 28 as a single digit then number of numbers = ${}^4P_4 = 24$

If we take 82 as a single digit then number of numbers = ${}^4P_4 = 24$

So the total numbers when 2 and 8 are next to each other = 24 + 24 = 48

(ii) Number of total permutation = ${}^5P_5 = 120$
thus number of numbers when 2 and 8 are not next to each other = 120 – 48 = 72

Question # 10
How many $6-digit$ numbers can be formed, without repeating any digit from the digits $0,1,2,3,4,5$? In how many of them will 0 be at the ten places?
Solution
Since number of permutation of 6 digits = ${}^6P_6 = 720$
But 0 at extreme left is meaning less
so number of permutation when 0 is at extreme left = ${}^5P_5 = 120$
Thus the number formed by 6 digits = 720 – 120 = 600
Now if we fix 0 at ten place then number formed = ${}^5P_5 = 120$

Question # 11
How many $5-digit$ multiples of 5 can be formed from the digits $2,3,5,7,9$, when no digit is repeated.
Solution
Number of digits = 5
For multiple of 5 we must have 5 at extreme right so number formes = ${}^4P_4 = 24$

Question # 12
In how many ways can 8 books including 2 books on English be arranged on a shelf in such a way that the English books are never together.
Solution
Total numbers of books = 8
Total number of permutation = ${}^8P_8 = 40320$
Let E_1 and E_2 denotes two English books then
Number of permutation when E_1E_2 place together = ${}^7P_7 = 5040$
Number of permutation when E_2E_1 place together = ${}^7P_7 = 5040$
So total permutation when E_1 and E_2 together = 5040 + 5040 = 10080
Required permutation when English books are not together = 40320 – 10080
= 30240

Question # 13
Find the number of arrangement of 3 books on English and 5 books on Urdu for placing them on a shelf such that the books on the same subjects are together.
Solution
Let E_1, E_2, E_3 be the book on English and U_1, U_2, U_3, U_4, U_5 be the book on Urdu
Then the permutation when
books are arranged as $E_1, E_2, E_3, U_1, U_2, U_3, U_4, U_5$ = ${}^3P_3 \times {}^5P_5 = 6 \times 120 = 720$

books are arranged as $U_1, U_2, U_3, U_4, U_5, E_1, E_2, E_3 = {}^5P_5 \times {}^3P_3 = 120 \times 6 = 720$
so total permutation when books of same subject are together = 720 + 720
= 1440

Question # 14
In how many ways can 5 boys and 4 girls be seated on bench so that the girls and the boys occupy alternative seats?
Solution
Let the five boys be B_1, B_2, B_3, B_4, B_5 and the four girls are G_1, G_2, G_3, G_4 then there seats plane is $B_1, G_1, B_2, G_2, B_3, G_3, B_4, G_4, B_5$

Then the permutations $= {}^5P_1 \times {}^4P_1 \times {}^4P_1 \times {}^3P_1 \times {}^3P_1 \times {}^2P_1 \times {}^2P_1 \times {}^1P_1 \times {}^1P_1$
$= 5 \times 4 \times 4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1 = 2880$
