

**Question # 1**

Evaluate the following:

(i)  ${}^{12}C_3$

(ii)  ${}^{20}C_{17}$

(iii)  ${}^nC_4$

**Solution**

$$(i) \quad {}^{12}C_3 = \frac{12!}{(12-3)!3!} = \frac{12!}{9!3!} = \frac{12 \cdot 11 \cdot 10 \cdot 9!}{9!3!} = \frac{12 \cdot 11 \cdot 10}{3!} = \frac{1320}{6} = 220$$

$$(ii) \quad {}^{20}C_{17} = \frac{20!}{(20-17)!17!} = \frac{20!}{3!17!} = \frac{20 \cdot 19 \cdot 18 \cdot 17!}{3!17!} = \frac{20 \cdot 19 \cdot 18}{3!} = \frac{6840}{6} = 1140$$

$$(iii) \quad {}^nC_4 = \frac{n!}{(n-4)!4!} = \frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!4!} = \frac{n(n-1)(n-2)(n-3)}{4!}$$


---

**Question # 2**Find the value of  $n$ , when

(i)  ${}^nC_5 = {}^nC_4$

(ii)  ${}^nC_{10} = \frac{12 \times 11}{2!}$

(iii)  ${}^nC_{12} = {}^nC_6$

**Solution**

(i)

Since  ${}^nC_5 = {}^nC_4$

$\Rightarrow {}^nC_{n-5} = {}^nC_4$

$\because {}^nC_r = {}^nC_{n-r}$

$\Rightarrow n-5=4 \quad \Rightarrow n=4+5 \quad \Rightarrow \boxed{n=9}$

$$(ii) \quad {}^nC_{10} = \frac{12 \times 11}{2!}$$

$$\Rightarrow {}^nC_{10} = \frac{12 \cdot 11 \cdot 10!}{2!10!}$$

$$\Rightarrow {}^nC_{10} = \frac{12!}{(12-10)!10!}$$

$$\Rightarrow {}^nC_{10} = {}^{12}C_{10}$$

$$\Rightarrow \boxed{n=12}.$$

(iii) *Do yourself as Q # 2 (i)***Question # 3**Find the values of  $n$  and  $r$ , when

(i)  ${}^nC_r = 35$  and  ${}^nP_r = 210$

(ii)  ${}^{n-1}C_{r-1} : {}^nC_r : {}^{n+1}C_{r+1} = 3:6:11$

**Solution**

$$(i) \quad {}^nC_r = 35 \quad \text{and} \quad {}^nP_r = 210$$

$$\text{Since } {}^nC_r = 35 \Rightarrow \frac{n!}{(n-r)! r!} = 35 \Rightarrow \frac{n!}{(n-r)!} = 35 \cdot r! \dots\dots\dots (i)$$

$$\text{Also } {}^nP_r = 210 \Rightarrow \frac{n!}{(n-r)!} = 210 \dots\dots\dots (ii)$$

Comparing (i) and (ii)

$$\begin{aligned} 35 \cdot r! &= 210 \\ \Rightarrow r! &= \frac{210}{35} \Rightarrow r! = 6 \Rightarrow r! = 3! \Rightarrow \boxed{r = 3} \end{aligned}$$

Putting value of  $r$  in equation (ii)

$$\begin{aligned} \frac{n!}{(n-3)!} &= 210 \\ \Rightarrow \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} &= 210 \\ \Rightarrow n(n-1)(n-2) &= 210 \\ \Rightarrow n(n-1)(n-2) &= 7 \cdot 6 \cdot 5 \\ \Rightarrow \boxed{n = 7} \end{aligned}$$

$$(ii) \quad {}^{n-1}C_{r-1} : {}^nC_r : {}^{n+1}C_{r+1} = 3 : 6 : 11$$

First consider

$$\begin{aligned} {}^{n-1}C_{r-1} : {}^nC_r &= 3 : 6 \\ \Rightarrow \frac{(n-1)!}{(n-1-r+1)! (r-1)!} : \frac{n!}{(n-r)! r!} &= 3 : 6 \\ \Rightarrow \frac{(n-1)!}{(n-r)! (r-1)!} : \frac{n!}{(n-r)! r!} &= 3 : 6 \\ \Rightarrow \frac{(n-1)!}{\frac{(n-r)! (r-1)!}{\frac{n!}{(n-r)! r!}}} &= \frac{3}{6} \\ \Rightarrow \frac{(n-1)!}{(n-r)! (r-1)!} \times \frac{(n-r)! r!}{n!} &= \frac{1}{2} \\ \Rightarrow \frac{(n-1)!}{(r-1)!} \times \frac{r!}{n!} &= \frac{1}{2} \\ \Rightarrow \frac{r}{n} = \frac{1}{2} \Rightarrow n = 2r \dots\dots\dots (i) \end{aligned}$$

Now consider  ${}^nC_r : {}^{n+1}C_{r+1} = 6:11$

$$\Rightarrow \frac{n!}{(n-r)! r!} : \frac{(n+1)!}{(n+1-r-1)! (r+1)!} = 6:11$$

$$\Rightarrow \frac{n!}{(n-r)! r!} : \frac{(n+1)!}{(n-r)! (r+1)!} = 6:11$$

$$\Rightarrow \frac{\frac{n!}{(n-r)! r!}}{\frac{(n+1)!}{(n-r)! (r+1)!}} = \frac{6}{11}$$

$$\Rightarrow \frac{n!}{(n-r)! r!} \times \frac{(n-r)! (r+1)!}{(n+1)!} = \frac{6}{11}$$

$$\Rightarrow \frac{n!}{r!} \times \frac{(r+1)!}{(n+1)!} = \frac{6}{11}$$

$$\Rightarrow \frac{n!}{r!} \times \frac{(r+1)r!}{(n+1)n!} = \frac{6}{11}$$

$$\Rightarrow \frac{(r+1)}{(n+1)} = \frac{6}{11}$$

$$\Rightarrow 11(r+1) = 6(n+1)$$

$$\Rightarrow 11(r+1) = 6(2r+1) \quad \because n = 2r$$

$$\Rightarrow 11r + 11 = 12r + 6$$

$$\Rightarrow 11r - 12r = 6 - 11 \Rightarrow -r = -5 \Rightarrow \boxed{r = 5}$$

Putting value of  $r$  in equation (ii)

$$\Rightarrow \boxed{n = 10}$$

#### Question # 4

How many (a) diagonals and (b) triangles can be formed by joining the vertices of the polygon having:

(i) 5 sides

(ii) 8 sides

(iii) 12 sides

**Solution**

(i)

(a) 5 sided polygon has 5 vertices,

so joining two vertices we have line segments =  ${}^5C_2 = 10$

Number of sides = 5

So number of diagonals =  $10 - 5 = 5$

(b) 5 sided polygon has 5 vertices,

so joining any three vertices we have triangles =  ${}^5C_3 = 10$

(ii)

- (a) 8 sided polygon has 8 vertices  
 So joining any two vertices we have line segments =  ${}^8C_2 = 28$   
 Number of sides = 8  
 So number of diagonals =  $28 - 8 = 20$
- (b) 8 sided polygon has 8 vertices,  
 so joining any three vertices we have triangles =  ${}^8C_3 = 56$ .

(iii) *Do yourself as above.*

**Question # 5**

The members of a club are 12 boys and 8 girls. In how many ways can a committee of 3 boys and 2 girls be formed?

**Solution**

Number of boys = 12  
 So committees formed taking 3 boys =  ${}^{12}C_3 = 220$   
 Number of girls = 8  
 So committees formed by taking 2 girls =  ${}^8C_2 = 28$   
 Now total committees formed including 3 boys and 2 girls =  $220 \times 28$   
 $= 6160$

**Question # 6**

How many committees of 5 members can be chosen from a group of 8 persons when each committee must include 2 particular persons?

**Solution**

Number of persons = 8  
 Since two particular persons are included in every committee so we have to find combinations of 6 persons 3 at a time =  ${}^6C_3 = 20$   
 Hence number of committees = 20

**Question # 7**

In how many ways can a hockey team of 11 players be selected out of 15 players? How many of them will include a particular player?

**Solution**

The number of player = 15  
 So combination, taking 11 player at a time =  ${}^{15}C_{11} = 1365$   
 Now if one particular player is in each collection  
 then number of combination =  ${}^{14}C_{10} = 1001$

**Question # 8**

Show that:  ${}^{16}C_{11} + {}^{16}C_{10} = {}^{17}C_{11}$

**Solution**

$$\begin{aligned} \text{L.H.S} &= {}^{16}C_{11} + {}^{16}C_{10} \\ &= \frac{16!}{(16-11)! 11!} + \frac{16!}{(16-10)! 10!} = \frac{16!}{5! 11!} + \frac{16!}{6! 10!} \end{aligned}$$

$$\begin{aligned}
&= \frac{16!}{5! \cdot 11 \cdot 10!} + \frac{16!}{6 \cdot 5! \cdot 10!} = \frac{16!}{10! \cdot 5!} \left( \frac{1}{11} + \frac{1}{6} \right) \\
&= \frac{16!}{10! \cdot 5!} \left( \frac{6+11}{66} \right) = \frac{16!}{10! \cdot 5!} \left( \frac{17}{66} \right) = \frac{16!}{10! \cdot 5!} \left( \frac{17}{11 \cdot 6} \right) \\
&= \frac{17 \cdot 16!}{11 \cdot 10! \cdot 6 \cdot 5!} = \frac{17!}{11! \cdot 6!} = \frac{17!}{11! (17-11)!} = {}^{17}C_{11} = \text{R.H.S}
\end{aligned}$$

**Alternative**

$$\text{L.H.S} = {}^{16}C_{11} + {}^{16}C_{10} = 4368 + 8008 = 12276 \dots\dots\dots (i)$$

$$\text{R.H.S} = {}^{17}C_{11} = 12376 \dots\dots\dots (ii)$$

From (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

**Question # 9**

There are 8 men and 10 women members of a club. How many committees of numbers can be formed, having;

(i) 4 women

(ii) at the most 4 women

(iii) at least 4 women

**Solution**

Number of men = 8

Number of women = 10

(i) We have to form combination of 4 women out of 10 and 3 men out of

$$= {}^{10}C_4 \times {}^8C_3 = 210 \times 36 = 11760$$

(ii) At the most 4 women means that women are less than or equal to 4, which implies the following possibilities (1W, 6M), (2W, 5M), (3W, 4M), (4W, 3M), (7M)

$$= {}^{10}C_1 \times {}^8C_6 + {}^{10}C_2 \times {}^8C_5 + {}^{10}C_3 \times {}^8C_4 + {}^{10}C_4 \times {}^8C_3 + {}^8C_7$$

$$= (10)(28) + (45)(56) + (120)(70) + (210)(56) + (8)$$

$$= 280 + 2520 + 8400 + 11760 + 8 = 22968$$

(iii) At least 4 women means that women are greater than or equal to 4, which implies the following possibilities (4W, 3M), (5W, 2M), (6W, 1M), (7W)

$$= {}^{10}C_4 \times {}^8C_3 + {}^{10}C_5 \times {}^8C_2 + {}^{10}C_6 \times {}^8C_1 + {}^{10}C_7$$

$$= (210)(56) + (252)(28) + (210)(8) + 120$$

$$= 11760 + 7056 + 1680 + 120$$

$$= 20616$$

**Question # 10**

Prove that;  ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

**Solution**

$$\text{L.H.S} = {}^nC_r + {}^nC_{r-1} = \frac{n!}{(n-r)! r!} + \frac{n!}{(n-(r-1))! (r-1)!}$$

$$\begin{aligned}
&= \frac{n!}{(n-r)! \, r!} + \frac{n!}{(n-r+1)! \, (r-1)!} \\
&= \frac{n!}{(n-r)! \, r(r-1)!} + \frac{n!}{(n-r+1)(n-r)! \, (r-1)!} \\
&= \frac{n!}{(n-r)! \, (r-1)!} \left( \frac{1}{r} + \frac{1}{(n-r+1)} \right) \\
&= \frac{n!}{(n-r)! \, (r-1)!} \left( \frac{n-r+1+r}{r(n-r+1)} \right) \\
&= \frac{n!}{(n-r)! \, (r-1)!} \left( \frac{n+1}{r(n-r+1)} \right) \\
&= \frac{(n+1)n!}{(n-r+1)(n-r)! \, r(r-1)!} \\
&= \frac{(n+1)!}{(n-r+1)! \, r!} = \frac{(n+1)!}{(n+1-r)! \, r!} \\
&= {}^{n+1}C_r = \text{R.H.S}
\end{aligned}$$


---