

Chapter 8

Exercise 8.2

Binomial Theorem

If a and x are two real number and n is a positive integer then

$$(a+x)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}x^1 + \binom{n}{2}a^{n-2}x^2 + \dots + \binom{n}{n-1}ax^{n-1} + \binom{n}{n}x^n$$

Proof

We will use mathematical induction to prove this so let $S(n)$ be the given statement.

Put $n=1$

$$S(1): (a+x)^1 = \binom{1}{0}a^1 + \binom{1}{1}a^{1-1}x^1 = (1)a + (1)(1)x \Rightarrow a+x = a+x$$

$S(1)$ is true so condition I is satisfied.

Now suppose that $S(n)$ is true for $n=k$.

$$S(k): (a+x)^k = \binom{k}{0}a^k + \binom{k}{1}a^{k-1}x^1 + \binom{k}{2}a^{k-2}x^2 + \dots + \binom{k}{k-1}ax^{k-1} + \binom{k}{k}x^k \dots \dots (i)$$

The statement for $n=k+1$

$$S(k+1): (a+x)^{k+1} = \binom{k+1}{0}a^{k+1} + \binom{k+1}{1}a^{k+1-1}x^1 + \binom{k+1}{2}a^{k+1-2}x^2 + \dots$$

$$+ \binom{k+1}{k+1-1}ax^{k+1-1} + \binom{k+1}{k+1}x^{k+1}$$

$$\Rightarrow (a+x)^{k+1} = \binom{k+1}{0}a^{k+1} + \binom{k+1}{1}a^kx^1 + \binom{k+1}{2}a^{k-1}x^2 + \dots$$

$$+ \binom{k+1}{k}ax^k + \binom{k+1}{k+1}x^{k+1}$$

Multiplying both sides of equation (i) by $(a+x)$

$$(a+x)^k(a+x) = \left(\binom{k}{0}a^k + \binom{k}{1}a^{k-1}x^1 + \binom{k}{2}a^{k-2}x^2 + \dots + \binom{k}{k-1}ax^{k-1} + \binom{k}{k}x^k \right)(a+x)$$

$$\Rightarrow (a+x)^{k+1} = \left(\binom{k}{0}a^k + \binom{k}{1}a^{k-1}x^1 + \binom{k}{2}a^{k-2}x^2 + \dots + \binom{k}{k-1}ax^{k-1} + \binom{k}{k}x^k \right)(a)$$

$$+ \left(\binom{k}{0}a^k + \binom{k}{1}a^{k-1}x^1 + \binom{k}{2}a^{k-2}x^2 + \dots + \binom{k}{k-1}ax^{k-1} + \binom{k}{k}x^k \right)(x)$$

$$\Rightarrow (a+x)^{k+1} = \binom{k}{0}a^{k+1} + \binom{k}{1}a^kx^1 + \binom{k}{2}a^{k-1}x^2 + \dots + \binom{k}{k-1}a^2x^{k-1} + \binom{k}{k}ax^k$$

$$+ \binom{k}{0}a^kx + \binom{k}{1}a^{k-1}x^2 + \binom{k}{2}a^{k-2}x^3 + \dots + \binom{k}{k-1}ax^k + \binom{k}{k}x^{k+1}$$

$$\Rightarrow (a+x)^{k+1} = \binom{k}{0}a^{k+1} + \left(\binom{k}{1} + \binom{k}{0} \right)a^kx^1 + \left(\binom{k}{2} + \binom{k}{1} \right)a^{k-1}x^2 + \dots$$

$$+ \left(\binom{k}{k} + \binom{k}{k-1} \right) ax^k + \binom{k}{k} x^{k+1}$$

Since $\binom{n}{0} = \binom{n+1}{0}$, $\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$ and $\binom{n}{n} = \binom{n+1}{n+1}$

$$\Rightarrow (a+x)^{k+1} = \binom{k+1}{0} a^{k+1} + \binom{k+1}{1} a^k x^1 + \binom{k+1}{2} a^{k-1} x^2 + \dots + \binom{k+1}{k} a x^k + \binom{k+1}{k+1} x^{k+1}$$

Thus $S(k+1)$ is true when $S(k)$ is true so condition II is satisfied and $S(n)$ is true for all

positive integral value of n .

Question # 1

Using binomial theorem, expand the following:

$$(i) (a+2b)^5$$

$$(ii) \left(\frac{x}{2} - \frac{2}{x^2} \right)^6$$

$$(iii) \left(3a - \frac{x}{3a} \right)^4$$

$$(iv) \left(2a - \frac{x}{a} \right)^7$$

$$(v) \left(\frac{x}{2y} - \frac{2y}{x} \right)^8$$

$$(vi) \left(\sqrt{\frac{a}{x}} - \sqrt{\frac{x}{a}} \right)^6$$

Solution

(i)

$$\begin{aligned} (a+2b)^5 &= \binom{5}{0} a^5 + \binom{5}{1} a^{5-1} (2b)^1 + \binom{5}{2} a^{5-2} (2b)^2 + \binom{5}{3} a^{5-3} (2b)^3 + \binom{5}{4} a^{5-4} (2b)^4 + \binom{5}{5} a^{5-5} (2b)^5 \\ &= (1) a^5 + (5) a^4 (2b) + (10) a^3 (4b^2) + (10) a^2 (8b^3) + (5) a^1 (16b^4) + (1) a^0 (32b^5) \\ &= a^5 + 10a^4 b + 40a^3 b^2 + 80a^2 b^3 + 80ab^4 + 32b^5 \quad \because a^0 = 1 \end{aligned}$$

$$\begin{aligned} (ii) \left(\frac{x}{2} - \frac{2}{x^2} \right)^6 &= \binom{6}{0} \left(\frac{x}{2} \right)^6 + \binom{6}{1} \left(\frac{x}{2} \right)^{6-1} \left(-\frac{2}{x^2} \right)^1 + \binom{6}{2} \left(\frac{x}{2} \right)^{6-2} \left(-\frac{2}{x^2} \right)^2 + \binom{6}{3} \left(\frac{x}{2} \right)^{6-3} \left(-\frac{2}{x^2} \right)^3 \\ &\quad + \binom{6}{4} \left(\frac{x}{2} \right)^{6-4} \left(-\frac{2}{x^2} \right)^4 + \binom{6}{5} \left(\frac{x}{2} \right)^{6-5} \left(-\frac{2}{x^2} \right)^5 + \binom{6}{6} \left(\frac{x}{2} \right)^{6-6} \left(-\frac{2}{x^2} \right)^6 \\ &= (1) \left(\frac{x}{2} \right)^6 - (6) \left(\frac{x}{2} \right)^5 \left(\frac{2}{x^2} \right) + (15) \left(\frac{x}{2} \right)^4 \left(\frac{2}{x^2} \right)^2 - (20) \left(\frac{x}{2} \right)^3 \left(\frac{2}{x^2} \right)^3 \\ &\quad + (15) \left(\frac{x}{2} \right)^2 \left(\frac{2}{x^2} \right)^4 - (6) \left(\frac{x}{2} \right)^1 \left(\frac{2}{x^2} \right)^5 + (1)(1) \left(\frac{2}{x^2} \right)^6 \\ &= \left(\frac{x^6}{64} \right) - 6 \left(\frac{x^5}{32} \right) \left(\frac{2}{x^2} \right) + 15 \left(\frac{x^4}{16} \right) \left(\frac{4}{x^4} \right) - 20 \left(\frac{x^3}{8} \right) \left(\frac{8}{x^6} \right) \\ &\quad + 15 \left(\frac{x^2}{4} \right) \left(\frac{16}{x^8} \right) - 6 \left(\frac{x}{2} \right) \left(\frac{32}{x^{10}} \right) + \left(\frac{64}{x^{12}} \right) \\ &= \frac{x^6}{64} - \frac{3x^5}{8} + \frac{15}{4} - \frac{20}{x^3} + \frac{60}{x^6} - \frac{96}{x^9} + \frac{64}{x^{12}} \end{aligned}$$

(iii) *Do yourself*(iv) *Do yourself*(v) *Do yourself*

$$\begin{aligned}
 \text{(vi)} \quad & \left(\sqrt[6]{\frac{a}{x}} - \sqrt[6]{\frac{x}{a}} \right)^6 = \binom{6}{0} \left(\sqrt[6]{\frac{a}{x}} \right)^6 + \binom{6}{1} \left(\sqrt[6]{\frac{a}{x}} \right)^{6-1} \left(-\sqrt[6]{\frac{x}{a}} \right)^1 + \binom{6}{2} \left(\sqrt[6]{\frac{a}{x}} \right)^{6-2} \left(-\sqrt[6]{\frac{x}{a}} \right)^2 + \binom{6}{3} \left(\sqrt[6]{\frac{a}{x}} \right)^{6-3} \\
 & \left(-\sqrt[6]{\frac{x}{a}} \right)^3 + \binom{6}{4} \left(\sqrt[6]{\frac{a}{x}} \right)^{6-4} \left(-\sqrt[6]{\frac{x}{a}} \right)^4 + \binom{6}{5} \left(\sqrt[6]{\frac{a}{x}} \right)^{6-5} \left(-\sqrt[6]{\frac{x}{a}} \right)^5 + \binom{6}{6} \left(\sqrt[6]{\frac{a}{x}} \right)^{6-6} \left(-\sqrt[6]{\frac{x}{a}} \right)^6 \\
 & = (1) \left(\sqrt[6]{\frac{a}{x}} \right)^6 - (6) \left(\sqrt[6]{\frac{a}{x}} \right)^5 \left(\sqrt[6]{\frac{x}{a}} \right)^1 + (15) \left(\sqrt[6]{\frac{a}{x}} \right)^4 \left(\sqrt[6]{\frac{x}{a}} \right)^2 - (20) \left(\sqrt[6]{\frac{a}{x}} \right)^3 \left(\sqrt[6]{\frac{x}{a}} \right)^3 - \\
 & \left(\sqrt[6]{\frac{x}{a}} \right)^3 + (15) \left(\sqrt[6]{\frac{a}{x}} \right)^2 \left(\sqrt[6]{\frac{x}{a}} \right)^4 - (6) \left(\sqrt[6]{\frac{a}{x}} \right)^1 \left(\sqrt[6]{\frac{x}{a}} \right)^5 + (1) \left(\sqrt[6]{\frac{a}{x}} \right)^0 \left(\sqrt[6]{\frac{x}{a}} \right)^6 = \\
 & \left(\sqrt[6]{\frac{a}{x}} \right)^6 - 6 \left(\sqrt[6]{\frac{a}{x}} \right)^5 \left(\sqrt[6]{\frac{a}{x}} \right)^{-1} + 15 \left(\sqrt[6]{\frac{a}{x}} \right)^4 \left(\sqrt[6]{\frac{a}{x}} \right)^{-2} - 20 \left(\sqrt[6]{\frac{a}{x}} \right)^3 \left(\sqrt[6]{\frac{a}{x}} \right)^{-3} \\
 & + 15 \left(\sqrt[6]{\frac{a}{x}} \right)^2 \left(\sqrt[6]{\frac{a}{x}} \right)^{-4} - 6 \left(\sqrt[6]{\frac{a}{x}} \right)^1 \left(\sqrt[6]{\frac{a}{x}} \right)^{-5} + 1(1) \left(\sqrt[6]{\frac{a}{x}} \right)^6 \\
 & = \left(\sqrt[6]{\frac{a}{x}} \right)^6 - 6 \left(\sqrt[6]{\frac{a}{x}} \right)^{5-1} + 15 \left(\sqrt[6]{\frac{a}{x}} \right)^{4-2} - 20 \left(\sqrt[6]{\frac{a}{x}} \right)^{3-3} + 15 \left(\sqrt[6]{\frac{a}{x}} \right)^{2-4} - 6 \left(\sqrt[6]{\frac{a}{x}} \right)^{1-5} + 1 \left(\sqrt[6]{\frac{a}{x}} \right)^6 \\
 & = \left(\sqrt[6]{\frac{a}{x}} \right)^6 - 6 \left(\sqrt[6]{\frac{a}{x}} \right)^4 + 15 \left(\sqrt[6]{\frac{a}{x}} \right)^2 - 20 \left(\sqrt[6]{\frac{a}{x}} \right)^0 + 15 \left(\sqrt[6]{\frac{a}{x}} \right)^2 - 6 \left(\sqrt[6]{\frac{a}{x}} \right)^4 + \left(\sqrt[6]{\frac{a}{x}} \right)^6 \\
 & = \left(\frac{a}{x}^{\frac{1}{2}} \right)^6 - 6 \left(\frac{a}{x}^{\frac{1}{2}} \right)^4 + 15 \left(\frac{a}{x}^{\frac{1}{2}} \right)^2 - 20(1) + 15 \left(\frac{x}{a}^{\frac{1}{2}} \right)^2 - 6 \left(\frac{x}{a}^{\frac{1}{2}} \right)^4 + \left(\frac{x}{a}^{\frac{1}{2}} \right)^6 \\
 & = \left(\frac{a}{x} \right)^3 - 6 \left(\frac{a}{x} \right)^2 + 15 \left(\frac{a}{x} \right) - 20 + 15 \left(\frac{x}{a} \right) - 6 \left(\frac{x}{a} \right)^2 + \left(\frac{x}{a} \right)^3 \\
 & = \frac{a^3}{x^3} - 6 \frac{a^2}{x^2} + 15 \frac{a}{x} - 20 + 15 \frac{x}{a} - 6 \frac{x^2}{a^2} + \frac{x^3}{a^3}
 \end{aligned}$$

Question # 2

Calculate the following by means of binomial theorem:

- (i) $(0.97)^3$ (ii) $(2.02)^4$ (iii) $(9.98)^4$ (iv) $(2.1)^5$

Solution (i) $(0.97)^3 = (1 - 0.03)^3$

$$\begin{aligned}
 &= \binom{3}{0}(1)^3 + \binom{3}{1}(1)^2(-0.03) + \binom{3}{2}(1)^1(-0.03)^2 + \binom{3}{3}(-0.03)^3
 \end{aligned}$$

$$\begin{aligned}
 &= (1)(1) + 3(1)(-0.03) + 3(1)(0.0009) + +(1)(-0.000024) \\
 &= 1 - 0.09 + 0.0027 - 0.000027 = 0.912673
 \end{aligned}$$

(ii) $(2.02)^4 = (2 + 0.02)^4$ Now do yourself.

$$\begin{aligned}
 \text{(iii)} \quad (9.98)^4 &= (10 - 0.02)^4 \\
 &= \binom{4}{0}(10)^4 + \binom{4}{1}(10)^3(-0.02) + \binom{4}{2}(10)^2(-0.02)^2 + \binom{4}{3}(10)^1(-0.02)^3 \\
 &\quad + \binom{4}{4}(10)^0(-0.02)^4 \\
 &= (1)(10000) + 4(1000)(-0.02) + 6(100)(0.0004) + 4(10)(-0.000008) \\
 &\quad + (1)(1)(0.00000016) \\
 &= 10000 - 80 + 0.24 - 0.00032 + 0.00000016 = 9920.23968
 \end{aligned}$$

(iv) $(2.1)^5 = (2 + 0.1)^5$ Now do yourself.

Question # 3

Expand and simplify the following:

$$\text{(i)} \quad (a + \sqrt{2}x)^4 + (a - \sqrt{2}x)^4 \quad \text{(ii)} \quad (2 + \sqrt{3})^5 + (2 - \sqrt{3})^5$$

$$\text{(iii)} \quad (2 + i)^5 - (2 - i)^5 \quad \text{(iv)} \quad (x + \sqrt{x^2 - 1})^3 + (x - \sqrt{x^2 - 1})^3$$

Solution (i) $(a + \sqrt{2}x)^4 + (a - \sqrt{2}x)^4$

We take

$$\begin{aligned}
 &(a + \sqrt{2}x)^4 \\
 &= \binom{4}{0}a^4 + \binom{4}{1}a^3(\sqrt{2}x)^1 + \binom{4}{2}a^2(\sqrt{2}x)^2 + \binom{4}{3}a^1(\sqrt{2}x)^3 + \binom{4}{4}a^0(\sqrt{2}x)^4 \\
 &= (1)a^4 + (4)a^3(\sqrt{2}x) + (6)a^2(2x^2) + (4)a(2\sqrt{2}x^3) + (1)(1)(4x^4) \\
 &\Rightarrow (a + \sqrt{2}x)^4 = a^4 + 4\sqrt{2}a^3x + 12a^2x^2 + 8\sqrt{2}ax^3 + 4x^4 \dots \dots \dots \text{(i)}
 \end{aligned}$$

Replacing $\sqrt{2}$ by $-\sqrt{2}$ in eq. (i)

$$\begin{aligned}
 (a - \sqrt{2}x)^4 &= a^4 + 4(-\sqrt{2})a^3x + 12a^2x^2 + 8(-\sqrt{2})ax^3 + 4x^4 \\
 &= a^4 - 4\sqrt{2}a^3x + 12a^2x^2 - 8\sqrt{2}ax^3 + 4x^4 \dots \dots \dots \text{(ii)}
 \end{aligned}$$

Adding (i) & (ii)

$$(a + \sqrt{2}x)^4 + (a - \sqrt{2}x)^4 = 2a^4 + 24a^2x^2 + 8x^4$$

(ii) Do yourself.

(iii) Since

$$\begin{aligned}
 (2+i)^5 &= \binom{5}{0} 2^5 + \binom{5}{1} 2^{5-1} i + \binom{5}{2} 2^{5-2} i^2 + \binom{5}{3} 2^{5-3} i^3 + \binom{5}{4} 2^{5-4} i^4 + \binom{5}{5} 2^{5-5} i^5 \\
 &= (1)2^5 + (5)2^4 i + (10)2^3 i^2 + (10)2^2 i^3 + (5)2^1 i^4 + (1)2^0 i^5 \\
 &= 32 + 80i + 80i^2 + 40i^3 + 10i^4 + i^5 \quad \dots \text{(i)}
 \end{aligned}$$

Replacing i by $-i$ in eq. (i)

$$\begin{aligned}
 (2+i)^5 &= 32 + 80(-i) + 80(-i)^2 + 40(-i)^3 + 10(-i)^4 + (-i)^5 \\
 &= 32 - 80i + 80i^2 - 40i^3 + 10i^4 - i^5 \quad \dots \text{(ii)}
 \end{aligned}$$

Subtracting (i) & (ii)

$$\begin{aligned}
 (2+i)^5 - (2-i)^5 &= 160i + 80i^3 + 2i^5 \\
 &= 160i + 80(-1) \cdot i + 2(-1)^2 \cdot i \\
 &= 160i - 80i + 2i = 82i
 \end{aligned}$$

$$(iv) \quad \left(x + \sqrt{x^2 - 1} \right)^3 + \left(x - \sqrt{x^2 - 1} \right)^3$$

Suppose $t = \sqrt{x^2 - 1}$ then

$$\left(x + \sqrt{x^2 - 1} \right)^3 + \left(x - \sqrt{x^2 - 1} \right)^3 = (x+t)^3 + (x-t)^3$$

$$\begin{aligned}
 &= ((x)^3 + 3(x)^2(t) + 3(x)(t)^2 + (t)^3) + ((x)^3 + 3(x)^2(-t) + 3(x)(-t)^2 + (-t)^3) \\
 &= x^3 + 3x^2t + 3xt^2 + t^3 + x^3 - 3x^2t + 3xt^2 - t^3 \\
 &= 2x^3 + 6xt^2 \\
 &= 2x^3 + 6x(\sqrt{x^2 - 1})^2 \quad \because t = \sqrt{x^2 - 1} \\
 &= 2x^3 + 6x(x^2 - 1) = 2x^3 + 6x^3 - 6x = 8x^3 - 6x
 \end{aligned}$$

Question # 4

Expand the following in ascending powers of x :

$$(i) (2+x-x^2)^4 \quad (ii) (1-x+x^2)^4 \quad (iii) (1-x-x^2)^4$$

Solution (i) $(2+x-x^2)^4$

Put $t = 2+x$ then

$$\begin{aligned}
 (2+x-x^2)^4 &= (t-x^2)^4 \\
 &= \binom{4}{0}(t)^4 + \binom{4}{1}(t)^3(-x^2) + \binom{4}{2}(t)^2(-x^2)^2 + \binom{4}{3}(t)^1(-x^2)^3 + \binom{4}{4}(t)^0(-x^2)^4 \\
 &= (1)(t)^4 - (4)(t)^3(x^2) + (6)(t)^2(x^4) - (4)(t)(x^6) + (1)(1)(x^8) \\
 &= t^4 - 4t^3x^2 + 6t^2x^4 - 4tx^6 + x^8 \quad \dots \text{(i)}
 \end{aligned}$$

Now

$$\begin{aligned}
 t^4 &= (2+x)^4 = \binom{4}{0}(2)^4 + \binom{4}{1}(2)^3(x) + \binom{4}{2}(2)^2(x)^2 + \binom{4}{3}(2)^1(x)^3 + \binom{4}{4}(2)^0(x)^4 \\
 &= (1)(16) + (4)(8)(x) + (6)(4)(x^2) + (4)(2)(x^3) + (1)(1)(x^4) \\
 &= 16 + 32x + 24x^2 + 8x^3 + x^4
 \end{aligned}$$

Also

$$\begin{aligned} t^3 &= (2+x)^3 = (2)^3 + (3)(2)^2(x) + (3)(2)^1(x)^2 + (x)^3 \\ &= 8 + 12x + 6x^2 + x^3 \\ t^2 &= (2+x)^2 = 4 + 4x + x^2 \end{aligned}$$

Putting values of t^4, t^3, t^2 and t in equation (i)

$$\begin{aligned} (2+x-x^2)^4 &= (16+32x+24x^2+8x^3+x^4)-4(8+12x+6x^2+x^3)x^2 \\ &\quad +6(4+4x+x^2)x^4-4(2+x)x^6+x^8 \\ &= 16+32x+24x^2+8x^3+x^4-32x^2-48x^3-24x^4-4x^5 \\ &\quad +24x^4+24x^5+6x^6-8x^6+4x^7+x^8 \\ &= 16+32-8x^2-40x^3+x^4+20x^5-2x^6-4x^7-x^8 \end{aligned}$$

- (ii) Suppose $t = 1-x$ Do yourself
 (iii) Suppose $t = 1-x$ Do yourself
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Question # 5

Expand the following in descending powers of x :

$$(i) (x^2 + x - 1)^3 \qquad (ii) \left(x - 1 - \frac{1}{x}\right)^3$$

Solution (i) Suppose $t = x-1$ Do yourself

$$(ii) \left(x - 1 - \frac{1}{x}\right)^3$$

Suppose $t = x-1$ then

$$\begin{aligned} \left(t - \frac{1}{x}\right)^3 &= (t)^3 + 3(t)^2\left(-\frac{1}{x}\right) + 3(t)\left(-\frac{1}{x}\right)^2 + \left(-\frac{1}{x}\right)^3 \\ &= t^3 - 3t^2 \cdot \frac{1}{x} + 3t \cdot \frac{1}{x^2} - \frac{1}{x^3} \dots \dots \dots (i) \end{aligned}$$

Now

$$\begin{aligned} t^3 &= (x-1)^3 = (x)^3 + 3(x)^2(-1) + 3(x)(-1)^2 + (-1)^3 \\ &= x^3 - 3x^2 + 3x - 1 \end{aligned}$$

$$t^2 = (x-1)^2 = x^2 - 2x + 1$$

Putting values of t^3, t^2 and t in equation (i)

$$\begin{aligned} \left(x - 1 - \frac{1}{x}\right)^3 &= (x^3 - 3x^2 + 3x - 1) - 3(x^2 - 2x + 1) \cdot \frac{1}{x} + 3(x-1) \cdot \frac{1}{x^2} - \frac{1}{x^3} \\ &= x^3 - 3x^2 + 3x - 1 - 3x + 6 - 3\frac{1}{x} + 3\frac{1}{x} - 3\frac{1}{x^2} - \frac{1}{x^3} \\ &= x^3 - 3x^2 + 5 - \frac{3}{x^2} - \frac{1}{x^3} \end{aligned}$$

Question # 6

Find the term involving:

- (i) x^4 in the expansion of $(3-2x)^7$
- $$\left(x-\frac{2}{x^2}\right)^{13}$$
- (ii) x^{-2} in the expansion of
- (iii) a^4 in the expansion of $\left(\frac{2}{x}-a\right)^9$
- (iv) y^3 in the expansion of $(x-\sqrt{y})^{11}$

Solution (i) Since

$$T_{r+1} = \binom{n}{r} a^{n-r} x^r$$

Here $a=3$, $x=-2x$, $n=7$ so

$$T_{r+1} = \binom{7}{r} (3)^{7-r} (-2x)^r = \binom{7}{r} (3)^{7-r} (-2)^r (x)^r$$

For term involving x^4 we must have

$$x^r = x^4 \Rightarrow r=4$$

So

$$T_{4+1} = \binom{7}{4} (3)^{7-4} (-2)^4 (x)^4$$

$$\Rightarrow T_5 = (35)(3)^3(-2)^4 (x)^4 = (35)(27)(16)(x)^4 \\ = 15120x^4$$

- (ii) Since $T_{r+1} = \binom{n}{r} a^{n-r} x^r$

Here $a=x$, $x=-\frac{2}{x^2}$, $n=13$ so

$$T_{r+1} = \binom{13}{r} (x)^{13-r} \left(-\frac{2}{x^2}\right)^r = \binom{13}{r} (x)^{13-r} (-2)^r (x)^{-2r} \\ = \binom{13}{r} (x)^{13-r-2r} (-2)^r = \binom{13}{r} (x)^{13-3r} (-2)^r$$

For term involving x^{-2} we must have

$$x^{13-3r} = x^{-2} \Rightarrow 13-3r = -2 \Rightarrow -3r = -2-13 \\ \Rightarrow -3r = -15 \Rightarrow r = 5$$

So

$$T_{5+1} = \binom{13}{5} (x)^{13-3(5)} (-2)^5$$

$$\Rightarrow T_6 = (1287)(x)^{13-15} (-32) = -41184x^{-2}$$

- (iii) Since

$$T_{r+1} = \binom{n}{r} a^{n-r} x^r$$

Here $a = \frac{2}{x}$, $x = -a$, $n = 9$ so

$$T_{r+1} = \binom{9}{r} \left(\frac{2}{x}\right)^{9-r} (-a)^r = \binom{9}{r} \left(\frac{2}{x}\right)^{9-r} (-1)^r (a)^r$$

For term involving a^4 we must have

$$a^r = a^4 \Rightarrow r = 4$$

$$\text{So } T_{4+1} = \binom{9}{4} \left(\frac{2}{x}\right)^{9-4} (-1)^4 (a)^4$$

$$\Rightarrow T_5 = (126) \left(\frac{2}{x}\right)^5 (1) a^4 = (126) \left(\frac{32}{x^5}\right) a^4 = 4032 \frac{a^4}{x^5}$$

(iv) Here $a = x$, $x = -\sqrt{y}$, $n = 11$ so

Since

$$\begin{aligned} T_{r+1} &= \binom{n}{r} a^{n-r} x^r \\ T_{r+1} &= \binom{11}{r} (x)^{11-r} (-\sqrt{y})^r = \binom{11}{r} (x)^{11-r} \left(-y^{\frac{1}{2}}\right)^r \\ &= \binom{11}{r} (x)^{11-r} (-1)^r \left(y^{\frac{r}{2}}\right) \end{aligned}$$

For term involving y^3 we must have

$$y^{\frac{r}{2}} = y^3 \Rightarrow \frac{r}{2} = 3 \Rightarrow r = 6$$

$$\text{So } T_{6+1} = \binom{11}{6} (x)^{11-6} (-1)^6 \left(y^{\frac{6}{2}}\right)$$

$$\Rightarrow T_7 = (462) (x)^5 (1) (y^3) = 462 x^5 y^3$$

Question # 7

Find the coefficient of;

$$(i) x^5 \text{ in the expansion of } \left(x^2 - \frac{3}{2x}\right)^{10} \quad (ii) x^n \text{ in the expansion of } \left(x^2 - \frac{1}{x}\right)^{2n}$$

Solution (i) Here $a = x^2$, $x = -\frac{3}{2x}$, $n = 10$ so

Since

$$\begin{aligned} T_{r+1} &= \binom{n}{r} a^{n-r} x^r \\ T_{r+1} &= \binom{10}{r} (x^2)^{10-r} \left(-\frac{3}{2x}\right)^r = \binom{10}{r} (x)^{2(10-r)} (-1)^r \frac{(3)^r}{(2)^r (x)^r} \\ &= \binom{10}{r} (x)^{20-2r} (-1)^r (3)^r (2)^{-r} (x)^{-r} = \binom{10}{r} (x)^{20-2r-r} (-1)^r (3)^r (2)^{-r} \end{aligned}$$

$$= \binom{10}{r} (x)^{20-3r} (-1)^r (3)^r (2)^{-r}$$

For term involving x^5 we must have

$$\begin{aligned} x^{20-3r} &= x^5 \Rightarrow 20-3r=5 \Rightarrow -3r=5-20 \\ \Rightarrow -3r &= -15 \Rightarrow r=5 \end{aligned}$$

$$\text{So } T_{5+1} = \binom{10}{5} (x)^{20-3(5)} (-1)^5 (3)^5 (2)^{-5}$$

$$\begin{aligned} \Rightarrow T_6 &= 252(x)^{20-15} (-1)^5 (3)^5 \frac{1}{2^5} = -252(x)^5 (243) \frac{1}{32} \\ &= -\frac{61236}{32} x^5 = -\frac{15309}{8} x^5 \end{aligned}$$

$$\text{Hence coefficient of } x^5 = -\frac{15309}{8}$$

- (ii) Here $a=x^2$, $x=-\frac{1}{x}$, $n=2n$ so

Since

$$\begin{aligned} T_{r+1} &= \binom{n}{r} a^{n-r} x^r \\ T_{r+1} &= \binom{2n}{r} (x^2)^{2n-r} \left(-\frac{1}{x}\right)^r = \binom{2n}{r} (x)^{2(2n-r)} (-1)^r \frac{1}{x^r} \\ &= \binom{2n}{r} (x)^{4n-2r} (-1)^r x^{-r} = \binom{2n}{r} (x)^{4n-2r-r} (-1)^r \\ &= \binom{2n}{r} (x)^{4n-3r} (-1)^r \end{aligned}$$

For term involving x^n we must have

$$\begin{aligned} x^{4n-3r} &= x^n \Rightarrow 4n-3r=n \Rightarrow -3r=n-4n \\ \Rightarrow -3r &= -3n \Rightarrow r=n \end{aligned}$$

$$\begin{aligned} \text{So } T_{n+1} &= \binom{2n}{n} (x)^{4n-3n} (-1)^n \\ &= \frac{(2n)!}{(2n-n)! \cdot n!} (x)^n (-1)^n = \frac{(2n)!}{n! \cdot n!} (x)^n (-1)^n \\ &= (-1)^n \frac{(2n)!}{(n!)^2} x^n \end{aligned}$$

$$\text{Hence coefficient of } x^n = (-1)^n \frac{(2n)!}{(n!)^2}$$

Question # 8

Find 6th term in the expansion of $\left(x^2 - \frac{3}{2x}\right)^{10}$

Solution Here $a = x^2$, $x = -\frac{3}{2x}$, $n = 10$ and $r+1=6 \Rightarrow r=5$ so

Since

$$\begin{aligned} T_{r+1} &= \binom{n}{r} a^{n-r} x^r \\ T_{5+1} &= \binom{10}{5} (x^2)^{10-5} \left(-\frac{3}{2x}\right)^5 \\ \Rightarrow T_6 &= 252 (x^2)^5 \left(-\frac{3^5}{(2x)^5}\right) = 252 x^{10} \left(-\frac{243}{32x^5}\right) \\ &= -\frac{61236}{32} x^{10-5} = -\frac{15309}{8} x^5 \end{aligned}$$

Question # 9

Find the term independent of x in the following expansions..

$$(i) \left(x - \frac{2}{x}\right)^{10} \quad (ii) \left(\sqrt{x} - \frac{1}{2x^2}\right)^{10} \quad (iii) \left(1 + x^2\right)^3 \left(1 + \frac{1}{x^2}\right)^4$$

Solution (i) Do yourself as Q # 9 (ii)

(ii) Here $a = \sqrt{x}$, $x = \frac{1}{2x^2}$, $n = 10$ so

Since

$$\begin{aligned} T_{r+1} &= \binom{n}{r} a^{n-r} x^r \\ T_{r+1} &= \binom{10}{r} \left(\sqrt{x}\right)^{10-r} \left(\frac{1}{2x^2}\right)^r = \binom{10}{r} \left(x^{\frac{1}{2}}\right)^{10-r} \left(\frac{1}{2^r x^{2r}}\right) \\ &= \binom{10}{r} \left(x^{\frac{1}{2}(10-r)} \frac{1}{2^r} x^{-2r}\right) = \binom{10}{r} \left(x^{5-\frac{r}{2}} \frac{1}{2^r} x^{-2r}\right) \\ &= \binom{10}{r} \left(x^{5-\frac{r}{2}-2r} \frac{1}{2^r}\right) = \binom{10}{r} \left(x^{5-\frac{5r}{2}} \frac{1}{2^r}\right) \end{aligned}$$

For term independent of x we must have

$$x^{5-\frac{5r}{2}} = x^0 \Rightarrow 5 - \frac{5r}{2} = 0 \Rightarrow -\frac{5r}{2} = -5$$

$$\Rightarrow r = (-5) \left(-\frac{2}{5}\right) \Rightarrow r = 2$$

$$\text{So } T_{2+1} = \binom{10}{2} \left(x^{5-\frac{5(2)}{2}} \frac{1}{2^2}\right)$$

$$\begin{aligned} \Rightarrow T_3 &= 45 \left(x^{5-5} \frac{1}{4}\right) = 45 x^0 \frac{1}{4} \\ &= 45 (1) \frac{1}{4} = \frac{45}{4} \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad (1+x^2)^3 \left(1+\frac{1}{x^2}\right)^4 &= (1+x^2)^3 \left(\frac{x^2+1}{x^2}\right)^4 \\
 &= (1+x^2)^3 \frac{(x^2+1)^4}{(x^2)^4} = (1+x^2)^3 \frac{(1+x^2)^4}{x^8} \\
 &= x^{-8}(1+x^2)^{3+4} = x^{-8}(1+x^2)^7
 \end{aligned}$$

Now $T_{r+1} = x^{-8} \binom{n}{r} a^{n-r} x^r$

Where $n=7, a=1, x=x^2$

$$\begin{aligned}
 T_{r+1} &= x^{-8} \binom{7}{r} (1)^{7-r} (x^2)^r = x^{-8} \binom{7}{r} (1) x^{2r} \\
 &= \binom{7}{r} x^{2r-8}
 \end{aligned}$$

For term independent of x we must have

$$x^{2r-8} = x^0 \Rightarrow 2r-8=0 \Rightarrow 2r=8 \Rightarrow r=4$$

So

$$\begin{aligned}
 T_{4+1} &= \binom{7}{4} x^{2(4)-8} \\
 \Rightarrow T_5 &= 35 x^{8-8} = 35 x^0 = 35
 \end{aligned}$$

Question # 10

Determine the middle term in the following expansions:

$$\begin{array}{lll}
 \text{(i)} \left(\frac{1}{x} - \frac{x^2}{2}\right)^{12} & \text{(ii)} \left(\frac{3}{2}x - \frac{1}{3x}\right)^{11} & \text{(iii)} \left(2x - \frac{1}{2x}\right)^{2m+1}
 \end{array}$$

Solution (i) $\left(\frac{1}{x} - \frac{x^2}{2}\right)^{12}$

Since $n=12$ is an even so middle terms is $\frac{n+2}{2} = \frac{12+2}{2} = 7$

Therefore $r+1=7 \Rightarrow r=7-1=6$

And $a=\frac{1}{x}, x=-\frac{x^2}{2}$ and $n=12$

Now

$$\begin{aligned}
 T_{r+1} &= \binom{n}{r} a^{n-r} x^r \\
 \Rightarrow T_{6+1} &= \binom{12}{6} \left(\frac{1}{x}\right)^{12-6} \left(-\frac{x^2}{2}\right)^6
 \end{aligned}$$

$$\Rightarrow T_7 = 924 \frac{1}{x^6} \frac{x^{12}}{64} = \frac{924}{64} x^{12-6} \\ = \frac{231}{16} x^6$$

Thus the middle terms of the given expansion is $\frac{231}{16} x^6$.

- (ii) Since $n=11$ is odd so the middle terms are $\frac{n+1}{2} = \frac{11+1}{2} = 6$ and

$$\frac{n+3}{2} = \frac{11+3}{2} = 7$$

So for first middle term

$$a = \frac{3}{2}x, \quad x = -\frac{1}{3x}, \quad n = 11 \text{ and } r+1 = 6 \Rightarrow r = 5$$

Now

$$T_{r+1} = \binom{n}{r} a^{n-r} x^r \Rightarrow T_{5+1} = \binom{11}{5} \left(\frac{3}{2}x\right)^{11-5} \left(-\frac{1}{3x}\right)^5$$

Now simplify yourself.

Now for second middle term

$$r+1 = 7 \Rightarrow r = 6$$

$$\text{so } T_{6+1} = \binom{11}{6} \left(\frac{3}{2}x\right)^{11-6} \left(-\frac{1}{3x}\right)^6 \quad \text{Now simplify yourself.}$$

- (iii) Since $n = 2m+1$ is odd so there are two middle terms

$$\text{First middle term} = \frac{n+1}{2} = \frac{2m+1+1}{2} = \frac{2m+2}{2} = m+1$$

$$\text{Second middle terms} = \frac{n+3}{2} = \frac{2m+1+3}{2} = \frac{2m+4}{2} = m+2$$

$$\text{Here } a = 2x, \quad x = -\frac{1}{2x} \text{ and } n = 2m+1$$

For first middle term $r+1 = m+1 \Rightarrow r = m$.

Since

$$T_{r+1} = \binom{n}{r} a^{n-r} x^r \\ \Rightarrow T_{m+1} = \binom{2m+1}{m} (2x)^{2m+1-m} \left(-\frac{1}{2x}\right)^m = \frac{(2m+1)!}{(2m+1-m)! \cdot m!} (2x)^{m+1} \left(-\frac{1}{2x}\right)^m \\ = \frac{(2m+1)!}{(m+1)! \cdot m!} (2)^{m+1} (x)^{m+1} (-1)^m \left(\frac{1}{2}\right)^m \left(\frac{1}{x}\right)^m \\ = \frac{(2m+1)!}{(m+1)! \cdot m!} (2)^{m+1} (x)^{m+1} (-1)^m (2)^{-m} (x)^{-m}$$

$$\begin{aligned}
 &= \frac{(2m+1)!}{(m+1)! \cdot m!} (2)^{m+1-m} (x)^{m+1-m} (-1)^m = \frac{(2m+1)!}{(m+1)! \cdot m!} (2)^1 (x)^1 (-1)^m \\
 &= \frac{(2m+1)!}{(m+1)! \cdot m!} 2x(-1)^m
 \end{aligned}$$

For second middle term

$$r+1 = m+2 \Rightarrow r = m+2-1 \Rightarrow r = m+1$$

$$\begin{aligned}
 \text{As } T_{r+1} &= \binom{n}{r} a^{n-r} x^r \\
 \Rightarrow T_{m+1+1} &= \binom{2m+1}{m+1} (2x)^{(2m+1)-(m+1)} \left(-\frac{1}{2x}\right)^{m+1}
 \end{aligned}$$

Now simplify yourself

Question # 11

Find $(2n+1)$ th term of the end in the expansion of $\left(x - \frac{1}{2x}\right)^{3n}$

Solution Here $a = x$, $x = -\frac{1}{2x}$,

Number of term from the end $= 2n + 1$

To make it from beginning we take $a = -\frac{1}{2x}$, $x = x$ and $r+1 = 2n+1$
 $\Rightarrow r = 2n$

$$\begin{aligned}
 \text{As } T_{r+1} &= \binom{n}{r} a^{n-r} x^r \\
 \Rightarrow T_{2n+1} &= \binom{3n}{2n} \left(-\frac{1}{2x}\right)^{3n-2n} (x)^{2n} = \frac{(3n)!}{(3n-2n)! \cdot (2n)!} \left(-\frac{1}{2x}\right)^n x^{2n} \\
 &= \frac{(3n)!}{(n)! \cdot (2n)!} (-1)^n \frac{1}{2^n \cdot x^n} x^{2n} = \frac{(3n)!}{n! \cdot (2n)!} (-1)^n \frac{1}{2^n} x^{2n-n} \\
 &= \frac{(-1)^n}{2^n} \frac{(3n)!}{n! \cdot (2n)!} x^n \quad \text{Answer}
 \end{aligned}$$

Note: If there are p term in some expansion and q th term is from the end then the term from the beginning will be $= p - q + 1$.

So in above you can use term from the end $= (3n+1) - (2n+1) + 1 = n + 1$

Question # 12

Show that the middle term of $(1+x)^{2n}$ is $\frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{n!} 2^n x^n$

Solution Since $2n$ is even so the middle term is $\frac{2n+2}{2} = n+1$ and

$$a = 1, \quad x = x, \quad n = 2n, \quad r+1 = n+1 \Rightarrow r = n$$

$$\text{Now } T_{r+1} = \binom{n}{r} a^{n-r} x^r$$

$$\begin{aligned}
\Rightarrow T_{n+1} &= \binom{2n}{n} (1)^{2n-n} x^n \\
\Rightarrow T_{n+1} &= \frac{(2n)!}{(2n-n)! \cdot n!} (1)^n x^n = \frac{(2n)!}{n! \cdot n!} x^n \\
&= \frac{2n(2n-1)(2n-2)(2n-3)(2n-4) \cdots 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{n! \cdot n!} x^n \\
&= \frac{[2n(2n-2)(2n-4) \cdots 4 \cdot 2][(2n-1)(2n-3) \cdots 5 \cdot 3 \cdot 1]}{n! \cdot n!} x^n \\
&= \frac{2^n [n(n-1)(n-2) \cdots 2 \cdot 1][(2n-1)(2n-3) \cdots 5 \cdot 3 \cdot 1]}{n! \cdot n!} x^n \\
&= \frac{2^n n! [(2n-1)(2n-3) \cdots 5 \cdot 3 \cdot 1]}{n! \cdot n!} x^n \\
&= \frac{2^n [1 \cdot 3 \cdot 5 \cdots (2n-1)]}{n!} x^n \\
&= \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{n!} 2^n x^n
\end{aligned}$$

Question # 13

Show that:

$$\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \cdots + \binom{n}{n-1} = 2^{n-1}$$

Solution Consider

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \binom{n}{4}x^4 + \binom{n}{5}x^5 + \cdots + \binom{n}{n-1}x^{n-1} + \binom{n}{n}x^n \quad \dots \text{(i)}$$

Put $x=1$

$$\begin{aligned}
(1+1)^n &= \binom{n}{0} + \binom{n}{1}(1) + \binom{n}{2}(1)^2 + \binom{n}{3}(1)^3 + \binom{n}{4}(1)^4 + \binom{n}{5}(1)^5 + \cdots + \binom{n}{n-1}(1)^{n-1} + \binom{n}{n}(1)^n \\
\Rightarrow 2^n &= \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \binom{n}{4} + \binom{n}{5} + \cdots + \binom{n}{n-1} + \binom{n}{n} \\
\Rightarrow 2^n &= \left[\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots + \binom{n}{n} \right] + \left[\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \cdots + \binom{n}{n-1} \right] \quad \dots \text{(ii)}
\end{aligned}$$

Now put $x=-1$ in equation (i)

$$\begin{aligned}
(1-1)^n &= \binom{n}{0} + \binom{n}{1}(-1) + \binom{n}{2}(-1)^2 + \binom{n}{3}(-1)^3 + \binom{n}{4}(-1)^4 + \binom{n}{5}(-1)^5 + \cdots \\
&\quad \cdots + \binom{n}{n-1}(-1)^{n-1} + \binom{n}{n}(-1)^n
\end{aligned}$$

If we consider n is even then

$$\Rightarrow (0)^n = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \binom{n}{4} - \binom{n}{5} + \cdots - \binom{n}{n-1} + \binom{n}{n}$$

$$\Rightarrow 0 = \left[\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \dots + \binom{n}{n} \right] - \left[\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n-1} \right]$$

$$\Rightarrow \left[\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n-1} \right] = \left[\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \dots + \binom{n}{n} \right]$$

Using it in equation (ii)

$$2^n = \left[\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n-1} \right] + \left[\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \dots + \binom{n}{n} \right]$$

$$\Rightarrow 2^n = 2 \left[\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n-1} \right]$$

$$\Rightarrow \frac{2^n}{2} = \left[\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n-1} \right]$$

$$\Rightarrow 2^{n-1} = \left[\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n-1} \right]$$

$$\Rightarrow \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n-1} = 2^{n-1}$$

Question # 14

Show that:

$$\binom{n}{0} + \frac{1}{2} \binom{n}{1} + \frac{1}{3} \binom{n}{2} + \frac{1}{4} \binom{n}{3} + \dots + \frac{1}{n+1} \binom{n}{n} = \frac{2^{n+1} - 1}{n+1}$$

Solution

$$\begin{aligned} \text{L.H.S} &= \binom{n}{0} + \frac{1}{2} \binom{n}{1} + \frac{1}{3} \binom{n}{2} + \frac{1}{4} \binom{n}{3} + \dots + \frac{1}{n+1} \binom{n}{n} \\ &= \left[\binom{n}{0} + \frac{1}{2} \left(\frac{n!}{(n-1)! \cdot 1!} \right) + \frac{1}{3} \left(\frac{n!}{(n-2)! \cdot 2!} \right) + \frac{1}{4} \left(\frac{n!}{(n-3)! \cdot 3!} \right) + \dots + \frac{1}{n+1} \binom{n}{n} \right] \\ &= \frac{n+1}{n+1} \left[1 + \frac{1}{2} \left(\frac{n!}{(n-1)! \cdot 1!} \right) + \frac{1}{3} \left(\frac{n!}{(n-2)! \cdot 2!} \right) + \frac{1}{4} \left(\frac{n!}{(n-3)! \cdot 3!} \right) + \dots + \frac{1}{n+1} \cdot 1 \right] \\ &= \frac{1}{n+1} \left[(n+1) + \frac{1}{2} \left(\frac{(n+1)n!}{(n-1)! \cdot 1!} \right) + \frac{1}{3} \left(\frac{(n+1)n!}{(n-2)! \cdot 2!} \right) + \frac{1}{4} \left(\frac{(n+1)n!}{(n-3)! \cdot 3!} \right) + \dots + \frac{n+1}{n+1} \cdot 1 \right] \\ &= \frac{1}{n+1} \left[(n+1) + \left(\frac{(n+1)!}{(n-1)! \cdot 2 \cdot 1!} \right) + \left(\frac{(n+1)!}{(n-2)! \cdot 3 \cdot 2!} \right) + \left(\frac{(n+1)!}{(n-3)! \cdot 4 \cdot 3!} \right) + \dots + 1 \right] \\ &= \frac{1}{n+1} \left[(n+1) + \left(\frac{(n+1)!}{(n+1-2)! \cdot 2!} \right) + \left(\frac{(n+1)!}{(n+1-3)! \cdot 3!} \right) + \left(\frac{(n+1)!}{(n+1-4)! \cdot 4!} \right) + \dots + 1 \right] \\ &= \frac{1}{n+1} \left[\binom{n+1}{1} + \binom{n+1}{2} + \binom{n+1}{3} + \binom{n+1}{4} + \dots + \binom{n+1}{n+1} \right] \\ &= \frac{1}{n+1} \left[-1 + 1 + \binom{n+1}{1} + \binom{n+1}{2} + \binom{n+1}{3} + \binom{n+1}{4} + \dots + \binom{n+1}{n+1} \right] \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{n+1} \left[-1 + \binom{n+1}{0} + \binom{n+1}{1} + \binom{n+1}{2} + \binom{n+1}{3} + \binom{n+1}{4} + \dots + \binom{n+1}{n+1} \right] \\
 &= \frac{1}{n+1} \left[-1 + 2^{n+1} \right] \\
 &= \frac{2^{n+1} - 1}{n+1} = \text{R.H.S}
 \end{aligned}$$

Remember

$$\binom{n+1}{0} = 1, \quad \binom{n+1}{1} = n+1 \quad \text{and} \quad \binom{n+1}{n+1} = 1$$

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Book: **Exercise 8.2**

*Text Book of Algebra and Trigonometry Class XI
Punjab Textbook Board, Lahore.*

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