Question # 1

Express the following sexagesimal measures of angles in radians:

(i) $30°$ (ii) $45°$ (x) $10°15'$ (xii) $75°6'30''$

Solutions

(i) $30° = 30 \times \dfrac{\pi}{180}$ radian $= \dfrac{\pi}{6}$ radian

(ii) $45° = 45 \times \dfrac{\pi}{180}$ radian $= \dfrac{\pi}{4}$ radian

(x) $10°15' = 10.25° = 10.25 \times \dfrac{\pi}{180}$ radian $= \dfrac{1025\pi}{18000}$ radian $= \dfrac{41\pi}{720}$ radian

(xii) $75°6'30'' = 75.1083° = 75.1083 \times \dfrac{\pi}{180}$ radian

$\qquad = 751083 \times \dfrac{\pi}{1800000}$ radian $= 9013 \times \dfrac{\pi}{21600}$ radian

Remaining do your self

Question # 2

Convert the following radian measures of angles into the measures of sexagesimal system:

(i) $\dfrac{\pi}{8}$ rad   (xii) $\dfrac{13}{16}\pi$ rad

Solutions

(i) $\dfrac{\pi}{8}$ rad. $= \left(\dfrac{180}{8}\right)° = 22.5°$ $\because \pi$ rad $= 180°$

(xii) $\dfrac{13}{16}\pi$ rad. $= \left(\dfrac{13}{16} \times 180\right)° = 146.25°$ $\because \pi$ rad $= 180°$

Remaining do your self

Question # 3

What is the circular measure of the angle between the heads of the watch at 4'O clock?

Solution

Since total angle in watch $= 2\pi$ rad.

Angel made by hands in 1 hour $= \dfrac{2\pi}{12} = \dfrac{\pi}{6}$ rad.

Thus angle made by hand in 4 hours = $4 \times \dfrac{\pi}{6} = \dfrac{2\pi}{3}$ rad.

Question # 4

Find θ, when:

(i) $l = 1.5\,\text{cm}$, $r = 2.5\,\text{cm}$ (ii) $l = 3.2\,\text{m}$, $r = 2\,\text{m}$

Solution

(i) $l = 1.5\,\text{cm}$, $r = 2.5\,\text{cm}$

Since $\theta = \dfrac{l}{r}$ $\Rightarrow$ $\theta = \dfrac{1.5}{2.5} = 0.6$ rad.

(ii) $l = 3.2\,\text{m}$, $r = 2\,\text{m}$

Since $\theta = \dfrac{l}{r}$ $\Rightarrow$ $\theta = \dfrac{3.2}{2} = 1.6$ rad.

Question # 5

Find *l*, when:

(i) $\theta = \pi\,\text{rad.}$ $r = 6\,\text{cm}$ (ii) $\theta = 65°20'$, $r = 18\,\text{mm}$

Solutions

(i) $\theta = \pi\,\text{rad.}$ $r = 6\,\text{cm}$

Since $l = r\theta$

$\Rightarrow l = 6\pi = 6(3.14159) = 18.85$ cm.

(ii) $\theta = 65°20' = 65°.33 = 65.33 \times \dfrac{\pi}{180} = 65.33 \times \dfrac{3.14159}{180} = 1.1403$ rad. ,

$r = 18\,\text{mm}$

Since $l = r\theta$

$\Rightarrow l = 18 \times 1.1403 = 20.5254$ mm

Question # 6

(i) Find r, when; $l = 5\,cm$, $\theta = \dfrac{1}{2}$ radian

(ii) Find r, when; $l = 56\,cm$, $\theta = 45°$

Solutions

(i) $l = 5\,\text{cm}$, $\theta = \dfrac{1}{2}$ rad

Since $l = r\theta$ $\Rightarrow$ $5 = r \times \dfrac{1}{2}$ $\Rightarrow$ $r = 5 \times 2 = 10\,\text{cm}$

(ii) $l = 56\,\text{cm}$, $\theta = 45° = 45 \times \dfrac{\pi}{180} = \dfrac{\pi}{4}$ rad

Since $l = r\theta$ $\Rightarrow$ $56 = r \times \dfrac{\pi}{4}$ $\Rightarrow$ $r = 56 \times \dfrac{4}{\pi} = \dfrac{224}{3.14159} = 71.30\,\text{cm}$

Question # 7

What is the length of arc intercepted on a circle of radius 14*cm* by the arms of a central angle of 45°?

Solution

$$l = ? \ , \qquad r = 14\,\text{cm}\,, \qquad \theta = \frac{\pi}{4}\,\text{rad}$$

Since $l = r\theta$

$$\Rightarrow \ l = 14 \times \frac{\pi}{4} = 14 \times \frac{3.14159}{4} = 10.9956\,\text{cm}$$

Question # 8

Find the radius of the circle, in which the arms of a central angle of measure 1 radian cut off an arc of length 35*cm*?

Solution

$$r = ?, \qquad \theta = 1\,\text{rad.}\,, \qquad l = 35\,\text{cm}$$

Since $l = r\theta \ \Rightarrow \ 35 = r \times 1 \ \Rightarrow \ r = 35\,\text{cm}$

Question # 9

A railway train is running on a circular track of radius 500 meters at the rate of 30km per hour. Through what angle will it turn in 10sec.

Solution

$$r = 500 \ \text{m}$$

$$\text{Speed} = 30\,\text{km/h} = 30 \times \frac{1000}{60 \times 60}\,\text{m/s} = \frac{25}{3}\,\text{m/s}$$

$$\text{Distance} = l = \frac{25}{3} \times 10 = \frac{250}{3}\,\text{m}$$

$$\text{Now} \quad l = r\theta \ \Rightarrow \ \frac{250}{3} = 500 \times \theta$$

$$\Rightarrow \ \theta = \frac{250}{3 \times 500} = \frac{250}{1500} = \frac{1}{6} \ \text{rad.}$$

Question # 10

A horse is tethered to a peg by a rope of 9 meters length and it can move in a circle with the peg as centre. If the horse moves along the circumference of the circle, keeping the rope tight, how far will it have gone when the rope has turned through an angle of 70° ?
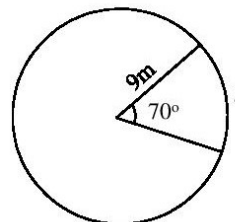
Solution

$$r = 9\,\text{m}\ , \quad \theta = 70° = 70 \times \frac{\pi}{180} = \frac{7}{18}\pi\,, \qquad l = ?$$

Now $l = r\theta$

$$\Rightarrow \ l = 9 \times \frac{7}{18}\pi = \frac{7}{2}(3.14159) = 10.9956\,\text{m} \approx 11\,\text{m}$$

Thus the horse will cover 11m distance.

Question # 11

The pendulum of a clock is $20cm$ long and its swings through an angle of 20° each second. How far does the tip of the pendulum move in 1 second?
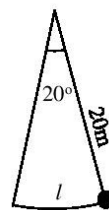
Solution

The pendulum of a clock is $20cm$ long and

$r = 20\,\text{cm}$, $\qquad \theta = 20^\circ = 20\times\dfrac{\pi}{180} = \dfrac{\pi}{9}\,\text{rad}$

Now $l = r\theta \quad \Rightarrow l = 20\times\dfrac{\pi}{9} = 20\times\dfrac{3.14159}{9} = 6.98$

Thus pendulum will move 6.98cm.

Question # 12

Assuming the average distance of the earth from the sun to be 148×10^6 km and an angle subtended by the sun at the eye of a person on the earth of measure 9.3×10^{-3} radian. Find the diameter of the sun.
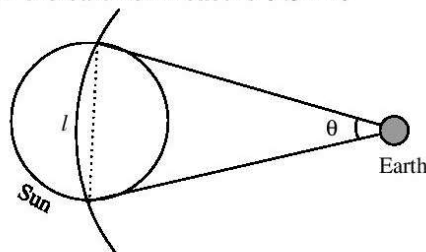
Solution.

Here $r = 148\times10^6\,\text{km}$,

$\qquad \theta = 9.3\times10^{-3}\,\text{rad}$

Since $l = r\theta$

$\qquad \Rightarrow l = \left(148\times10^6\right)\left(9.3\times10^{-3}\right) = 1376400\,\text{km}$

$\qquad\qquad\qquad\qquad = 1.3764\times10^6\,\text{km}$

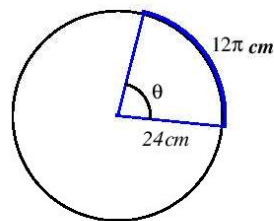Thus diameter of sun $= 1.3764\times10^6\,\text{km}$

Question # 13

A circular wire of radius 6 cm is cut straightened and then bent so as to lie along the circumference of a hoop of radius 24cm. Find the measure of the angle which it subtends at the centre of the hoop.

Solution.

Length of wire = circumference of circle

$\qquad\qquad = 2\pi r' = 2\pi(6) = 12\pi$

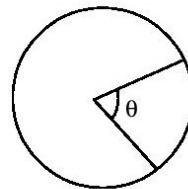i.e. $l = 12\pi\,\text{cm}$, $\quad r = 24\,\text{cm}$

Now $\theta = \dfrac{l}{r} \quad \Rightarrow \quad \theta = \dfrac{12\pi}{24} = \dfrac{\pi}{2}\text{rad}.$

Question # 14

Show that the area of a sector of a circular region of radius r is $\dfrac{1}{2}r^2\theta$, where θ is the circular measure of the central angle of the sector.

Solution

Since $\dfrac{Area\ of\ Sector}{Area\ of\ Circle} = \dfrac{Central\ angle\ of\ Sector}{Angel\ of\ Circle}$

$$\Rightarrow \quad \frac{Area\ of\ Sector}{\pi r^2} = \frac{\theta}{2\pi}$$

$$\Rightarrow \quad Area\ of\ Sector = \frac{\theta}{2\pi} \times \pi r^2$$

$$= \frac{1}{2} r^2 \theta$$

Question # 15

Two cities A and B lie on the equator such that there longitudes are $45^o E$ and $25^o W$ respectively. Find the distance between the two cities, taking radius of the earth as $6400 kms$.
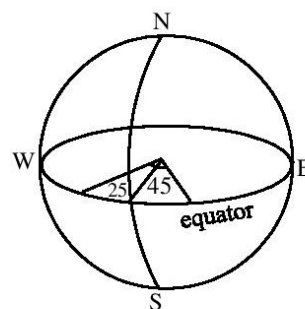
Solution

$r = 6400\,\text{km}$

$\theta = 45^\circ + 25^\circ = 70^\circ = 70 \times \dfrac{\pi}{180} = \dfrac{7}{18}\pi\,\text{rad}$

Now $l = r\theta$

$\Rightarrow \ l = (6400)\left(\dfrac{7}{18}\pi\right) = (6400)(0.3889 \times 3.14159)$

$\qquad\qquad\qquad = 7819.075$

Thus distance between cities $= 7819.075$ km

Question # 16

The moon subtended an angle of 0.5° at the eye of an observer on earth. The distance of the moon from the earth is 3.844×10^5 Km approx. taking radius of the earth as 6400 kms.

Solution

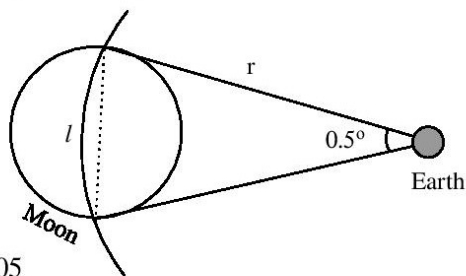$\theta = 0.5^\circ = 0.5 \times \dfrac{\pi}{180} = 0.5 \times (0.01745)$

$\qquad\qquad = 0.008727\,\text{rad}$

$r = 3.844 \times 10^5\,\text{km}$

Now $l = r\theta$

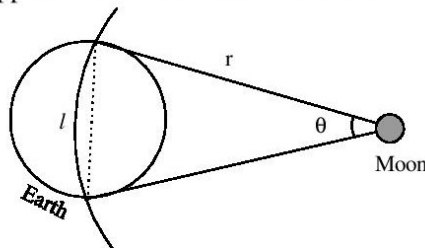$\Rightarrow \ l = \left(3.844 \times 10^5\right)(0.0087266) = 3354.505$

Thus diameter of the moon $= 3354.505\,\text{km}$

Question # 17

The angle subtended by the earth at the eye of a spaceman, landed on the moon is $1^\circ 54'$. The radius of the earth is 6400km. Find the approximate distance between the moon and earth.

Solution

$\theta = 1^\circ 54' = 1.9^\circ = 1.9 \times \dfrac{\pi}{180} = 0.03316\,\text{rad}$

$l = 2(6400) = 12800\,\text{km}$, $\qquad r = ?$

Now $l = r\theta \Rightarrow 12800 = r(0.03316)$

$$\Rightarrow \quad r = \frac{12800}{0.03316} = 386007.24\,\text{km}$$

Thus distance between earth and moon = 386007.24km