

Q No 1 L.H.S =  $\tan\theta + \cot\theta$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}$$

$$= \frac{1}{\cos\theta \cdot \sin\theta}$$

$$= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} = \sec\theta \cdot \operatorname{cosec}\theta$$

$$= \operatorname{cosec}\theta \cdot \sec\theta = R.H.S \text{ proved.}$$

Domain =  $\{\theta | \theta \in \mathbb{R} \text{ but } \theta \neq n\frac{\pi}{2}, n \in \mathbb{Z}\}$

Q No 2 L.H.S =  $\sec\theta \cdot \operatorname{cosec}\theta \cdot \sin\theta \cdot \cos\theta$

$$= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} \cdot \sin\theta \cdot \cos\theta$$

$$= 1 = R.H.S$$

Domain =  $\mathbb{R} - \{n\frac{\pi}{2}\}; n \in \mathbb{Z}$

Q No. 3 L.H.S =  $\cos\theta + \tan\theta \cdot \sin\theta$

$$= \cos\theta + \frac{\sin\theta}{\cos\theta} \cdot \sin\theta$$

$$= \cos\theta + \frac{\sin^2\theta}{\cos\theta} = \frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} = \sec\theta = R.H.S \text{ proved.}$$

Domain =  $\mathbb{R} - \{(2n+1)\frac{\pi}{2}\}; n \in \mathbb{Z}$

Q No. 4 L.H.S =  $\operatorname{cosec}\theta + \tan\theta \sec\theta$

$$= \frac{1}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\sin\theta}{\cos^2\theta} = \frac{\cos^2\theta + \sin^2\theta}{\sin\theta \cos^2\theta}$$

$$= \frac{1}{\sin\theta \cdot \cos^2\theta} = \frac{1}{\sin\theta} \cdot \frac{1}{\cos^2\theta}$$

$$= \operatorname{cosec}\theta \cdot \sec^2\theta = R.H.S \text{ proved}$$

Domain =  $\{\theta | \theta \in \mathbb{R} \text{ but } \theta \neq n\frac{\pi}{2}, n \in \mathbb{Z}\}$

$$\begin{aligned}
 Q\text{No}5. \quad L.H.S &= \sec^2\theta - \csc^2\theta && \because 1 + \cot^2\theta = \csc^2\theta \\
 &= (1 + \tan^2\theta) - (1 + \cot^2\theta) && 1 + \tan^2\theta = \sec^2\theta \\
 &= 1 + \tan^2\theta - 1 - \cot^2\theta \\
 &= \tan^2\theta - \cot^2\theta = R.H.S \quad \text{proved}
 \end{aligned}$$

$$\begin{aligned}
 Q\text{No}6. \quad L.H.S &= \cot^2\theta - \cos^2\theta \\
 &= \frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta = \cos^2\theta \left( \frac{1}{\sin^2\theta} - 1 \right) \\
 &= \cos^2\theta \left( \frac{1 - \sin^2\theta}{\sin^2\theta} \right) \\
 &= \cos^2\theta \left( \frac{\cos^2\theta}{\sin^2\theta} \right) && \because 1 - \sin^2\theta = \cos^2\theta \\
 &= \cos^2\theta \cdot \cot^2\theta = R.H.S
 \end{aligned}$$

$$\begin{aligned}
 Q\text{No}7. \quad L.H.S &= (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) \\
 &= \sec^2\theta - \tan^2\theta \\
 &= (1 + \tan^2\theta) - \tan^2\theta && \because 1 + \tan^2\theta = \sec^2\theta \\
 &= 1 + \tan^2\theta - \tan^2\theta \\
 &= 1 = R.H.S
 \end{aligned}$$

$$\begin{aligned}
 Q\text{No}8. \quad L.H.S &= 2\cos^2\theta - 1 && \because \sin^2\theta + \cos^2\theta = 1 \\
 &= 2(1 - \sin^2\theta) = 1 && \because \cos^2\theta = 1 - \sin^2\theta \\
 &= 2 - 2\sin^2\theta = 1 \\
 &= 1 - 2\sin^2\theta = R.H.S
 \end{aligned}$$

$$\begin{aligned}
 Q\text{No}9. \quad R.H.S &= \frac{1 - \tan^2\theta}{1 + \tan^2\theta} \\
 &= \frac{1 - \frac{\sin^2\theta}{\cos^2\theta}}{1 + \frac{\sin^2\theta}{\cos^2\theta}} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} \\
 &= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} = \frac{\cos^2\theta - \sin^2\theta}{1} \\
 &= \cos^2\theta - \sin^2\theta = R.H.S
 \end{aligned}$$

$$\begin{aligned}
 \text{Ques.10} \quad \text{R.H.S.} &= \frac{\cot\theta - 1}{\cot\theta + 1} \\
 &= \frac{\frac{\cos\theta}{\sin\theta} - 1}{\frac{\cos\theta}{\sin\theta} + 1} = \frac{\cos\theta - \sin\theta}{\sin\theta + \cos\theta} \\
 &= \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Ques.11} \quad \text{L.H.S.} &= \frac{\sin\theta}{1 + \cos\theta} + \cancel{\cos\theta} \cot\theta. \quad * \text{correction} \\
 &= \frac{\sin\theta + \cos\theta(1 + \cos\theta)}{1 + \cos\theta} = \frac{\sin\theta + \cos^2\theta}{1 + \cos\theta} \\
 &= \frac{\sin\theta + \cos\theta(1 + \cos\theta)}{(1 + \cos\theta)\sin\theta} \\
 &= \frac{\sin^2\theta + \cos\theta + \cos^2\theta}{(1 + \cos\theta)\sin\theta} \\
 &= \frac{(\sin^2\theta + \cos^2\theta) + \cos\theta}{(1 + \cos\theta)\sin\theta} \\
 &= \frac{1 + \cos\theta}{(1 + \cos\theta)\sin\theta} \quad \because \sin^2\theta + \cos^2\theta = 1 \\
 &= \frac{1}{\sin\theta} = \csc\theta = \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Ques.12} \quad \text{L.H.S.} &= \frac{\cot^2\theta - 1}{1 + \cot^2\theta} \\
 &= \frac{\frac{\cos^2\theta}{\sin^2\theta} - 1}{1 + \frac{\cos^2\theta}{\sin^2\theta}} = \frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta + \cos^2\theta} \\
 &= \frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta + \cos^2\theta} = \frac{\cos^2\theta - \sin^2\theta}{1} \\
 &= \cos^2\theta - \sin^2\theta = \cos^2\theta - (1 - \cos^2\theta) \\
 &= \cos^2\theta - 1 + \cos^2\theta \\
 &= 2\cos^2\theta - 1 = \text{R.H.S. proved}
 \end{aligned}$$

Q No.13 L.H.S =  $(\csc \theta + \cot \theta)^2$

$$= \left( \frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} \right)^2$$

$$= \left( \frac{1 + \cos \theta}{\sin \theta} \right)^2$$

$$= \frac{(1 + \cos \theta)^2}{\sin^2 \theta} = \frac{(1 + \cos \theta)^2}{1 - \cos^2 \theta}$$

$$= \frac{(1 + \cos \theta)(1 + \cos \theta)}{(1 - \cos \theta)(1 + \cos \theta)}$$

$$= \frac{1 + \cos \theta}{1 - \cos \theta} = \text{R.H.S proved}$$

Q No.14 L.H.S =  $(\sec \theta - \tan \theta)^2$

$$= \left( \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \right)^2 = \left( \frac{1 - \sin \theta}{\cos \theta} \right)^2$$

$$= \frac{(1 - \sin \theta)^2}{\cos^2 \theta} = \frac{(1 - \sin \theta)^2}{1 - \sin^2 \theta}$$

$$= \frac{(1 - \sin \theta)(1 - \sin \theta)}{(1 - \sin \theta)(1 + \sin \theta)}$$

$$= \frac{1 - \sin \theta}{1 + \sin \theta} = \text{R.H.S proved}$$

Q No.15

$$\text{L.H.S} = \frac{2 \tan \theta}{1 + \tan^2 \theta} = \frac{2 \tan \theta}{\sec^2 \theta}$$

$$= 2 \tan \theta \cdot \cos^2 \theta$$

$$= 2 \frac{\sin \theta}{\cos \theta} \cdot \cos^2 \theta = 2 \sin \theta \cos \theta = \text{R.H.S}$$

Q No.16

$$\text{L.H.S} = \frac{1 - \sin \theta}{\cos \theta}$$

$$= \frac{1 - \sin \theta}{\cos \theta} \cdot \frac{1 + \sin \theta}{1 + \sin \theta} \quad \text{Rationalizing}$$

$$= \frac{1 - \sin^2 \theta}{\cos \theta (1 + \sin \theta)} = \frac{\cos^2 \theta}{\cos \theta (1 + \sin \theta)}$$

$$= \frac{\cos \theta}{1 + \sin \theta} = \text{R.H.S proved}$$

(QNo.17) L.H.S =  $(\tan \theta \pm \sec \theta)^2$

$$= \left( \frac{\sin \theta}{\cos \theta} \pm \frac{\cos \theta}{\sin \theta} \right)^2$$

$$= \left( \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \right)^2$$

$$= \left( \frac{1}{\cos \theta \sin \theta} \right)^2 \quad \because \sin^2 \theta + \cos^2 \theta = 1$$

$$= (\sec \theta \csc \theta)^2$$

$$= \sec^2 \theta \csc^2 \theta = R.H.S \quad proved$$

(QNo.18) L.H.S =  $\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1}$

**See Alternative Solution at the end.**

$$= \frac{\frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} - 1}{\frac{\sin \theta}{\cos \theta} - \frac{1}{\cos \theta} + 1} = \frac{\frac{\sin \theta + 1 - \cos \theta}{\cos \theta}}{\frac{\sin \theta - 1 + \cos \theta}{\cos \theta}}$$

$$= \frac{\sin \theta + 1 - \cos \theta}{\sin \theta - 1 + \cos \theta}$$

$$= \frac{\sin \theta + 1 - \cos \theta}{\sin \theta - 1 + \cos \theta} \cdot \frac{\sin \theta + 1 + \cos \theta}{\sin \theta + 1 + \cos \theta}$$

$$= \frac{(\sin \theta + 1) - \cos \theta}{(\sin \theta + \cos \theta) - 1} \cdot \frac{(\sin \theta + 1) + \cos \theta}{(\sin \theta + \cos \theta) + 1}$$

$$= \frac{(\sin \theta + 1)^2 - (\cos \theta)^2}{(\sin \theta + \cos \theta)^2 - (1)^2}$$

$$= \frac{\sin^2 \theta + 2 \sin \theta + 1 - \cos^2 \theta}{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1}$$

$$= \frac{\sin^2 \theta + 2 \sin \theta + \sin^2 \theta}{1 + 2 \sin \theta \cos \theta - 1} = \frac{2 \sin^2 \theta + 2 \sin \theta}{2 \sin \theta \cos \theta}$$

$$= \frac{2 \sin^2 \theta}{2 \sin \theta \cos \theta} + \frac{2 \sin \theta}{2 \sin \theta \cos \theta}$$

$$= \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}$$

$$= \tan \theta + \sec \theta = R.H.S \quad proved$$

FSc-I- Ex 9.4 - 6

$$\begin{aligned}
 \text{Qno 19 L.H.S} &= \frac{1}{\csc \theta - \cot \theta} - \frac{1}{\sin \theta} \\
 &= \frac{1}{\csc \theta - \cot \theta} \cdot \frac{\csc \theta + \cot \theta}{\csc \theta + \cot \theta} - \frac{1}{\sin \theta} \\
 &= \frac{\csc \theta + \cot \theta}{\csc^2 \theta - \cot^2 \theta} - \frac{1}{\sin \theta} \quad \therefore 1 + \cot^2 \theta = \csc^2 \theta \\
 &= \frac{\csc \theta + \cot \theta}{1} - \frac{1}{\sin \theta} \quad \therefore 1 = \csc^2 \theta - \cot^2 \theta \\
 &= \frac{\csc \theta + \cot \theta}{\sin \theta} - \frac{1}{\sin \theta} = \frac{1}{\sin \theta} + \frac{\cot \theta}{\sin \theta} - \frac{1}{\sin \theta} \\
 &= \frac{\csc \theta}{\sin \theta} = \cot \theta \quad (\text{i})
 \end{aligned}$$

$$\begin{aligned}
 \text{R.H.S} &= \frac{1}{\sin \theta} - \frac{1}{\csc \theta + \cot \theta} \\
 &= \frac{1}{\sin \theta} - \frac{1}{\csc \theta + \cot \theta} \cdot \frac{\csc \theta - \cot \theta}{\csc \theta - \cot \theta} \\
 &= \frac{1}{\sin \theta} - \frac{\csc \theta - \cot \theta}{\csc^2 \theta - \cot^2 \theta} \\
 &= \frac{1}{\sin \theta} - \frac{(\csc \theta - \cot \theta)}{1} \quad \therefore 1 + \cot^2 \theta = \csc^2 \theta \\
 &= \frac{1}{\sin \theta} - (\csc \theta - \cot \theta) = \frac{1}{\sin \theta} - \csc \theta + \cot \theta \\
 &= \frac{1}{\sin \theta} - \frac{1}{\sin \theta} + \frac{\cot \theta}{\sin \theta} - \frac{\csc \theta}{\sin \theta} = \cot \theta \quad (\text{ii})
 \end{aligned}$$

From (i) & (ii) L.H.S = R.H.S proved

$$\begin{aligned}
 \text{Qno 20 L.H.S} &= \sin^3 \theta - \cos^3 \theta \\
 &= (\sin \theta)^3 - (\cos \theta)^3 \\
 &= (\sin \theta - \cos \theta)(\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta) \\
 &= (\sin \theta - \cos \theta)(1 + \sin \theta \cos \theta) \\
 &= \text{R.H.S}
 \end{aligned}$$

Q No. 21 L.H.S =  $\sin^6\theta - \cos^6\theta$

$$\begin{aligned}
 &= (\sin^2\theta)^3 - (\cos^2\theta)^3 \\
 &= (\sin^2\theta - \cos^2\theta)((\sin^2\theta)^2 + (\cos^2\theta)^2 + \sin^2\theta \cos^2\theta) \\
 &= (\sin^2\theta - \cos^2\theta)((\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta \cos^2\theta \\
 &\quad - \sin^2\theta \cos^2\theta) \\
 &= (\sin^2\theta - \cos^2\theta)((\sin^2\theta + \cos^2\theta)^2 - \sin^2\theta \cos^2\theta) \\
 &= (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta \cos^2\theta) \\
 &= \text{R.H.S proved}
 \end{aligned}$$

Q No. 22 L.H.S =  $\sin^6\theta + \cos^6\theta$

$$\begin{aligned}
 &= (\sin^2\theta)^3 + (\cos^2\theta)^3 \\
 &= (\sin^2\theta + \cos^2\theta)((\sin^2\theta)^2 + (\cos^2\theta)^2 - \sin^2\theta \cos^2\theta) \\
 &= (1)((\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta \cos^2\theta - 3\sin^2\theta \cos^2\theta) \\
 &= ((\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta \cos^2\theta) \\
 &= ((1)^2 - 3\sin^2\theta \cos^2\theta) = 1 - 3\sin^2\theta \cos^2\theta = \text{R.H.S}
 \end{aligned}$$

Q No. 23 L.H.S =  $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$

$$\begin{aligned}
 &= \frac{1-\sin\theta + 1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)} = \frac{2}{1-\sin^2\theta} \\
 &= \frac{2}{\cos^2\theta} = 2\sec^2\theta = \text{R.H.S proved}
 \end{aligned}$$

Q No. 24 L.H.S =  $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} + \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

$$\begin{aligned}
 &= \frac{(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)} \\
 &= \frac{\cos^2\theta + \sin^2\theta + 2\cos^2\theta \sin^2\theta + \cos^2\theta + \sin^2\theta - 2\cos^2\theta \sin^2\theta}{\cos^2\theta - \sin^2\theta} \\
 &= \frac{2\cos^2\theta + 2\sin^2\theta}{\cos^2\theta - \sin^2\theta} = \frac{2(\cos^2\theta + \sin^2\theta)}{(1 - \sin^2\theta) - \sin^2\theta} \\
 &= \frac{2(1)}{1 - \sin^2\theta - \sin^2\theta} = \frac{2}{1 - 2\sin^2\theta} = \text{R.H.S proved}
 \end{aligned}$$

~~Not solved~~

**Question # 18** [by Hafiz Syed Rizwan (FSc, Session: 2007-2009) Punjab College of Science, Lahore.]

$$\text{L.H.S} = \frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1}$$

×ing and ÷ing by  $\tan \theta + \sec \theta$

$$\begin{aligned} &= \frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} \times \frac{\tan \theta + \sec \theta}{\tan \theta + \sec \theta} \\ &= \frac{(\tan \theta + \sec \theta - 1)(\tan \theta + \sec \theta)}{\tan^2 \theta - \tan \theta \sec \theta + \tan \theta + \tan \theta \sec \theta - \sec^2 \theta + \sec \theta} \\ &= \frac{(\tan \theta + \sec \theta - 1)(\tan \theta + \sec \theta)}{\tan^2 \theta - \sec^2 \theta + \tan \theta + \sec \theta} \quad \because 1 + \tan^2 \theta = \sec^2 \theta \\ &= \frac{(\tan \theta + \sec \theta - 1)(\tan \theta + \sec \theta)}{-1 + \tan \theta + \sec \theta} \quad \therefore \tan^2 \theta - \sec^2 \theta = -1 \\ &= \tan \theta + \sec \theta = \text{R.H.S} \end{aligned}$$